A

Practical Instruction

for

Nestler's Duo Multiplic and Duo Log. Slide Rules.

Special Slide Rules
for
Engineering
&
Technical Calculations.

Published by
ALBERT NESTLER A.-G.
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Preface.

Among the requirements of modern professional life, we find an urgent need for time saving devices for computing and calculating. Wonderful apparatuses have been constructed, which make calculations easy, apparatuses which are the acme of the engineering art. The merits of these complicated machines cannot be denied, and what we intend on the following pages is to give a full explanation of the working of our

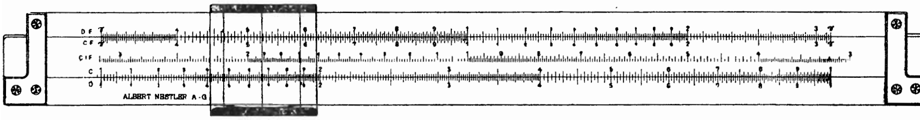
logarithmic slide rules

which are devised especially for technical calculations and which can also be used for checking accounts and for any secretarial work. For the latter calculations we have constructed special types of slide rules, which are available at moderate prices.

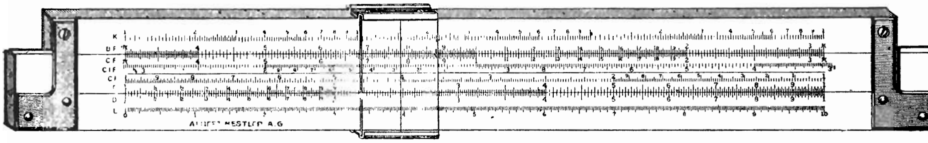
In order to become a good slide-rule calculator, it is sufficient to study thoroughly the following pages and to get acquainted with the different scales and their subdivisions. It is understood, that we cannot give a complete treatise on practical mathematics on these few pages and that our task must be limited to a thorough explanation of all the calculating possibilities our slide rules offer.

Mathematical handbooks and daily professional work will supply examples in profusion and offer to the user of our instruments the possibility to acquire the mastership in calculation, which cannot fail if the present instructions are followed line by line. Thus the student will realize, that it is true what we promise: The slide rule reduces tiresome calculations to a mere reading off of results. Its use is not limited to the drafting room, it can be employed everywhere under any circumstances. It is so inexpensive, that everybody can afford to buy an instrument either of our beginners or ordinary type.

The highest and lasting precision of our slide rules is fully guaranteed.



Duo Multiplic



Duo Log.

First Part.

§ 1. General description of the slide rules.

Both types differ from our normal construction, as the scales appear on both sides of the body and of the slide and that setting of values and reading of results is facilitated by a double face glass runner.

The Duo Log. Slide Rule has the following scales:

A on the body and *B* on the slide, two identical scales traced in two logarithmical units of 12·5 cm. each.

S scale of the sines and *T* scale of the tangents in the middle of the slide; used in conjunction with scales *A* and *B*.

C a logarithmic scale of 25 cm.

LLO a scale of power for the values less than unity running from the right to the left and from 0·05 up to 0·97.

LL1, *LL2* and *LL3* a power scale at the bottom of the rule in three sections and running from the left to the right and from 1·01 to 20000.

K at the upper edge of the rule, a scale of cubes in three logarithmical units.

Two identical scales *DF* and *CF* beginning with the value of $\pi = 3{\cdot}142$.

CIF a reciprocal scale running from the right to the left and beginning with the same value.

CI a scale of the reciprocal values traced in a logarithmic unit of 25 cm.

C and *D* two identical scales traced in a length of 25 cm.

L a scale of mantissae of the logarithms of the Brigg's system.

On the scales *A* and *B* we have the gauge points $\pi = 3{\cdot}142$, further a special mark at 0·785 giving the value of $\frac{\pi}{4}$ for calculations in connection with circles.

On the Duo Multiplic Rule the scales of powers are missing and raising to powers and extracting of roots must be accomplished by means of the *L* scale exlusively.

On this slide rule the other scales are arranged in a slightly different manner, i. e.:

Front: *DF, CF, CIF, C* and *D*.
Back: *K, A, B, S, T, CI, D, L*.

§ 2. Reading the Scales, Multiplication and Division.

Experience proves that the learner's greatest difficulty lies in correctly reading the scales. It is the key-stone of accuracy and we cannot too strongly recommend this all-important point to the beginner and assure him that time spent in scale reading will amply repay him for his trouble.

Briefly speaking, the graduations on the scales *C & D* are as follows: They may be classified into three groups, viz:

Those between 1 & 2
Those between 2 & 4
Those between 4 & 10.

All divisions on the Slide Rule are decimal.

The *Graduations between 1 & 2.*

The primary graduations are numbered:

1 1 2 3 4 5 6 7 8 9 **2**

and represent the numbers:

1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 and 2·0

Each of these divisions is divided into *ten* sub-divisions, representing:

1·0 1·01 1·02 1·03 1·04 1·05 1·06 1·07 1·08 1·09
1·1 1·11 1·12 1·13 1·14 1·15 1·16 1·17 1·18 1·19
1·2 1·21 and so on to 1·98 1·99 2·00

The *Graduations between 2 and 4.*

Only the primary intervals are numbered, viz.:

2 3 4

Each of these numbers has *ten* secondary divisions.

Each secondary division has *five* sub-divisions, which, commencing at 2, are read as follows:

2 2·02 2·04 2·06 2·08 2·1
2·12 2·14 2·16 2·18 2·2 and so on up to
3·9 3·92 3·94 3·96 3·98 and 4.

The *Graduations between 4 and 10.*

The primary divisions 4 5 6 7 8 9 1 only are numbered. These numbers have, each, *ten* secondary divisions.

Each of these secondary divisions has *one* sub-division, which, commencing at 4, are read as follows:

4 4·05 4·1 4·15 4·2 4·25 4·3 4·35

and so on up to 9·9 9·95 and 10.

The Graduations on the inverted (CI) scale are exactly the same, reading from the *Right* to the *Left.*

The Graduations on the Scales *A & B* are as follows:

They may be classified into three groups, viz.:

Those between 1 & 2
Those between 2 & 5
Those between 5 & 10.

Only the primary divisions on these scales are numbered, viz.:

1 2 3 4 5 6 7 8 9 10 etc.

Between 1 and 2 there are *Ten* secondary divisions, thus:

1 1·1 1·2 1·3 1·4 1·5 to 1·9 and 2.

Each of these secondary divisions is divided into *five* tertiary divisions, viz.:

1 1·02 1·04 1·06 1·08 1·1 1·12 1·14 1·16 and so on up to 1·9 1·92 1·94 1·96 1·98 and 2

Between 2 and 5 there are *ten* secondary divisions, viz.:

2 2·1 2·2 2·3 2·4 2·5 2·6 2·7 2·8 and so on up to 4·8 4·9 and 5.

Each of these secondary divisions is divided into *two* tertiary divisions, viz:

2 2·05 2·1 2·15 2·2 2·25 2·3 2·35 and so on up to 4·7 4·75 4·8 4·85 4·9 4·95 and 5.

Between 5 and 10 there are *no* tertiary divisions, as in the previous cases, but only *ten* secondary divisions in all, which are read, thus:

5·1 5·2 5·3 5·4 5·5 5·6 5·7 5·8 and so on up to 9·5 9·6 9·7 9·8 9·9 10.

Scales *CF, DF, CIF* have the same subdivision as scales *C* and *D*. The *L* scale is divided into equal parts. *K* has the same subdivisions as *A & B*.

The Sine and Tangent scales show the following subdivisions: The Sine Scale begins at 0° 34′ 23″ and the first division line gives the value of 35′; subdivision lines here proceed from 1′ to 1′; from 1° to 2° each division line represents 2′ and from 2 to 10° the intervals are 5′; from 10 up to 20° the scale progresses from 10 to 10′ and from 20 to 30 each interval is 20′; between 30° and 40° 30′, between 40° and 70° we have as interval 1°; from 70 to 80—2° and from 80 to 90—5°.

The subdivisions of the Tangent Scale are different. This scale begins at 5° 42′ 38″ and runs to 45°. The intervals are up to 20°, 5 minutes and from 20 to 45°, 10 minutes.

While the ordinary scales of the slide rule give only the series of digits without the number of places or the position of the particular decimal point, the *LL* scales give any value to be set or read within its limits exactly with its number of places and position of the decimal point. Determining of numbers at the right of the decimal point is of course limited to the subdivision which the space on the scales allows and to the eventual interpolation. By taking into consideration the numbers engraved on these scales, the value of each subdivision line will become quite clear to the user and need no further explanation.

Generally speaking, values not represented by a division line on the scales must be interpolated or determined exactly by procedures, which we will explain later on.

Multiplication: Integers & Fractions.

The greatest exactitude is obtained by using the scales *C* and *D*.

Method: Set the initial index (or final index, as required) of the scale *C* to the multiplicand on the scale *D*, and read the answer on the scale *D* under the multiplier read on the scale *C*.

Whole numbers.

Example No. 1. (With Initial Index)

$$175 \times 46 = 8050.$$

Working: Set the initial index of the scale *C* to 175 on the scale *D*, and read the answer on the scale *D* under 46 on the scale *C*.

Example No. 2. (With Final Index)

$$475 \times 24 = 11400.$$

Working: Set the final index of the scale *C* to 475 on the scale *D*, and read the answer on the scale *D* at 24 on the scale *C*.

Fractional Numbers.

Example No. 1. (With Initial Index)

$$1{\cdot}04 \times 1{\cdot}75 = 1{\cdot}82.$$

Working: As in example No. 1, under Whole Numbers, set the initial index of the scale *C* to 1·04 on the scale *D*, and read the answer on the scale *D* at 1·75 on the scale *C*.

Example No. 2. (With Final Index)

$$7{\cdot}5 \times 1{\cdot}64 = 12{\cdot}3.$$

Working: As in example No. 2, under Whole Numbers, set the final index of the scale *C* to 7·5 on the scale *D*, and read the answer on the scale *D* at 1·64 on the scale *C*.

Number of Figures in the Answer.

The number of figures in the answer of a multiplication can be obtained in two ways, viz.:

By Estimation (or Inspection)

By Rule

with ease and accuracy.

Rule: If the answer of a multiplication is read to the *Left* of the setting, it contains as many digits as there are in the *sum* of the digits in the multiplier and the multiplicand.

See example No. 2 under Whole Numbers.

The sum of the digits in the multiplier and the multiplicand is 5 (3 + 2), therefore, the answer will contain 5 figures, thus 11400.

If the answer of a multiplication is read to the *Right* of the setting, it will contain as many digits as there are in the *sum* of the digits in the multiplier and the multiplicand *less* 1.

See example No. 1 under Whole Numbers.

The sum of the digits in the multiplier and the multiplicand is 5 (3 + 2), but as the result is read to the *Right* of the setting we must deduct 1, therefore the answer contains 5 — 1 = 4 digits and is 8050.

In the case of Fractional Numbers the same rule applies; the integral part only may be considered. or the fraction and the integral part may be considered together, and the decimal point fixed by count afterwards.

See example No. 2 under Fractional Numbers

Considering only the integral part, that is to say ignoring the decimal fraction, we have 1 + 1 = 2 digits as the integral part of the answer, thus 12·3. Considered without regard to the decimal point, we have, in this instance, 2 + 3 = 5 figures, and as the slide rule gives the significant figures 1 2 3 we must add two zeros in order to obtain five figures in the answer, thus 12300. Now, count the three decimal places shown in the problem, and get the correct answer 12·3.

See example No. 1 under Fractional Numbers.

The integral part of the problem contains 1 + 1 = 2 figures, which, *less* 1 (as already explained) = 1 digit in the answer, thus 1·82.

Ignoring the decimal point in both numbers we get, in this instance, 3 + 3 figures = 6 figures, and 6 — 1 (as previously explained) gives 5 figures in the result, thus 18200, the significant figures read on the slide rule being 1 8 2. Now, count the 4 decimal places shown in the problem, and get the correct answer 1·82.

Estimation (or Inspection).

In estimating results, we reduce the problem to its simplest expression by factorizing with powers of 10, thus:

See example No. 2 under Whole Numbers.

$475 \times 24 = 4{\cdot}75 \times 10^2 \times 2{\cdot}4 \times 10^1$ which we can express as $4{\cdot}75 \times 2{\cdot}4 \times 10^3$. For practical purposes we can round this up to:

$$4 \times 3 \times 10^3 = 12000,$$

thus it is certain that the answer is very near to

12000,

and, therefore, in reading the significant figures

1 1 4

on the slide rule, it requires two zeros adding to bring it up to a 5 figure number, thus

11400

is the correct answer.

See example No. 1 under Whole Numbers.

$175 \times 46 = 1{\cdot}75 \times 10^2 \times 4{\cdot}6 \times 10^1 = 1{\cdot}75 \times 4{\cdot}6 \times 10^3$. Rounding this up, as shown in the previous example, gives

$$4 \times 2 \times 10^3 = 8000$$

so that the answer is near to 8000,

and as the slide rule gives the significant figures

8 0 5,

we have only to add one zero in order to obtain the correct answer

8050.

As regards the examples No. 1 & 2 given under Fractional Numbers, the magnitude of the answer is self-evident from the problem itself.

The significant figures 1 8 2 indicated on the rule could not in any case be considered as 0·182

or even as 18·2.

In cases where the magnitude of the answer is not evident, it must be borne in mind that the Slide Rule gives only the *significant* figures, and in cases where interpolation is necessary. considerable practice is required to get results correct to more than 4 figures, but the fourth figure can always be obtained by inspection even though the reading on the slide rule may be doubtful to the reader.

Example No. 1.

$$0{\cdot}000221 \times 0{\cdot}017 = 0{\cdot}000003757.$$

There should be *no* possible doubt in reading the answer to such a problem, whatever method be employed in order to fix the decimal point.

Factorized, it may be expressed thus:

$$2{\cdot}21 \times 10^{-4} \times 1{\cdot}7 \times 10^{-2}$$

which equals $2{\cdot}21 \times 1{\cdot}7 \times 10^{-6}$

Rounding this up for practical purposes gives

$$2 \times 2 \times 10^{-6} = 0{\cdot}000004$$

so that the correct answer is somewhere near this figure, and as we read 3 7 5 7 as the significant figures on the slide rule, the answer must be

$$0{\cdot}000003757.$$

On the Rule Method we get minus 3 and minus 1 in the multiplier and the multiplicand respectively, and minus 1 (for reasons already explained) gives 5 cipher places.

The correct reading of the fourth figure as a 7 is absolutely certain from the problem itself, as will be apparent to the reader.

It is not of any great importance, therefore, whether the Rule Method or the Estimating Method is employed in reading correct results, but care must be taken in interpolating. As the user gains confidence and becomes more proficient in using the Rule, he will often be able to read four figure results with the greatest ease, in spite of the fact that it is generally admitted that the slide rule gives exact results to three figures only, considered as sufficient for all practical purposes.

When proficiency in interpolating has been achieved, and four figure results read with accuracy, it will often be possible to obtain further figures by inspection of the problem, even though they may not be readable on the rule.

Example No. 2.

$$0{\cdot}000815 \times 0{\cdot}0175 = 0{\cdot}0000142625.$$

Factorized, in the manner already explained, gives

$$8{\cdot}15 \times 10^{-4} \times 1{\cdot}75 \times 10^{-2}$$

which equals $8{\cdot}15 \times 1{\cdot}75 \times 10^{-6}$.

Rounding this up gives $8 \times 2 \times 10^{-6} = 0{\cdot}000016$.

Sharp reading on the slide rule gives the following significant figures 1 4 2 6, and multiplying the last two figures in the problem (5×5) gives 25 which, placed to the right of the significant figures read on the rule, gives 142625, and as the estimation gave four ciphers before the first significant figure, we get 0·0000142625 as the correct answer.

It is interesting to note that the Rule Method gives the decimal places in the answer even quicker, viz.: $-3 + -1 = -4$ places. The answer is wholly decimal because the problem is in both factors

Example No. 3. $815 \times 17 = 13855.$

Proceed as previously explained.

Read the significant figures on the rule as

1 3 8 5.

But, there are 5 figures in the answer because there are 5 digits in the multiplier and the multiplicand $(3 + 2 = 5)$. Inspection shows that the last figure must be a 5 because the final digits produce 5. Consequently, it is an easy matter to give the answer correct to five figures, viz.: 13855.

Division: Integers & Fractions.

The greatest exactitude is obtained by using the scales *C* and *D*.

Method: Set the divisor read on scale *C* into coincidence with the dividend on scale *D*, and read the quotient on the scale *D* at whichever index is within the rule.

In dividing, the answer can never fall outside the stock of the rule, as sometimes occurs in multiplying with the scales *C* and *D*.

Whole Numbers.

Example No. 1. (Reading to the *Left* of the setting.) 450 ÷ 15 = 30.

Working: Set the number 15 on the scale *C* into coincidence with the number 450 read on the scale *D*, and read the answer on the scale *D* at the initial index of the scale *C*.

Example No. 2. (Reading to the *Right* of the setting.) 736 ÷ 8 = 92.

Working: Set the cursor to 736 on the scale *D*, bring the number 8 of the scale *C* into coincidence, and read the result on the scale *D* at the final index on the scale *C*.

Fractional Numbers.

Example No. 1. (Reading to the *Left* of the setting.) 1·68 ÷ 1·4 = 1·2.

Working: Set the cursor to 1·68 on the scale *D*, bring the number 1·4 on the scale *C* into coincidence, and read the answer 1·2 on the scale *D* at the initial index of the scale *C*.

Example 2. (Reading to the *Right* of the setting.) 0·0136 ÷ 0·016 = 0·85.

Working: Set the cursor to 136 on the scale *D*, bring the number 16 on the scale *C* into coincidence, and read the answer on the scale *D* at the final index of the scale *C*.

Number of Figures in the Answer.

As in multiplication, the number of digits in the answer of a division can be obtained in two ways, viz.: by Estimation, and by Rule.

Rule: If the answer of a division is read to the *Right* of the setting, it will contain as many digits as the difference between the number of digits in the numerator and the denominator.

See example No. 2 under Whole Numbers.

The numerator contains	3 digits
The denominator contains	1 digit
Difference	2 digits.

Therefore, the answer will contain 2 digits which gives 92.

If the answer of a division is read to the *Left* of the setting, it will contain as many digits as the difference between the number of digits in the numerator and the denominator, *plus 1*.

See example No. 1 under Whole Numbers.

The numerator contains	3 digits
The denominator contains	2 digits
Difference	1 digit
add *plus 1* =	1 digit
as explained above, and get	2 digits in the answer which gives 30.

Decimal Fractions are considered as minus quantities according to the number of ciphers following the decimal point.

See example No. 2 under Fractional Numbers.

In the numerator there is	1 cipher
In the denominator there is	1 cipher
Difference	none,

and as the slide rule gives the significant figures 8 5, the answer will be 0·85.

See example No. 1 under Fractional Numbers.

In the case of compound fractions composed of whole numbers and fractions, as in this instance, consider only the Whole Numbers.

In the numerator there is	1 digit
In the denominator there is	1 digit
Difference	none,

but as the quotient is read to the *Left* of the setting 1 digit must be added. The significant figures indicated on the slide are 1 2, and as the answer must contain 1 digit, the result is 1·2

Estimation: Rough estimating in the case of small fractions can be done mentally as the magnitude of the answer is self-evident from the problem.

In the case of numbers where the magnitude of the answer is not apparent, or may be doubtful, it must be factorized, that is to say reduced to its simplest form by powers of 10.

For instance, if we had to deal with a problem such as, say,

$$\frac{0{\cdot}00000285}{0{\cdot}000197} = 0{\cdot}01446,$$

answer is obtained by interpolation, thus requiring care in setting and reading. Factorizing this, brings it to

$$\frac{2{\cdot}85 \times 10^{-6}}{1{\cdot}97 \times 10^{-4}}$$

which may be expressed as

$$\frac{2{\cdot}85}{1{\cdot}97} \times 10^{-2}.$$

We can see at a glance that this is about $\frac{3}{2} = 1{\cdot}5$ which multiplied by 10^{-2} will move the decimal point two places to the left, thus 1·5 becomes 0·015.

The significant figures read on the Slide Rule being 1 4 4 6, the answer is 0·01446.

§ 3. Proportion, Compound Multiplication & Division.

Problems such as $d = \frac{b \cdot c}{a}$

and

$$x = \frac{a \cdot b \cdot c \cdot d \cdot e}{f \cdot g \cdot h \cdot i}$$

are easily solved with the slide rule.

Example No. 1.

$$\frac{0{\cdot}00275 \times 4350}{0{\cdot}0369} = 324.$$

Number of digits in the answer, by Rule, as explained on pages 7—9, is $-2 + 4 - (-1) = +3$.

Example No. 2.

$$\frac{\overset{(1)}{0{\cdot}00376} \times \overset{(9)}{0{\cdot}853} \times \overset{(7)}{11270} \times \overset{(3)}{53{\cdot}2} \times \overset{(5)}{0{\cdot}987}}{\underset{(6)}{0{\cdot}0165} \times \underset{(2)}{0{\cdot}422} \times \underset{(4)}{955000} \times \underset{(8)}{18{\cdot}33}} = 0{\cdot}01556.$$

The working of such problems is best accomplished by the scales *C & D* and the reciprocal scale. Care should be taken in the selection of the order of the multiplications and divisions to avoid transposing the slide.

Example No. 2 is worked with one crossing of the slide. The small figures in brackets indicate the order in which the problem should be worked on the scales *C & D.*

Number of digits in the answer, by Rule as explained on pages 7—9, is

$$\begin{aligned} -2 + 0 + 5 + 2 + 0 &= +5 \\ -1 + 0 + 6 + 2 \quad &= -7 \\ = \text{minus} \quad & \quad\;\; 2 \end{aligned}$$

and minus 2 plus 1 = minus 1 place in the answer, as shown.

§ 4. Squares & Square Roots.

Squares and Square Roots are calculated by means of the cursor in combination with the scales *A* and *D* (and *B* and *C*). The two units of the scale *A* are half the length of the scale *D,* consequently, any number read on the scale *A* is the square of the number with which it coincides under the cursor hair-line read on the scale *D*. It follows, therefore, that the numbers on the scale *D* are the square roots of those read on the scale *A*. (The scales *B* and *C* being in the same relation give identical results.)

Example No. 1. (Squares.)

$$0{\cdot}0204^2 = 0{\cdot}000416 \ (0{\cdot}00041616).$$

Working: Set the cursor hair-line to 204 on the scale *D*, and read the answer in the first log. unit on the scale *A*.

Example No. 2. (Squares.)

$$40{\cdot}8^2 = 1664 \ (1664{\cdot}64).$$

Working. Set the cursor hair-line to 408 on the scale *D*, and read the answer in the second log. unit on the scale *A*.

Number of Figures in the Answer.

The numbers read in the first log. unit of the scale *A* are odd-place numbers, and contain twice the number of places in the original number, *less 1*.

See example No. 1.

This number is a (minus) one-place number. Therefore, $2 \times (-1) = -2$, and minus $2 + (-1) = -3$.

The significant figures read on the slide rule are

4 1 6

which, minus 3 places, gives 0·000416 as the answer. (See note below.)

See example No. 2.

The integral part of this number has 2 digits, consequently it is a two-place number. Numbers read in the second log. unit of the scale *A* are even-place numbers and contain twice the number of places as there are places in the original number. This gives $2 \times (+2) = +4$ in the example in question, therefore the integral part of the answer must contain 4 figures, and these four significant figures 1 6 6 4 can be read on rule. (See note below.)

Note: In the examples No. 1 & 2 given under Squares, the answers read on the scales *A* have been given to the nearest readable figure with the naked eye; to three significant figures in example No. 1, and to four significant figures in example No. 2 the full answer in example No. 1 contains five significant figures as is shown by the answer placed in brackets.

The full answer in example No. 2 contains 6 significant figures, as shown by the answer in brackets.

Generally speaking, the three figures, or four figures, read on the scale of squares are sufficient for all practical purposes, but the trained eye of the expert slide rule user will often "see" the complete answer. For instance, the significant figures of the square of 204 read on the first log. unit of the scale *A* are 4 1 6, but a glance at the figures 204 shows 204×204 must have 5 figures in the full answer. The final digit in the problem is 4 which proves that the final figure in the answer must also be a 6. Therefore, to obtain the complete answer, the square of the final digit $(4) = 16$ must be added to the right of the 3 significant figures read on the rule, to make 5 digits, and considering the decimal point gives 0·00041616 as the full answer.

The square of 40·8 (Example No. 2) must contain six digits, four of which are readable on the second log. unit of the scale *A*. A glance at the figures shows that the sixth digit must be a 4. Adding the square of $8 = 64$ to the right of the significant figures read on the rule, and fixing the decimal point gives the correct, full answer, viz.: 1664·64. This would be taken as 1665 in general practice.

The value of such exercises in reading nearest figure results, and full figure results wherever possible, cannot be denied.

Should any doubt exist in the reader's mind as to the importance of such reading practice in interpolating results, let him convince himself by the following two methods, viz.:

(1) Set the initial index of the scale *C* to 204 on the scale *D*, now move the cursor to 2 on the scale *C*, and thus obtain a sharp setting for the number 408 on the scale *D*, and read the square of the number on the scale *A*.

(2) Leave the cursor in this position, and reverse the slide, bringing the final index into coincidence; move the cursor to 408 on the scale *C,* and read the result on the scale *D.*

Owing to the peculiar nature of logarithmic scales, interpolation becomes more and more a question of proficiency as we proceed to the right of the unit for which reason we have endeavoured to make it clear to the reader by examples easily understood, how such proficiency is best attained. The reader will appreciate that it would serve no useful purpose to give examples which could not be readily understood

Example No. 3. (Square Roots.) (Whole Numbers.)

$$\sqrt[2]{538} = 23{\cdot}195.$$

Working: In order to find out in which log. unit a whole number should be set in order to extract its square root, we divide the number into groups of *two figures* to the *Left* of the decimal point, thus:

5'38'.

We see that this gives two groups, the group to the extreme left being the one which determines the setting. This group has only one figure, consequently, the number is an odd-place one. Odd-place group numbers are set in the *First log. unit* of the scale *A,* and the answer read under the cursor hair-line on the scale *D.*

Example No. 4 (Square Roots.) (Decimal Fractions.)

$$\sqrt[2]{0{\cdot}0000697} = 0{\cdot}008349.$$

Working: In order to find out in which log. unit a decimal fraction number should be set in order to extract its square root, we divide the number into groups of *two figures* to the *Right* of the decimal point, thus

0·00'00'69'7

In the case of numbers wholly decimal the determining group is the *First* group to the *Right* which is *not* a complete cipher group.

We see that the determining group in this instance is *two-place,* consequently, the number must be set in the *second* log. unit of the scale *A*, and the answer read under the cursor hair-line on the scale *D.*

Number of Places in the Answer.

The number of digits in the answer of a square root, in the case of whole numbers, is determined by the number of groups to the *Left* of the decimal point, and by the number of complete cipher groups to the *Right* of the decimal point in the case of numbers less than 1.

See example No. 3.

This example has *two* groups to the *Left* of the decimal point, consequently the answer is two-place. Reading the significant figures

2 3 1 9 5

on the rule then gives the correct answer as

23 · 195.

See example No. 4.

This example has *two* complete cipher groups to the right of the decimal point. Consequently, the answer will contain two ciphers after the decimal point. The significant figures read on the slide rule are

8 3 4 9,

the answer then is, as per the above explanation

0·008349.

§ 5. Cubes & Cube Roots.

CUBES:

Cubes on the Duo Slide Rule are calculated in the following manner:

Set the hair-line of the cursor to the number of which it is desired to find the cube on scale *D,* and read the answer on the cube scale *K* under the same cursor line.

Example No. 1. (Cubes.)

$$1{\cdot}64^3 = 4{\cdot}41.$$

Working: Set the cursor line to 1·64 read on the scale *D*, and read the answer in the first log. unit of the cube scale under the cursor line.

Example No. 2. (Cubes.)

$$0{\cdot}0043^3 = 0{\cdot}0000000795.$$

Working: Set the middle cursor line to 43 read on the scale *D*, and read the answer in the second log. unit of the cube scale, in the manner above described.

Example No. 3. (Cubes.)

$$655^3 = 281\,000\,000.$$

Working: Set the middle cursor line to 655 read on the scale *D*, and read the answer in the third log. unit, as previously explained.

The reader is advised to acquire proficiency in interpolating on the cube scale by checking — either by multiplying the square of the number on the scales *A* and *B*, or by triple multiplication on the scales *C* and *D;* or the latter in combination with the reciprocal scale to minimise slide movement. Don't forget:

Patience + Practice = Proficiency.

Number of Figures in the Answer.

It is obvious from the nature of a logarithmic cube scale on a 10 inch rule that the maximum number of readable significant figures is 3.

The number of figures in the answer of a cube number depends upon the log. unit in which the significant figures indicated on the rule are read.

If they appear in the first log. unit, the result will contain three times the number of places less 2 as there are digits in the problem.

If they appear in the second log. unit the result will contain three times the number of places less 1 as there are digits in the problem.

If the answer appears in the third log. unit it will contain three times the number of places as there are digits in the problem.

See example No. 1. (Cubes.)

The reading of the answer is in the first log. unit. The problem has one digit to the left of the decimal point, therefore $3 \times (+1) = +3$, and $+3 + (-2) = +1$. The significant figures read on the rule are

4 4 1.

The answer then is 4·41 (to the nearest figure).

See example No. 2. (Cubes.)

As the reading of the significant figures on the rule appears in the second log. unit, the answer will contain three times the number of places, minus 1 as there are places in the problem. The problem has minus 2 decimal places, therefore thrice minus $2 = -6$ and $-6 +$ minus $1 = -7$. The answer then is 0·0000000795.

See example No. 3. (Cubes.)

The significant figures in the answer to this problem appear in the third log. unit and are

2 8 1.

The answer being read in the third log. unit will contain three times as many digits as there are digits in the problem. The answer then has $3 \times 3 = 9$ figures and is

281 000 000.

CUBE ROOTS.

Cube roots are calculated in the following manner:

To find the cube root of an integral number, divide it into groups of 3 figures from *Right* to *Left.*

To find the cube root of a wholly decimal number, divide it into groups of 3 figures from the decimal point to the *Right.*

In the case of a whole number the final group to the *Left* will indicate in which log. unit on the cube scale it must be set, i. e., first, second or third, depending upon the number of digits contained in the final *(Left)* group.

In the case of a wholly decimal number, the first group to the *Right* of the decimal point which is *not* a complete cipher group will indicate in which log. unit on the cube scale the number must be set, depending upon the number of digits contained in the said group.

The cube root is read on the scale D under the cursor line.

Example No. 1. (Cube Root.)

$$\sqrt[3]{5832000} = 180.$$

Dividing this number up into groups of 3 figures from the decimal point to the *Left* gives 5'832'000'.

The last group contains one figure, consequently the number 5 832 000 must be set in the *first* log. unit of the cube scale.

The answer will contain as many digits as there are numbers of groups in the number itself. It will be seen that there are 3 groups, therefore the answer will contain 3 figures.

Example No. 2. (Cube Root.)

$$\sqrt[3]{28094464} = 304.$$

Divide the number into groups of 3 figures from *Right* to *Left* in the manner explained above, thus:

28'094'464.

This shows that the number 28 094 464 must be set in the second log. unit of the cube scale. The number being divided into 3 groups, there will be 3 digits in the answer.

Example No. 3. (Cube Root.)

$$\sqrt[3]{340\,068\,392} = 698.$$

Dividing this number into groups of 3 figures from *Right* to *Left* gives

340'068'392

which indicates that the number 340 068 392 must be set in the third log. unit of the cube scale.

The answer is three-place because there are 3 groups of figures in the problem.

Example No. 4. (Cube Root.)

$$\sqrt[3]{0{\cdot}00000144} = 0{\cdot}01128.$$

Dividing this number into groups of 3 figures from the decimal point to the *Right* gives 0·000'001'44.

This shows that the number 144 requires setting in the first log. unit of the cube scale, and that the answer is a minus one-place number by reason of its having one complete cipher group.

Example No. 5. (Cube Root.)

$$\sqrt[3]{0{\cdot}0349} = 0{\cdot}327.$$

Dividing this number into groups, as mentioned above, gives

0·'034'9.

This shows that it requires setting in the second log. unit of the Cube scale. The answer will be no-place because the number itself contains *no* complete cipher group.

Example No. 6. (Cube Root.)

$$\sqrt[3]{0{\cdot}000\,000\,000\,329} = 0{\cdot}000\,691.$$

Dividing this number into groups, as above explained, gives

0·'000'000'000'329.

This shows that the number must be set in the third log. unit on the cube scale. The answer will contain 3 ciphers after the decimal point because there are three complete cipher groups in the problem.

§ 6. The Reciprocal Scale *CI*.

This scale runs from *Right* to *Left* ←, that is to say, in the opposite direction to the ordinary scales, and has red figures.

The advantage of this is that, in addition to giving the reciprocal $\left(\frac{1}{n}\right)$ of any number, it facilitates multiplication in that the answer is always immediately readable at whichever index of the slide is within the rule, and that it eliminates any hesitation in settings by simply bringing any two factors into coincidence.

It is also extremely useful for continued multiplication, or combined multiplication and division in that dividing by the reciprocal of a number is equal to multiplying by the number itself, and inversely, multiplying by a reciprocal is equal to dividing by the number in question.

Example No. 1. (Reciprocal.)

$$736 \times 225 = 165\,600.$$

Example No 2. (Reciprocal.)

$$296 \times 275 = 81\,400.$$

The number of figures in the answer is obtained in the manner already explained on pages 7—10.

Example No 3. (Combined Reciprocal and Ordinary Scales.)

$$344 \times 375 \times 469 = 60\,500\,000.$$

Example No. 4. (Combined Reciprocal and Ordinary Scales.)

$$\frac{386 \times 246}{143} = 664.$$

Workings:

Example No. 1.

Set the cursor to 736 on the scale *D*, and 225 on the reciprocal scale *CI* into coincidence. Read the answer at the initial index of the scale *C* on the scale *D*.

Example No. 2.

Set the cursor to 296 on the scale *D* and 275 on the reciprocal scale into coincidence. Read the answer at the final index of the scale *C* on the scale *D*.

Example No 3.

Set the cursor to 386 on the scale *D*, bring 246 on the reciprocal scale into coincidence, move the cursor to 143 on the reciprocal scale, and read the answer on the scale *D* under the cursor line.

§ 7. Use of the Trigonometrical Scales.

Scale *S* of the sines is used in conjunction with scale *A* or *B* as the case may be and scale *T* in conjunction with scale *C* or *D*. We are giving hereafter a range of examples for setting and reading the trigonometrical values, and practical examples of trigonometrical calculations will follow in the second part of this booklet.

Number of Place of the trigonometrical functions: The values at the left of the middle index of scale *A*, are minus one place (till 0° 30′ minus two places) and those at the right of this index zero place. The Tangent values are zero places.

Examples of Setting.

On Sine Scale	On Scale *A*	On Tangent Scale	On Scale *C* or *D*
1° 10′	0·0204	6° 20′	0·111
2° 30′	0·0436	7° 10′	0·126
3° 20′	0·0581	8° 40′	0·152
3° 50′	0·0668	9° 30′	0·167
12° 10′	0·2108	12° 10′	0·216
14° 20′	0·2476	14° 20′	0·255
38° 40′	0·625	38° 40′	0·800

As the values of little angles vary only imperceptibly for the Sine, Tangent and Arc values, the values of such angles can be expressed for all practical purposes by the measures of the arc as explained page 47.

§ 8. The *L* and the *LL* Scales.

With the Duo Log. slide rule, powers and roots can be determined:

1. by means of scale *L*
2. „ „ „ scales *LL*.

We give first some examples for using the *L* scale. By bringing into coincidence any value on *C* or *D* with any on scale *L*, we can read on the latter scale the corresponding mantissa without its characteristic and inversely.

Examples for setting and reading:

Scale *D*	2·	2·4	4·3	6·	8·	8·6	9·2
Scale *L*	0·301	0·38	0·633	0·778	0·903	0·934	0·964

For the determination of numbers of more places and for those of less than unity, the corresponding mantissae must be added.

Example: $34^{2\cdot8} = 19\,320$. First determine the mantissa of 34 = 1·531 (531 on scale *L*, 1 as characteristic), then set 1·531 on scale *D* with the initial index of the slide, cursor line on 2·8 of scale *C* and read the result on scale *D* = 4·287. To this log. we have to determine the anti-log. As the number at the left of the decimal point is 4, the result must be a 5 place number, as shown in the second part of this booklet, and the series of digits corresponding to the log. 287 will be 1935 and the definitive result 19355. (Values only to be read and interpolated on the slide rule up to 4 places.) This procedure gives more precise results than a direct reading of the log. log. scales.

Example: $0\cdot0489^{5\cdot8} = 0\cdot00000002455$. First determine the mantissa of the basic number thus: log. $0\cdot0489 = \overline{2}\cdot6895 = -2 + 0\cdot6895 = -1\cdot3105$. This value must be multiplied by the exponent 5·8 in the manner already explained getting $-7\cdot61$. This number brought into the required form gives $-8\cdot0 + 0\cdot39 = \overline{8}\cdot39$. Now we have to determine the series of digits corresponding to the mantissa 0·39 reading on scale *D* 2455. The number of places will be one more than the characteristic, therefore minus 7.

Example: $\sqrt[3\cdot2]{724} = 7\cdot82$. First determine the mantissa of 724 reading on *L* scale the value of 8597; adding to this series of digits the characteristic 2 and consequently as logarithm 2·8597. This number must be divided by the root 3·2 in the manner fully explained unter "Multiplication", and the result obtained will be 0·8936. Determining the anti-logarithm of this log. will give the value of 7·82.

Example: $\sqrt[3\cdot34]{0\cdot000000745} = 0\cdot0713$. First determine the log. of the radicand reading on *L* scale 8722, which series of digits give, after having added the characteristic, $\overline{7}\cdot8722 = -6\cdot1278$. Then divide this log. by the root and get as result $-1\cdot835 = -2 + 0\cdot165$ or $\overline{2}\cdot165$. Determining the number of this mantissa gives 1462, and the number of places is 1 more than the characteristic thus -1.

The Scales *LL1, LL2, LL3*.

These scales are traced in continuation and in distances that are based on the decimal mantissae of the natural or hyperbolic logarithms proportional to scale *C*. Using of these scales reduces raising to powers and extraction of roots to the same setting and reading of results as described for multiplication and division. The scales can also be used for determining the natural logarithms to any number. For this operation we set the number, of which the log. *n* is to be determined, on one of the *LL* scales, and the corresponding natural logarithm is read under the hair line of the cursor on scale *C*. As for the number of places of the log. nat. thus determined, we have to note that numbers from 1·01 1·105 are minus one place

„ 1·105 to $e = 2\cdot718$ zero places

„ $e = 2\cdot71828$ to 21900 one place

„ 21 900 upwards two places.

Examples of setting: After having brought the initial indexes of the different scales into coincidence, we set the cursor successively on the different values.

On scale *LL1*	1·015	on scale *C*	0·01489
„ „ *LL1*	1·02	„ „ *C*	0·01979
„ „ *LL1*	1·054	„ „ *C*	0·0526
„ „ *LL2*	1·11	„ „ *C*	0·1043
„ „ *LL2*	1·27	„ „ *C*	0·239
„ „ *LL3*	4·25	„ „ *C*	1·447
„ „ *LL2*	2·3	„ „ *C*	0·833
„ „ *LL3*	7·9	„ „ *C*	2·066
„ „ *LL3*	50	„ „ *C*	3·912

The *LL* scales can also be used for determining the artificial logarithms by setting the initial or final index of scale *C* in coincidence with 10 of the *LL3* scale. Under the cursor line we then have the corresponding values on one of the *LL* scales and on scale *C*. Care must be taken in properly setting the decimal point of the log. and its characteristic.

Examples for setting with the initial index of scale C:

log.	20	45	125	600	1000	10000
	1·301	1·653	2·097	2·778	3	4

Examples for setting with the final index of C:

log.	4	3·3	1·11	1·02	1·2	1·258
	0·602	0·518	0·04532	0·0086	0·0791	0·09968.

Involution and Evolution by Means of the Scales *LL1, LL2, LL3.*

As we have already explained on page 6, the values on the log. log. scales are traced with their decimal points and it is, therefore, necessary to pay attention to this peculiarity of these scales, which distinguishes them from the other logarithmic divisions.

For those who have followed closely our explanation of multiplication and division, the proceeding for involution and evolution described in the following lines will be quite clear.

If we have to determine the power to any base, we set the initial index of scale *C* to the base on one of the scales *LL* and read the result under the cursor line set on the exponent on scale *C* on the same scale on which we have determined the base. Now the attentive reader will have noticed that sometimes the result would fall outside of Scale *LL,* and in such cases we must set the final index, but then the result appears on the next scale above. On the *LL3* scale, setting with the final index is not possible, and involutions which would give results beyond the range of our *LL3* scale must be treated otherwise, i. e. by means of the *L* scale or by factorizing, as will be explained later on.

An analogous proceeding is to be observed for evolutions. Generally speaking, the root, read on scale *C,* must be brought for evolution into coincidence with the number of which the root is to be extracted, and results appearing at the initial line of *C* are read on the same scale. On the contrary, if setting of the two corresponding values shows that the result must be read at the end line, it will appear on the scale thereunder, i. e. for values set on scale *LL3* on scale *LL2* and for those set on *LL2* on scale *LL1.*

For extracting of roots of values read on scale *LL1* we can only make use of the initial line of *C,* and results falling outside of the scale must be obtained otherwise, as we shall explain on pages 19 and 20.

The Scale *LL0.*

This scale covers the range of 0·05 to 0·97 and is in coincidence with scales *A & B* so that $e^{-\cdot 1}$ coincides with the right and left index of *A* and *B* and e^{-1} with the central index of these scales. It runs from the right to the left. Thus the hyperbolic co-logarithms of the numbers read on scale *LL0* can be read directly on scale *A* or *B* if the latter is in alignment with scale *A.*

Examples of settings.

Scale *LL 0*	0·82	0·755	0·653	0·09	0·97	0·958
Co-log. on Scale *A*	0·197	0·281	0·425	0·0242	0·305	0·428

Examples for Using the Scales *LL 1, 2 & 3.*

Involution. Setting with the initial index:

$1{\cdot}02^{3{\cdot}04}$	= 1·062.	Initial line of *C* to 1·02 of scale *LL 1*, result at 3·04 of *C* on *LL 1*
$1{\cdot}0545^{1{\cdot}76}$	= 1·098	„ „ „ *C* „ 1·0545 „ „ *LL 1*, „ „ 1·76 „ *C* „ *LL 1*
$1{\cdot}23^{3{\cdot}4}$	= 2·02	„ „ „ *C* „ 1·23 „ „ *LL 2*, „ „ 3·4 „ *C* „ *LL 2*
$1{\cdot}435^{2{\cdot}52}$	= 2·485	„ „ „ *C* „ 1·435 „ „ *LL 2*, „ „ 2·52 „ *C* „ *LL 2*
$7{\cdot}2^{1{\cdot}84}$	= 37·8	„ „ „ *C* „ 7·2 „ „ *LL 3*, „ „ 1·84 „ *C* „ *LL 3*
$5{\cdot}2^{2{\cdot}04}$	= 28·9	„ „ „ *C* „ 5·2 „ „ *LL 3*, „ „ 2·04 „ *C* „ *LL 3*

Setting with the final index:

$1{\cdot}09^{3{\cdot}8}$	= 1·387.	Final index of *C* to 1·09 on scale *LL 1*, result at 3·8 of *C* on *LL 2*
$1{\cdot}025^{7{\cdot}3}$	= 1·1975	„ „ „ *C* „ 1·025 „ „ *LL 1*, „ „ 7·3 „ *C* „ *LL 2*
$1{\cdot}0154^{6{\cdot}75}$	= 1·1087	„ „ „ *C* „ 1·0154 „ „ *LL 1*, „ „ 6.75 „ *C* „ *LL 2*
$2{\cdot}14^{6{\cdot}5}$	= 140	„ „ „ *C* „ 2·14 „ „ *LL 2*, „ „ 6·5 „ *C* „ *LL 3*
$1{\cdot}645^{5{\cdot}25}$	= 13·62	„ „ „ *C* „ 1·645 „ „ *LL 2*, „ „ 5·25 „ *C* „ *LL 3*
$1{\cdot}24^{9{\cdot}55}$	= 7·79	„ „ „ *C* „ 1·24 „ „ *LL 2*, „ „ 7·79 „ *C* „ *LL 3*

Evolution:

$\sqrt[1{\cdot}02]{1{\cdot}03}$	= 1·0294,	1·02 of *C* to 1·03 of *LL 1*, result at initial index of *C* on *LL 1*
$\sqrt[1.75]{1{\cdot}055}$	= 1·031	1·75 „ *C* „ 1·055 „ *LL 1*, „ „ „ „ „ *C* „ *LL 1*
$\sqrt[3]{1{\cdot}071}$	= 1·02305	3 „ *C* „ 1·071 „ *LL 1*, „ „ „ „ „ *C* „ *LL 1*
$\sqrt[1{\cdot}3]{1{\cdot}25}$	= 1·187	1·3 „ *C* „ 1·25 „ *LL 2*, „ „ „ „ „ *C* „ *LL 2*
$\sqrt[7]{1{\cdot}45}$	= 1·0545	7 „ *C* „ 1·45 „ *LL 2*, „ „ final „ „ *C* „ *LL 1*
$\sqrt[1{\cdot}9]{1{\cdot}13}$	= 1·0665	1·9 „ *C* „ 1·13 „ *LL 2*, „ „ „ „ „ *C* „ *LL 1*
$\sqrt[2]{30}$	= 5·477	2 „ *C* „ 30 „ *LL 3*, „ „ initial „ „ *C* „ *LL 3*
$\sqrt[6{\cdot}4]{2000}$	= 3·28	6·4 „ *C* „ 2000 „ *LL 3*, „ „ „ „ „ *C* „ *LL 3*
$\sqrt[8{\cdot}1]{250}$	= 1·977	8·1 „ *C* „ 250 „ *LL 2*, „ „ final „ „ *C* „ *LL 2*

For negative exponents we have to bear in mind, that $a^{-n} = \frac{1}{a^n}$ and $\sqrt[-n]{a} = \sqrt[n]{a}^{\,1}$ ... i.e. $\sqrt[-n]{a} = \frac{1}{\sqrt[n]{a}}$. Solving of these problems need, therefore, no further explanation. Therefore, for negative exponents we first determine the power in the usual manner and then form the reciprocal value of the number thus obtained.

Examples: $3{\cdot}4^{-4{\cdot}2} = 0{\cdot}00581$. Initial index of *C* on 3·4 of *LL 3* at 4·2 of *C* we read on *LL 3* 172. Then set the hair line of the cursor at 172 on scale *C* and read the reciprocal of this value on scale *CI*. For such cases, care must be taken to determine correctly the position of the decimal point.

$\sqrt[-4{\cdot}6]{256} = 0{\cdot}300$. Bring 4·6 on scale *C* into coincidence with 256 on scale *LL 3* and read on this same scale at the initial index of *C* 3·33. Then determine the reciprocal value, setting cursor line at 3·33 on *D* or *C* and read the result on scale *CI*.

Examples for Using the Scale *LL 0*.

Only results falling within the limits of 0·05 to 0·97 can be read directly.

Involution:

$0{\cdot}75^{2{\cdot}4} = 0{\cdot}502$,	initial index	of *B*	to	0·75	of *LL 0*,	result at	2·4	of *B*	on	*LL 0*
$0{\cdot}46^{3{\cdot}1} = 0{\cdot}09006$	„	„ *B*	„	0·46	„ *LL 0*,	„ „	3·1	„ *B*	„	*LL 0*
$0{\cdot}75^{2{\cdot}45} = 0{\cdot}4942$	„	„ *B*	„	0·75	„ *LL 0*,	„ „	2·45	„ *B*	„	*LL 0*
$0{\cdot}95^{3{\cdot}6} = 0{\cdot}832$	final	„ *B*	„	0·95	„ *LL 0*,	„ „	3·6	„ *B*	„	*LL 0*
$0{\cdot}97^{4{\cdot}15} = 0{\cdot}881$	„	„ *B*	„	0·97	„ *LL 0*,	„ „	4·15	„ *B*	„	*LL 0*

The results of bases set in the right section from *e* to 0·97 are read on the continuation at the left, after having set the final index of scale *B*, if results cannot be read at the right.

Evolution:

$\sqrt[5]{0{\cdot}20} = 0{\cdot}7248$. Setting: 5 of *B* into coincidence with 0·20 on *LL 0*, result at initial index of *B* on *LL 0*.

$\sqrt[5]{0{\cdot}75} = 0{\cdot}944$. Setting: 5 of *B* into coincidence with 0·75 on *LL 0*, result at final index of *B* on *LL 0*.

$\sqrt[2{\cdot}1]{0{\cdot}62} = 0{\cdot}796$. Setting: 2·1 of *B* into coincidence with 0·62 on *LL 0*, result at initial index of *B* on *LL 0*.

$\sqrt[3{\cdot}4]{0{\cdot}55} = 0{\cdot}8387$. Setting: 3·4 of *B* into coincidence with 0·55 on scale *LL 0*, result at initial index of *B* on *LL 0*.

$\sqrt[3]{0{\cdot}15} = 0{\cdot}528$. Setting: 3 of *B* into coincidence with 0·15 on scale *LL 0*, result at initial index of *B* on *LL 0*.

It follows from the definition of *LL* scales that the values of scale *LL2* are the 10th power of those traced on *LL1*; the same ratio exists between scales *LL2* and *LL3*. Therefore, it will be clear that, when raising of powers with decimal fractions, we must read results not on the scale where same should appear when having an integral number as exponent, but on the scale below.

Example:

$3^{0{\cdot}13} = 1{\cdot}158$. Initial index of *C* on 3 of *LL 3*, result at 13 on *LL 2*

$15^{0{\cdot}02} = 1{\cdot}0556$ „ „ „ *C* „ 15 „ *LL 3*, „ „ 2 „ *LL 1* as there are two decimals in the exponent.

For evolution the inverse procedure must be followed.

Example:

$\sqrt[0{\cdot}3]{1{\cdot}7} = 5{\cdot}845$. 3 on *C* into coincidence with 1·7 on *LL 2*, result at the initial index of *C* on *LL 3*

$\sqrt[0{\cdot}4]{1{\cdot}4} = 2{\cdot}318$. 4 on *C* into coincidence with 1·4 on *LL 2*, result at the initial index of *C* on *LL 2*. ($\sqrt[4]{1{\cdot}4}$ would be read on *LL 1*) as explained page 17.

General Remarks.

In cases where the result of any power sought falls beyond the limit of the scale, i. e. 20000, we proceed as follows:

We divide the number in two factors of which the powers fall within the scale. The two powers thus obtained are multiplied one with another, and this operation gives the result sought.

Examples.

$8^5 = 32768 = 2^5 \cdot 4^5 = 32 \cdot 1024 = 32768$

$52^6 = 19770000000 = 5{\cdot}2^6 \cdot 10^6 = 19777 \cdot 10^6 = 19777000000.$

It will also be clear that the root of any number of greater magnitude than 20 000 cannot be read directly on the log. log. scales. Nevertheless, it is also possible to solve easily such problems in the following way: We divide the number by such a figure, which brings the quotient within the limits of the scale. Then extract the root of the quotient and of the divisor and multiply the root of the quotient by that of the divisor, thus getting the result.

Examples:

$\sqrt[4]{43620} = 14{\cdot}43 \cdot 43620 : 10^3 = 43{\cdot}620 \cdot \sqrt[4]{43{\cdot}620} \cdot \sqrt[4]{10^3} = 2{\cdot}566 \cdot 5{\cdot}624 = 14{\cdot}43$

$\sqrt[2\cdot4]{107000} = 122{\cdot}5 \cdot 107000 : 10^3 = 107 \cdot \sqrt[2\cdot4]{107} \cdot \sqrt[2\cdot4]{1000} = 7{\cdot}0 \cdot 17{\cdot}5 = 122{\cdot}5.$

As for the setting, we may refer to the examples previously given. We have to bear in mind for the use of scale *C* and scale *B* in conjunction with the log. log. scales, that for such combinations the numbers traced on scales *C* and *B* need not to be considered as values with any number of places to which the decimal point is to be ascribed as required by the corresponding problem, but the numbers must be given their exact value and the subdivision thus represents the corresponding decimals. Thus, number 2 in this case means exactly this value and never 20 or 200 as we may consider it when using the scales for multiplication.

Second Part.

After having given a description of the slide rule and a clear explanation how to use and to read the different scales, we shall give in continuation a range of advanced examples and a brief explanation of the theory.

§ 9. The Theory of Logarithms briefly explained.

The reader who has gone carefully through the preceding pages and had some experience, certainly will be in a position to make use of the slide rule for all current operations, but he should also get acquainted with the underlying principle and not only rely on practice. Otherwise the want of knowledge of the theory may engender an uncertainty and lack of self-confidence, which may become an obstacle to proficiency and mastering not only difficult but also simple problems.

The logarithm of the number to any base is the index of the power to which the base must be raised to equal the given number. Here we are principally concerned with the common or logarithms of Briggs, for which the base is 10. The general explanation given above, therefore, must be modified in the following sense: The common logarithm of a number is the index of the power to which 10 must be raised in order to get the given number. Thus we write for 100 log. 2, seeing that 10 must be squared in order to get 100. For 1000 we write log. 3, as the base 10 must be raised to the cube in order to get 1000. Thus we have the following table:

10000	1000	100	10	1
4	3	2	1	0

This table shows that logarithms of 1—10 are wholly decimal, from 10 to 100, 1 is followed by the decimals, from 100—1000, 2 is followed by the decimals, from 1000—10000, 3 is followed by the decimals and so on. The exact values of the decimals are found in the tables of logarithms, and to these decimals, called mantissae, must be added the characteristic thus:

Numbers	3	30	300	3000	30000
Logarithms	0·477	1·477	2·477	3·477	4·477

If we extend this rule to the numbers less than 1 we get:

Numbers	1	0·1	0·01	0·001	0·0001
Logarithms	0	—1	—2	—3	—4

In continuation of the above given table we have thus:

Numbers	3	0·3	0·03	0·003	0·0003
Logarithms	0·477	$\bar{1}$·477	$\bar{2}$·477	$\bar{3}$·477	$\bar{4}$·477

After having read these brief explanations, the reader will understand the underlying principle of the logarithmic scales. Adding or subtracting two exponents, as we do in the case of logarithmic scales, always will give the result of a multiplication or a division. And as the values traced on the logarithmic scales are not the respective numbers, but their mantissae, the proceeding will be quite clear in principle to the reader.

Advanced Examples.

After having learned how to use the slide rule, the reader may study also the possibilities to extend the use of the slide rule to more complicated examples and to get familiar with the methods of obtaining a higher degree of precision for slide rule operations than the instrument seems to offer at first sight.

Calculating all the following examples and adding similar ones from his own professional or scientific experience will give the true mastership in calculation and procure a degree of efficiency in calculating work, impossible to attain with any other calculating device.

§ 10. Multiplication.

Example No. 1. Determine the surface of a parcel of land measuring 27·5 × × 39·2 yards. Setting of this problem, as shown in the first part, will require using of the final index; and for such examples where it is doubtful if the result will fall beyond the scales, when set with the initial index, we recommend to make use of the scales *DF* and *CF*, where the result never can fall beyond the scales, as the index is in the centre and coincides with $\sqrt{10}$ of scales *C* and *D*. Thus, after setting of the middle index 1 of scale *CF* to 27·5 of scale *DF*, we read the result 1078 square yards in coincidence with 39·2 of *CF* on *DF*. When calculating yards and feet or feet and inches, the values must be multiplied separately, or they must be converted into decimals of the next higher unit. For the calulator's convenience we have given the measures in this example and in the following ones in yards and decimals.

Example No. 2. Determine the total surface of the following floors measuring respectively: 4·2 × 3·75; 4·2 × 6·15; 6·15 × 3·85; 3·75 × 3·85. In order to avoid any possible error in setting of the decimal point, we first make an estimate of the result, which shows to be some 80 square yards. Then to reduce the settings required for the solution of the different problems, the reader should bear in mind that those containing the same factor can be solved with the same setting. This problem, therefore, requires only two movements of the slide, as we set first the index 1 of scale *C* to 4·2 of *D* reading successively the results of the multiplication of this factor with 3·75 and 6·15 as previously explained. For the two other problems the index is set to 3·85 in the manner already explained, and the four results are added, giving as exact result 79·7 sq. yards.

Example No. 3. Determine the total length of a travel executed with the following means of communication:

Branch line	with	28	km. per hour, time	43 minutes
Fast train	„	85	„ „ „ , „	1 hour 12 minutes
Autobus	„	35	„ „ „ , „	13 minutes
Aeroplane	„	130	„ „ „ , „	52 „

First we have to convert the minutes into decimals of hours, multiplying by $\frac{1}{60} = 0{\cdot}01667$.

Thus we shall get the following table:

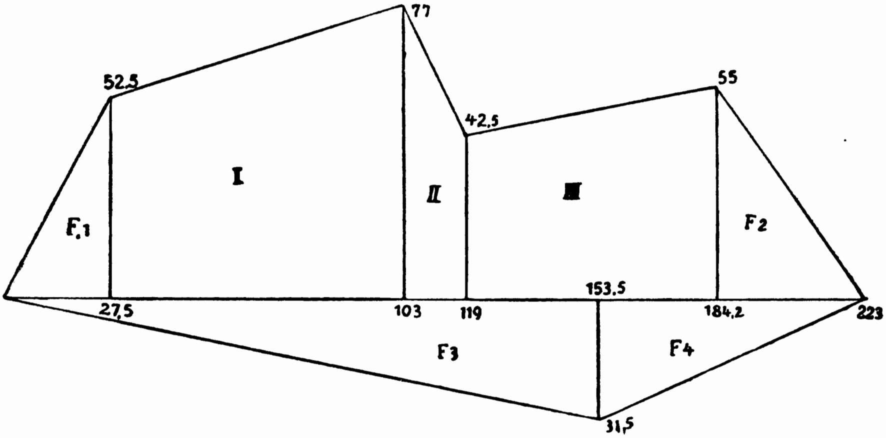
Means of communication	Velocity	Time	Length of Way
Branch line	28	0·717	20·1
Express	85	1·2	102·—
Autobus	35	0·217	7·6·
Aeroplane	130	0·867	112·7
			Total 242·4 km.

Rough estimating would give us the following values:

$$30 \times 0{\cdot}7 + 90 \times 1 + 35 \times 0{\cdot}2 + 130 \times 0{\cdot}9 = 21 + 90 + 7 + 117 = 235 \text{ km.}$$

Example No. 4. A parcel of land of the form and dimension shown by the diagram is to be measured exactly. For this purpose, the surface must be divided into three triangles and three trapezoids. The area of the triangle is found multiplying the base with the height and dividing the result by 2. We, therefore, have to multiply the following values: $F_1 = 1/2 \times 27{\cdot}5 \times 52{\cdot}5$; $F_2 = 1/2 \times (223-184{\cdot}2) \times 55$; $F_3 = 1/2 \times 153{\cdot}5 \times 31{\cdot}5$; $F_4 = 1/2 \times (223-153{\cdot}5) \times 31{\cdot}5$, thus $F_1 = 13{\cdot}75 \times 52{\cdot}5 = 722$ m.², $F_2 = 1/2 \times 38{\cdot}8 \times 55 = 19{\cdot}4 \times 55 = 1067$ m.²; $F_3 = 76{\cdot}75 \times 31{\cdot}5 = 2418$ m.²; $F_4 = 1/2 \times 69{\cdot}5 \times 31{\cdot}5 = 34{\cdot}75 \times 31{\cdot}5 = 1095$ m.².

fig. 1.



Thus, the sum of areas of the triangles will be 5302 m.².

The area of the trapezoids is determined by multiplying the half of the parallel sides with the base. For the present example we get the following table:

Trapezoid	Half of Parallels	Base	Area
" I	1/2 · (52·5 + 77) = 61·75	103—27·5 = 75·5	4890
" II	1/2 · (77 + 42·5) = 59·75	119—103 = 16	956
" III	1/2 · (42·5 + 55) = 48·75	184·2—119 = 65·2	3180

Total area of the trapezoids: 9026 m.².

The total area, therefore, will be 5302 + 9026 m.².

Setting of the different values will be quite clear, and we recommend to solve the problem successively by means of the scales *DF* and *CF* and *DF* and *CIF*.

Example No. 5. Determine the percentage and the total quantity of alcohol in a composition of the following quantities of spirit:

9·7 kilos of 42%; 11·5 kilos of 73%; 2·4 kilos of 65%; 21 kilos of 83%; 13·24 kilos auf 38%; and 5·25 kilos of 96 %.

The total quantity evidently will be 9·7 + 11·5 + 2·4 + 21 + 13·24 + 5·25 = = 63·09 kg.

The total amount of pure alcohol must be calculated multiplying the different weights by the percentages, i. e. 9·7 × 0·42 etc., and thus we get the following amounts: 4·075; 8·4; 1·56; 17·42; 5·02; 5·04; the total weight of pure alcohol contained in the composition thus being 41·515 kilos. Considering that 63·05 kilos of the composition contain 41·515 kilos of alcohol, the percentage will be 65·84. Result obtained by the division 41·515: 63·05.

Example No. 6. Determine if the mixture of 0·350 m.³ of water of 15° with 0·24 m.³ of water of 72° gives the right temperature for a bath. The first quantity gives 350 × 15 calories, the second 240 · 72 calories, or a total of 22530 calories. The division of 22530 : 590 gives 38·2 calories or 38·2° of the Celsius thermometer. The water therefore is too hot. Setting of these easy problems needs no further explanation, if the reader has studied carefully the explanations given in the first part of this instruction.

As we have already shown, the slide rule can also be used for determining tables of values with one setting of the slide, if one factor is constant. This is the case, for instance, when we have to derive the circumference of the circle from the diameter. Setting the middle index of scale *CF* to the initial or final gauge point π on scale *DF* gives the circumferences to any diameter. Of course, we must convert the measures with their fractions in twelfth into decimals. Thus, we can read for the diameter of 2″ the circumference of 6·284 of 3″ of 9·43 (setting to the gauge point π to the left) of 4″ 12·56 of 5″ 15·71 (setting to the gauge point π to the right). Similar tables can also be established by means of the scales *A, B* and *C, D* for any conversion of measures, weights, moneys, workmen's salaries, calculations of cost and similar problems, where one or more factors are constant. We have only to set the initial, middle or end index to the constant factor and then to read the different results without any further movement of the slide, only using the cursor for bringing the different values into close coincidence.

Example No. 7. If we have to convert any amount of $ into Marks, we set initial or final index of scale *C* to 4·20 on scale *D*. On scale *D* we then have the amounts in Marks and on scale *C* those in $. Thus:

on scale *D* 137·6; 21·71; 3·57; 768·60
„ „ *C* 32·80; 5·17; 0·85; 183·00.

Example No. 8. 1 HP. equals 0·736 KW. Converting of these values gives the following table with setting of the end index of *C* to 736 to *D*.

Scale *C* HP.	2·5	816	92·8;	0·672
„ *D* KW.	1·84	600	68·3;	0·495.

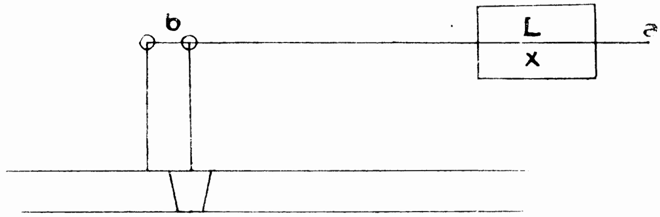
Example No 9. A boiler has a valve of $q = 1{\cdot}5$ cm.² opening. The total length of the lever bar a is 52·5 cm., its weight 180 gr. The turning point has the distance of 4·5 cm. from the valve, the sliding weight has 1·8 kilos. Determine its position, if the boiler shall have a pressure of 1, 2, 3, 4, 5 atmospheres respectively. The pressure p in the boiler acts on the valve with a force $q \times p = 1{\cdot}5\,p$ kilos. The static momentum of this force, therefore, is 0·0675 mkg. (0,045 × 0·015 × 0·015 × p). To this counteracts the static momentum of the lever (0·2625 × 0·18 = 0·04725) and the static momentum of the slide weight $x \times L$, being the distance from the turning point. Thus:

$$1{\cdot}8 \cdot x + 0{\cdot}04725 = 0{\cdot}0675\, p$$
$$x = 0{\cdot}0375\, p - 0{\cdot}02625$$

from which we derive the following table:

p	1	2	3	4	5 atm.
x	1.1	4.9	8.6	12.4	16.1 cm.

fig. 2.



Example No. 10. Determine the forces produced by the free fall of a stone of 5·35 kilos from different heights, i. e. 1 m.; 19·4 m.; 92 m.; 300 m.; 1350 m. The results for such problems are obtained by multiplying the weight with the height of the fall, thus getting 5·35; 103·8; 492; 1605; 7220 kilos respectively.

Example No. 11. For transforming of forces into heat we have for the metric system the formula 1 mkg. $= \frac{1}{427} = 0{\cdot}00234$ kg. cal.

Determine the amounts resulting from the conversion of the values of the previous examples.

Results must be 0·01253; 0·243; 1·153; 3·76; 16·92.

The slide rule shows its superiority especially for the solution of problems of the following form: Suppose an importer has to calculate some hundreds of selling prices all with the same percentages of profit and cost. The execution of a long series of such operations would mean a tiresome work, even when using a calculating machine, as each problem, according to the following example, is composed of multiplications and additions. With the slide rule it is quite sufficient to calculate one example only in order to find the ratio which then remains constant for all the other prices to determine, provided that the different percentages do not vary. Invariable amounts, which cannot be brought into the ratio, must be added to the results after reading of the same on the slide rule.

Example No. 12.	Price of the article free continental seaport	$	5·85
	Customs duty 33 1/3 %	„	1·95
	Freight 2 %	„	0·10
	Other expenses 8 %	„	0·47
	Thus cost landed	$	8·37
	Profit 25 % of $ 8·37	„	2·10
	Selling price	$	10·47

Thus, we get the final ratio of 5·85 : 10·47, and bringing into coincidence these two values on scales *CF* and *DF* allows of immediate reading of any selling price corresponding to any cost price based on the above given example.

Thus:	Scale *CF*	10·47	13·—	22·—	26·—	7·05	8·95
	Scale *DF*	5·85	7·25	12·30	14·50	3·95	5·—

The important saving of time will be evident.

Repeated Multiplication. For these operations we preferably use the scales *CI* and *D* or *DF* and *CIF*, as these combinations require only one setting of the slide for two consecutive multiplications.

Example No. 13. Determine the cubic measure of a room having $15\,^1/_4{}'$ of length; $14\,^3/_4{}'$ of width and $11\,^1/_2{}'$ of height. For direct multiplication with the slide rule, we must write units with their fractions converted into decimals, thus $15{\cdot}25 \times 14{\cdot}75 \times 11{\cdot}50$. Using scales *CIF* and *DF* we proceed as follows: 14·75 on scale *CIF* is brought into coincidence with 15·25 on scale *DF*. Then the result would appear opposite the middle index of *CIF* on *DF*. Without reading same we set cursor on 11·5 of scale *CF*, $2586{\cdot}8\,^1/_4{}'$ will be the result.

Example No. 14. Determine the area of an ellipse having the following measures: Length 3·7; width 1·9 m. The formula for calculating the area of an ellipse being $a \times b \times \pi$ we have in this example $2\,a = 3{\cdot}7$, $a = 1{\cdot}85$, $2\,b = 1{\cdot}90$, $b = 0{\cdot}95$. Thus the area will be:

$1{\cdot}85 \times 0{\cdot}95 \times 3{\cdot}14 = 5{\cdot}52$. For this example, we bring 3·14 of scale *CI* into coincidence with 185 of scale *D*, set cursor to 95 of *C* and read the result under the cursor line on *D*.

Example No. 15. $35^4 = 1\,500\,625$ (to calculate without making use of the mantissa or *LL* scales, only using the *A* and *B* scales). Set initial index of *B* to 35 of *A*, then cursor line to 35 of *B;* this gives the square on scale *A* = = 1225. Now the cursor is set to this same number on scale *B*, thus obtaining the third power = 42 875. Finally a further move of the cursor to 42 875 on scale *B* gives the result 1 500 635. This problem, therefore, could be executed with one setting of the slide, only moving the cursor.

Example No. 16. $423 \times 18 \times 23 \times 14 = 2\,451\,708$. The slide rule gives the digits only up to three, and for the number of places in the result we have reference to the explanation given on pages 7—10. For this example we set 18 of scale *CI* into coincidence with 423 of scale *D*, the cursor line to 23 of scale *C* and finally initial index of *C* under cursor line in this position. Then the result will appear opposite 14 of *C* on *D*. Thus, multiplication of 4 factors can be executed with two settings of the slide.

Example No. 17. A pulley of 325 mm. of diameter makes 1450 rotations per minute. Determine the length of way any point of the circumference is making per second. For one rotation per second the way should be $d \times \pi =$ $= 0{\cdot}325 \times 3{\cdot}142$. As we have to calculate with $\frac{1450}{60} = 24{\cdot}2$ rotations per second the velocity will be $24{\cdot}2 \times 0{\cdot}325 \times 3{\cdot}142 = 24{\cdot}7$ m./sec. The setting is the same as for the preceding example, combining the scales *CI* and *C* with *D*.

§ 11. Increasing the Degree of Precision for Multiplications.

Although the precision, which the slide rule gives, is sufficient for almost any case of technical and secretarial work, it is sometimes desirable to determine results with more places than the slide rule allows. For such cases we indicate herewith various proceedings, which will show that the slide rule is not inferior in efficiency to costly machines in the hand of an expert calculator.

Example No. 18. Calculate the exact amount which 92 pieces at $ 1.37 each will cost. Setting 1.37 × (100—8) gives 137—8 × 1.37 = 137—10.96 = 126.04. The slide rule gives the result with three places, the fourth is found by checking, considering the two end digits of the factors 7 × 8 = 56. The subtraction should not cause any difficulty, as results of similar operations can be determined by checking.

Example No. 19. 12 hours at $ 1.82 each amount to how much? We calculate first 12 × 2 = 24 deducting then 12 × 18 =

```
  2.16 getting as result
 21.84.
```

§ 12. Abridged Multiplication, Splitting up in Groups.

Example No. 20. Determine the natural logarithm of log. 3 = 0.4771 In the abridged form the example looks as follows:

```
 0.4771
 2.3026
-----------
 9542|
 1431|3
    9|542
    2|8626
-----------
1·0985 7046 or for practical purposes 1.0986.
```

This proceeding offers the advantage that only the first row of digits must be calculated in the usual way, and that for the other the slide rule may be used.

Example No. 21. We have to derive from the equatorial diameter of the earth (12755 km.) the equatorial circumference. Consequently, calculate 12755 × × 3.14159. 3 × 12755 = 38265 is calculated in the usual way 0.1416 × 12760 = 1806 with the slide rule, as explained above, gives the result of 40071.

These examples lead us to another procedure for determining exactly the results of multiplications of numbers with many digits.

Example No. 22. 2137 × 29 = 61973. The multiplicand is split up into groups of 2, thus 21'37 × 29, each group is multiplied separately, and the results are added in the usual way

```
2100 × 29 = 60900
  37 × 29 =  1073
            ------
             61973.
```

As the fourth digit always can be determined exactly by checking, the partial results can be determined exactly with the slide rule.

Example No. 23. 31325 × 4126 = 129246950. Splitting up both factors gives 3'13'25 × 41'26, and the partial multiplication can be executed with two settings of the slide only. We have only to take care to determine the fourth digit by checking and to set the results properly, as shown in the example. For the beginner we recommend to set also the zeros, but when the necessary efficiency for similar calculations is obtained, we can refrain from doing so and write only the numbers:

```
  26 × 25     =       650
  26 × 1300   =     33800
  26 × 30000  =    780000
4100 × 25     =    102500
4100 × 1300   =   5330000
4100 × 30000  = 123000000
                ---------
                129246950
```

Example No. 24. Determine the exact value in Reichsmark of $ 344.28 at the exchange rate of 4.198. Splitting up of the factors gives 3'44'28 × 41'98, and the multiplication executed with the slide rule has the following form:

98 × 28	=	2744
98 × 4400	=	4 31200
98 × 30000	=	29 40000
4100 × 28	=	1 14800
4100 × 4400	=	180 40000
4100 × 30000	=	1230 00000
		1445.28744

§ 13. Division.

Advanced examples:

Example No. 25. Convert the common fractions 3/17, 5/19, 1/7, 5/16, 5/29 into decimal fractions.

Setting the common fractions, numerator on *D,* denominator on *C,* gives the corresponding decimal fractions at the initial or end line of *C* on *D.* Thus we have for the above given common fractions the decimal fractions:

3/17 = 0·1765; 5/19 = 0·263; 1/7 = 0·143; 5/16 = 0·3125; 5/29 = 0·1724.

Example No. 26. Various glass tubes have the section of 1 cm.^{2} and shall be filled with liquids to the exact weight of 10 g. each. What will be the height of the liquids, if the specific gravity *s* is given? *Solution:* 1 cm.^{3} weighs *s* grams, thus $s \times h = 10$ and $h = 10 : s$. Thus, the slide rule will give the following table:

Liquid	Mercury,	Sulfuric Acid,	Muriatic Acid,	Water,	Alcohol,	Ether
Specific Gravity	13·6	1·84	1·19	1·—	0·79	0·73
Height	0·735	5·43	8·40	10·—	12.65	13·70

These results are obtained by bringing into coincidence the different specific gravities on scale *C* with the initial or final index of scale *D.*

Example No. 27. The chemical formula for sulphuric acid is H_2SO_4, the atomic weight of hydrogen *H* is 1, that of sulphur *S* 32, that of oxygen *O* 16. Determine the percentage of quantity of each component contained in sulphuric acid.

Solution: As the atomic weight of hydrogen = 1, 2 atoms = 2; 1 atom of sulphur = 32; 4 atoms of oxygen = 4 × 16 = 64, one molecule sulphuric acid, therefore, 2 plus 32 plus 64 = 98. This ratio remains constant for any quantity of sulphuric acid, 98 parts always will contain, 2 parts of hydrogen, 32 parts of sulphur, 64 parts of oxygen. The percentage thus will be $\frac{200}{98} = 2{\cdot}04\%$ of hydrogen, $\frac{3200}{98} = 32{\cdot}7\%$ of sulphur, $\frac{6400}{98} = 65{\cdot}3\%$ of oxygen. Setting of these problems will not need any further explanation.

Example No. 28. Having found sine 55° to be 0·8192, sine 56° to be 0·8290, we have to derive the value of sine 55° 27'. Considering the units of the fourth number of places in the given values, we state that the sine increases by 98 if the angle increases by 1° = 60', this gives us the ratio of $60' : 27' = 98 : x$. *x,* therefore, will be the value to add to the lesser value of sine, thus: $x = 44$; sin. 55° 27' = 0·8192 + 0·0044 = 0·8236.

Example No. 29. tg. 20° = 0·3640, tg. 21° = 0·3839, tg. α = 0·3710. Determine the value of α. Tg. α will be x' greater than 20°. If the angle increases by 60', the tangent increases by 199 units of the fourth decimal, thus $60 : x = 199 : 70$; $x = 21'$; $\alpha = 20° \, 21'$.

Also in this example we have a proportion to be set according to example 28.

Example No. 30. 1 £ has the standard value of Marks 20·40, 1 Pengö Hungarian money is worth 0·734 Marks. How many Pengö will give £ 2¾?

Here the proportion to set on the slide rule in the manner already explained will be $2{\cdot}75 : x = 0{\cdot}734 : 20{\cdot}4$. Making use of the scales *A* and *B,* we set 734 on *A* to 20·4 on *B,* then we read opposite 2·75 of *A* 76·4, Pengö on *B.*

Example No. 31. $ 9172.— at 4% yield $ 183·44 interest in 180 days. How much in 45, 125, 260 days respectively?

Setting of 180 on *A* opposite 183·44 on *B*, enables us to read opposite the various numbers of days on *A* the values sought on *B*, for 45 days £ 45·85; for 125 days £ 127·37; for 260 days 264·94.

Scales *A* and *B* can also be used for determining the values of fractions of the same denominator and of different numerators. Setting the denominator on *B* opposite the initial line of *A*, we can read opposite the multiples of the numerator set with the cursor on scale *A*, the corresponding values on scale *B*.

On *A*	1	2	2·5	1	2	3
„ *B*	2	4	5	3	6	9

on *B*	12	0·166	0·250	0·500.
„ *A*	1	2	3	6

The proceeding is of special usefulness if we have to convert twelfth into decimals.

Repeated Division.

For the calculation of expressions as $\frac{a}{b \times c}$, we have only to notice that $\frac{a}{b \times c} = \frac{a}{b} : c$ or $= \frac{a}{c} : b$. We calculate first $a : b$ without reading the intermediate result, setting it only with the cursor line. Bringing then *c* on scale *B* under the cursor line, gives us the final result at the initial, middle or end index of *B* on *A*.

Example No. 32. For a flower bed (elliptical) are provided the width $2b = 3{\cdot}6$ m. and the area of 14.5 m. Determine the lenght. If we set for the length $2a$, and for the area *F* we have the formula $a \times b \times \pi = F$

$$a = \frac{F}{b \times \pi}; a = \frac{14{\cdot}5}{1{\cdot}8 \times \pi} \text{ m. } \alpha) \frac{14{\cdot}5}{1{\cdot}8} = 8{\cdot}05; a = \frac{8{\cdot}05}{3{\cdot}142} = 2{\cdot}56; 2a = 5{\cdot}12 \text{ m. } \beta) \frac{14{\cdot}5}{\pi} =$$
$$= \underline{4{\cdot}62}; a = \frac{4{\cdot}62}{1{\cdot}8} = 2{\cdot}56 \text{ m. } \gamma) 1{\cdot}8 \times \pi = \underline{5{\cdot}65}; a = \frac{14{\cdot}5}{5{\cdot}65} = 2{\cdot}56 \text{ m.}$$

The underlined intermediate values need not to be read.

Example No. 33. A container has the base of 37·6 cm.². Determine its height if it shall contain 2 kilos of mercury (special gravity = 13·6). Solving of this problem gives the equation $37{\cdot}6 \times 13{\cdot}6 \times x = 2000$; $x = \frac{2000}{37{\cdot}6 \times 13{\cdot}6} = 3{\cdot}91$ cm.

§ 14. Examples of Combined Multiplication and Division.

When executing operations of this kind by means of scales *C* and *D*, always divide first and then multiply. This allows to make the two successive operations with one setting of the slide.

Example No. 34. A parcel of ground measuring 315 × 62 yards is to be exchanged for another which is 73 yards wide. Determine the length, which must be 315 × 62 : 73 yards. Bringing into coincidence 315 on *D* with 73 on *C* and then setting the cursor to 62 on *C* gives the result sought on *D* = 268 yards².

Example No. 35. Determine the cost in Francs of a meter of cloth, sold in England at 87 Pence per yard, if the laid down expenses amount to 7% and if the rate of exchange is Fs. 25.50.

$$1 \text{ meter costs in England } \frac{87}{0{\cdot}914}\ d, \text{ as } 1 \text{ m.} = 0{\cdot}914\ yd.$$

$$\frac{87}{0{\cdot}914}\ d = \frac{87}{0{\cdot}914 \times 240}\ £, \text{ as } 1\ £ = 240\ d; \frac{87}{0{\cdot}914 \times 240}\ £ = \frac{87 \times 25{\cdot}5}{0{\cdot}914 \times 240} \text{ fr.}$$

As we have to add 7% for the laid down expenses we must multiply the result by 1·07, getting finally $x = \frac{87 \times 25{\cdot}5 \times 1{\cdot}07}{0{\cdot}914 \times 240} = 10{\cdot}82$ fr.

Example No. 36. Determine the velocity of gyration of the earth around the sun, if the mean distance of 149·5 millions of kilometers is given. The orbit of the run of the earth during one year is $2r\pi = 299 \times \pi \times 10^6$ kilometers. Finding the distance for one day requires division of the result by

365·25, this result then must be divided by 24 in order to find the amount per hour and finally dividing the latter amount by $60 \times 60 = 3600$ gives the velocity in kilometers per second, thus:

$$v = \frac{299 \times 3{\cdot}142 \times 10^6}{365{\cdot}25 \times 24 \times 3600} = 29{\cdot}8 \text{ km/sec.}$$

§ 15. Solution of Linear Equations with Unknown Quantities.

If $a, b, c \ldots$ are kown, $x, y, z \ldots$ unknown $\begin{matrix} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{matrix}$, is the normal form of two linear equations from which we have to determine x and y.

From $\begin{matrix} a_1 x + b_1 y + c_1 z = d_1 \\ a_2 x + b_2 y + c_2 z = d_2, \\ a_3 x + b_3 y + c_3 z = d_3 \end{matrix}$ we can generally determine x, y and z.

Example No. 37. Determine x and y if the following equations are given.

$$\begin{aligned} 1)\ 11{\cdot}18\,x + 2{\cdot}17\,y &= 21{\cdot}17 \\ 2)\ 6{\cdot}25\,x - 3{\cdot}16\,y &= -3{\cdot}34 \end{aligned}$$

One of the equations does not undergo any modification, the other must be multiplied by a factor which allows the elimination of one of the unknown. If x shall be eliminated, the equation which has the lesser index of x is not modified for pratical reasons. Consequently we have to multiply the first equation by $-\frac{6{\cdot}25}{11{\cdot}18}$ getting:

$$\begin{aligned} 2)\quad 6{\cdot}25\,x - 3{\cdot}16\,y &= -3{\cdot}34 \\ 1a)\ -6{\cdot}25\,x - 1{\cdot}21\,y &= -11{\cdot}84 \end{aligned}$$

From the addition of 2 and 1a we get 3) $-4{\cdot}37\,y = -15{\cdot}18$; $y = \frac{15{\cdot}18}{4{\cdot}37} = \mathbf{3{\cdot}47}.$

Setting this value in the original equation No. 1, we have then:

$$11{\cdot}18\,x + 7{\cdot}53 = 21{\cdot}17;\ 11{\cdot}18\,x = 13{\cdot}64;\ x = \frac{13{\cdot}64}{11{\cdot}18} = \mathbf{1{\cdot}22}.$$

For examination, we introduce x and y in the equation No. 2:

$$6{\cdot}25\,x = 7{\cdot}63;\ 3{\cdot}16\,y = 10{\cdot}96;\ 6{\cdot}25\,x - 3{\cdot}16\,y = 7{\cdot}63 - 10{\cdot}96 = -3{\cdot}33.$$

Example No. 38.

$$\begin{aligned} 1)\quad 1{\cdot}23\,x + 2{\cdot}46\,y + 0{\cdot}324\,z &= -1{\cdot}002 \\ 2)\ -2{\cdot}17\,x + 1{\cdot}74\,y + 2{\cdot}33\,z &= 9{\cdot}33 \\ 3)\quad 3{\cdot}18\,x - 0{\cdot}28\,y + 1{\cdot}652\,z &= 10{\cdot}18 \end{aligned}$$

In order to eliminate x we do not modify the first equation, but multiply the second by $\frac{1{\cdot}23}{2{\cdot}17}$; and the third by $-\frac{1{\cdot}23}{3{\cdot}18}$.

Thus we shall get:

$$\begin{aligned} 1)\quad 1{\cdot}23\,x + 2{\cdot}46\,y + 0{\cdot}324\,z &= -1{\cdot}002; \\ 2a)\ -1{\cdot}23\,x + 0{\cdot}986\,y + 1{\cdot}321\,z &= 5{\cdot}29; \\ 3a)\ -1{\cdot}23\,x + 0{\cdot}1083\,y - 0{\cdot}639\,z &= -3{\cdot}94. \end{aligned}$$

After the addition [1) + 2a); 1) + 3a)] we find, abstracting from the superfluous numbers of places:

$$\begin{aligned} 4)\ 3{\cdot}45\,y + 1{\cdot}645\,z &= 4{\cdot}29 \\ 5)\ 2{\cdot}57\,y - 0{\cdot}315\,z &= -4{\cdot}94 \end{aligned}$$

y is eliminated multiplying No. 4 by $-\frac{2{\cdot}57}{3{\cdot}45}$.

$$\begin{aligned} 5)\quad 2{\cdot}57\,y - 0{\cdot}315\,z &= -4{\cdot}94 \\ 4a)\ -2{\cdot}57\,y - 1{\cdot}226\,z &= -3{\cdot}20 \\ 6)\ -1{\cdot}541\,z = -8{\cdot}14;\ z &= \frac{8{\cdot}14}{1{\cdot}541} = \mathbf{5{\cdot}28}. \end{aligned}$$

The value of z can be inserted in one of the equations 4 or 5, for instance, in 4:

$$3{\cdot}45\ y + 8{\cdot}69 = 4{\cdot}29;\ 3{\cdot}45\ y = -4{\cdot}40;\ y = -\frac{4{\cdot}40}{3{\cdot}45} = \mathbf{-1{\cdot}276.}$$

Finally we get x, inserting y and z in one of the primary equations, eventually in No. 3, getting:

$$3{\cdot}18\ x + 0{\cdot}357 + 8{\cdot}72 = 10{\cdot}18;\ 3{\cdot}18\ x = 1{\cdot}103;\ x = \mathbf{0{\cdot}347.}$$

The trial consists in inserting the values of x, y, and z in the two other primary equations, thus, No. 1 and 2:

$$1)\ \begin{aligned} 1{\cdot}23\ x &= 0{\cdot}427 \\ 2{\cdot}46\ y &= -3{\cdot}14 \\ 0{\cdot}324\ z &= 1{\cdot}710 \\ \hline & -1{\cdot}003 \end{aligned} \qquad 2)\ \begin{aligned} -2{\cdot}17\ x &= -0{\cdot}753 \\ 1{\cdot}74\ y &= -2{\cdot}22 \\ 2{\cdot}33\ z &= 12{\cdot}30 \\ \hline & 9{\cdot}327 \end{aligned}$$

We see that the results are exact within the limits of precision, the slide rule offers.

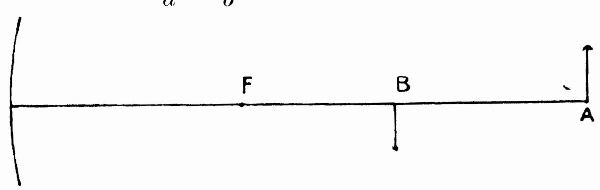
§ 16. Some Advanced Examples of Calculations with Reciprocal Values.

The use of the scales of reciprocals has already been explained on page 15; we can, therefore, restrict ourselves by giving some advanced examples in the following lines.

Example No. 39. A concave mirror casts a picture distant b yards from an object distant a yards. If f be the focal distance, we have the formula:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}.$$

fig. 3.



A catoptric telescope has the focal distance of $f = 1{\cdot}4$ yards. It is directed on a tree in the distance of 27·5 yards. Where will the picture be produced?

Solution: $\frac{1}{27{\cdot}5} + \frac{1}{b} = \frac{1}{1{\cdot}4}$; $\frac{1}{b} = \frac{1}{1{\cdot}4} - \frac{1}{27{\cdot}5} = 0{\cdot}715 - 0{\cdot}0364$; $\frac{1}{b} = 0{\cdot}679$; $b = 1{\cdot}474$ yards.

It results, therefore, that the objective must be set back by 0·074. For this example, we have only to determine the respective reciprocal values and o calculate the rest in the usual way.

Example No. 40. A photographic camera having the focal distance o 13·5 cm., we have to determine the distance of the disc from the lens for the following distances of the object to be taken:

a) 1 m., b) 2 m., c) 4 m., d) 6 m., e) 8 m., f) 10 m.

Making use of the preceding formula, we shall get the following results:

a) 0·156, b) 0·145, c) 0·140, d) 0·138, e) 0·137, f) 0·137 m.

Example No. 41. In an electric conductor are branched off 4 wires, which join again (parallel connection). The wires have to be replaced by a single wire, for which we have to determine the resistance; the individual wires being the following $w_1 = 12{\cdot}4$ ohms; $w_2 = 7{\cdot}94$ ohms; $w_3 = 43{\cdot}6$ ohms; $w_4 = 218$ ohms. For the solution of this operation we make use of the formula $\frac{1}{w} = \frac{1}{w_1} + \frac{1}{w_2} + \frac{1}{w_3} + \frac{1}{w_4}$. For the values above given the result will be $\frac{1}{w} = 0{\cdot}234$; $w = 4{\cdot}27$ ohms.

Example No. 42. Calculating the different parts of a triangle from the sides $a = 5$ cm., $b = 6$ cm., $c = 7$ cm., we find the heights h:

$$h_a = 5{\cdot}88 \text{ cm.}, \; h_b = 4{\cdot}90, \; h_c = 4{\cdot}20 \text{ cm.},$$

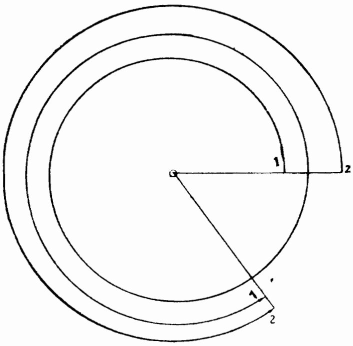
the outer radii $\varrho_a = 3{\cdot}675$ cm., $\varrho_b = 4{\cdot}90$ cm., $\varrho_c = 7{\cdot}35$ cm., the inner radius $\varrho = 1{\cdot}633$ cm. For examining we make use of the formulae:

$$\frac{1}{\varrho} = \frac{1}{\varrho_a} + \frac{1}{\varrho_b} + \frac{1}{\varrho_c}; \; \frac{1}{\varrho} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c};$$

$$(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = (h_a + h_b + h_c)\left(\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}\right); \; \frac{1}{h_a} = \frac{1}{2}\left(\frac{1}{\varrho_b} + \frac{1}{\varrho_c}\right) \text{ and}$$

so on. The examination will prove that the values given are precise.

fig. 4.



Example No. 43. An observer stands in the centre of a circular stadium. A cyclist rounds it in $t_1 = 42$ seconds, another in $t_2 = 53$ seconds. At a certain time the observer sees both cyclists on the same line. Determine the space of time when the same position of both will occur again. This will be the case after t seconds, when the faster cyclist has circled the course once more than the slower rider. The radius of vision to the cyclist turns in t_1 seconds by 360°, in 1sec. by $\frac{360}{t_1}$, in $t$sec. by $\frac{360\,t}{t_1}$. Thus, $\frac{360\,t}{t_1} - \frac{360\,t}{t_2} = 360$ or $\frac{1}{t_1} - \frac{1}{t_2} = \frac{1}{t}$. In our case $\frac{1}{t} = 0{\cdot}00495$, $t = 202$sec. $= 3$min. 22sec..

§ 17. Inverse Proportionality.

If we are speaking of proportionality, we set $a : b = a_1 : b_1$, for inverse proportionality we have $a : b = b_1 : a_1$. Solving of inverse proportions of the latter formula requires setting of a on D scale and bringing into coincidence a_1 on CI. Then setting cursor line on b of D, we get b_1 on CI under the cursor line in this position.

Examples No. 44. Various sportsmen make the distance of 100 yards in the following times: 11·5; 11·8; 12·5; 12·1; 11·9; 12·9; 12·4; 12·3; 11·7; 12·6 seconds. Determine the average speed. Setting the cursor to the different values given above on scale D gives the speed per yard/sec. on scale CI. The avarage speed will be 8·226 yards/sec.

Example No. 45. At the 400 meter course in Stockholm the maximum speed (1921) obtained was 48·2 sec. In Paris 1924 we had as best results: 48; 47·8; 47·6 sec. Determine the m./sec. made by the different men. Setting of the initial line of CI opposite 4 of D gives us the result sought opposite the respective values on CI on D.

Example No. 46. The product of atomic weight and specific heat is about the same for almost all chemical elements, i. e. 6·2. Derive the atomic weight from the specific heat of the following elements. Aluminum 0·217; Sulphur 0·171; Iron 0·113; Copper 0·09305; Silver 0·056; Gold 0·031; Lead 0·0309. The results will be the following in the above given rotation, and for comparison we also give those found in the experimental way. The differences are not due to the sometimes alleged lack of accuracy of the slide rule (which does not exist), but to the fact that 6·2 constitutes a mean value. Also, in this example, setting of the initial or final index of CI to 6·2 on scale D gives the results. We have only to bring into coincidence the values of specific heat on scale CI with the values sought on scale D by means of the cursor line.

28·6 (27·1); 36·25 (32·07); 54·9 (55·84); 66·6 (63·57); 110·7 (107·88); 200 (197·2); 200·6 (207·1).

§ 18. Abridged Division.

If we have to convert the value of an angle given in degrees, minutes and seconds into the measure of the arc, we first convert the value into seconds and then divide it by $\varrho'' = 206265$, for instance: $\alpha = 14° 23' 32{\cdot}4''$. $14° = 14 \times 3600 = 50400''$; $23' = 23 \times 60 = 1380''$, thus, $\alpha = 50400'' + 1380'' + 32{\cdot}4'' = 51812{\cdot}4''$. In the usual manner we calculate as follows:

:20626 5	
51812·4	0·25119
41253 0	
10559 4	0
10313 2	5
246 1	50
206 2	65
39 8	850
20 6	265
19 2	5850
18 5	6385 etc.

This manner of calculation is somewhat tiresome and not altogether unobjectionable, because there are supposed to be zero places behind the last figure (4) of the dividend, while in similar cases the last places are generally evened up.

By the abridged division both inconveniences are avoided. We begin exactly as in the previous example. But instead of dividing 1055940 by 206265 we divide 105594 by 20626·5 or 206265, as result we get 5 and calculate $5 \times 5 = 25$, $5 \times 6 = 30$, $30 + 3 = 33$; $5 \times 2 = 10$; $10 + 3 = 13$ etc., thus getting

105594
103133 The rest is not divided by 206265 but by 20627 (rounding off 6·5 gives 7)
2461.

The result is 1, thus, we get

2461
2063 this number is divided by 2063 etc., etc.
398

The new proceeding we are giving represents itself as follows:

: 206265	
51812·4	= 0·2511935.
412530	
105594	
103133	
2461	
2063	
398	
206	
192	
185	
7	

The figures of the divisor are underlined in the course of the operation, beginning from the right.

We see that the division could have been interrupted at 2461 : 2063 or at 398 : 206·3 and that the rest could have been executed with the slide rule. We now have the following form:

: 206265			: 206265	
51812·4	= 0·251194	or	51812·4	= 0·2511935.
412530			412530	
105594			105594	
103133			103133	
2461			2461	
			2063	
			398	

The underlined figures of the result have been determined with the slide rule.

Example No. 47. The terrestrial meridian has not the exact circumference of 40 000 000 meters, because the metric measure does not correspond exactly with its definition, but 40 003 423 meters. Determine the radius of a circle of the same circumference.

Solution: $2 r \pi = 40003423$; $r = 20001712 : 3{\cdot}141593$

```
|: 3·141593|
 20001712  = 6366742.
 18849558
  1152154
   942478
   209676
   188496
    21180
    18850
     2330
```

The underlined figures have been determined with the slide rule.

§ 19. Increasing of the Precision of Division by Serial Development.

If we want to get results of divisions with a precision beyond that which direct reading and estimating on the slide rule can give us, we cannot proceed in the same manner as for multiplication but we must follow the procedure as given hereafter.

If q be a proper fraction, a any number, we know, that $\frac{a}{1-q} = a + aq + aq^2 + aq^3 + \ldots$ For instance, $\frac{1}{1-0{\cdot}15} = \frac{1}{0{\cdot}85} = 1{\cdot}17647$.

Further we have (provided that $a = 1$, $q = 0.15$) $a = 1$, $aq = 0.15$; $aq^2 = 0{\cdot}0225$; $aq^3 = 0{\cdot}00338$; $aq^4 = 0{\cdot}00051$; $aq^5 = 0{\cdot}00008$; $aq^6 = 0{\cdot}00001$.

The further terms can be neglected, and the sum written down is 1·17648. As each term results from multiplying the preceding by q, we have to set the constant factor $q = 0{\cdot}15$ on D scale, and results of the different multiplications can be read without further moving of the slide.

As a further example we calculate $\frac{2}{1{\cdot}045} = 2 - 2 \times 0{\cdot}045 + 2 \times 0{\cdot}045^2 - 2 \times 0{\cdot}045^3 \pm \ldots = 2 - 0{\cdot}09000 + 0{\cdot}00405 - 0{\cdot}00018 + 0{\cdot}00001 = 1{\cdot}91388$.

This value is in perfect accordance with that obtained by direct calculation.

The lesser be the term q, the faster we get the result required, and often already the first term may suffice $\frac{a}{1-q} \backsim a + aq$.

Example No. 48. An ancient atmosphere (760 mm. of mercury) equals 1·0333 new atmospheres (1 kg./cm.²). Calculate the result of 1·35; 22·3; 0·627 of new atmospheres in ancient ones.

Solution: 1 new atmosphere $= \frac{1}{1{\cdot}0333} = 1 - 0{\cdot}0333 + 0{\cdot}0011$ ancient ones. Multiplying the given numbers with this factor, we get 1·3065; 21·582; 0·6068 ancient atmospheres.

Example No. 49. If a banker buys for $ 100 000 shares at the rate of exchange of 94·5%, which nominal amount will he receive?

Solution: $100000 \times \frac{100}{94{\cdot}5} = 100000 \times \frac{1}{1-0{\cdot}055} = 100000(1 + 0{\cdot}055 + 0{\cdot}003025 + 0{\cdot}0001664 + 0{\cdot}0000091 + 0{\cdot}0000005) =$ $ 105 820.10.

Example No. 50. Determine the diameter of a circle of which the circumference is 40 000 km.

Solution: $\frac{40000}{3{\cdot}1416} = \frac{40000}{3 \times 1{\cdot}0472} = \frac{40000}{3}(1 - 0{\cdot}0472 + 0{\cdot}0022 - 0{\cdot}0001) = \frac{40000}{3} \times 0{\cdot}9549 = 9549 \times (1 + 1/3) = 12732$ km.

§ 20. Logarithms.

Logarithms to any Base.

If $10^b = a$, we have, according to the definition of the logarithms, $b = \log. a$. Number 10 here is the base of the logarithmic system, and this base has been chosen, because our numeric system is decimal. For advanced problems we often make use of the logarithmic system based on number $e = 2{\cdot}71828\ldots\ldots$ For this we write $e^b = a$ and $b = ln\ a$, ln being the abbreviation for logarithmus naturalis. When making use of the artificial logarithms, we derive from $e^b = a$ the following equation: $b \times \log. e = \log. a;\ b = \frac{1}{\log. e} \times \log. a;\ b = = \frac{1}{0{\cdot}43429\ldots} \log. a = 2{\cdot}3025 \times \log. a$.

As $b = ln\ a$, it results from this that $ln\ a = 2{\cdot}3025 \times \log. a$.

Thus, we get the following rule for the determination of the natural logarithms. In order to determine the natural logarithm of any number, first determine the artificial one and then multiply it by 2·3025.

Thus, it will be easy to derive the following table:

a	1	2	3	4	5	6	7	8	9	10
log. a	0	0·301	0·477	0·602	0·699	0·778	0·845	0·903	0·954	1·000
$ln\ a$	0	0·693	1·099	1·386	1·609	1·792	1·946	2·08	2·20	2·30

The same rules apply for calculating with logarithms, whether we make use of the artificial (Briggs) system or of the natural one.

If the natural logarithm is given, we can transform it into the artificial one by dividing it by 2·303 or by multiplying by 0·434.

Increasing of the Precision when Determining the Logarithms.

For this we refer to the formula

$$1.\ ln\ (a+h) = ln\ a + 2\left[\frac{h}{2a+h} + \frac{1}{3}\left(\frac{h}{2a+h}\right)^3 + \frac{1}{5}\left(\frac{h}{2a+h}\right)^5 + \ldots\right]$$

which passes into the following one after the multiplication by 0·4342945

$$2.\ \log. (a+h) = \log. a + 0{\cdot}8685890\left[\frac{h}{2a+h} + \frac{1}{3}\left(\frac{h}{2a+h}\right)^3 + \frac{1}{5}\left(\frac{h}{2a+h}\right)^5 + \ldots\right]$$

In this formula "a" means a value of which the logarithm is exactly known, h a little additional value. From a table of logarithms which gives the mantissae with 7 places we can take the following values

a	1	2	3	4	5
log. a	0	0·3010300	0·4771213	0·6020600	0·6989700
a	6	7	8	9	10
log. a	0·7781513	0·8450980	0·9030900	0·9542425	1

If, for instance, we have to determine log. 1·5 we have $a = 1$, $h = 0{\cdot}5$, $\frac{h}{2a+h} = \frac{0{\cdot}5}{2{\cdot}5} = 0{\cdot}2$. Then we get:

$$\begin{aligned}
\log. a &= 0\\
0{\cdot}8685890 \times \frac{h}{2a+h} &= 0{\cdot}1737178\\
1/3 \times 0{\cdot}8685890 \times \left(\frac{h}{2a+h}\right)^3 &= 0{\cdot}0023162\\
1/5 \times 0{\cdot}8685890 \times \left(\frac{h}{2a+h}\right)^5 &= 0{\cdot}0000556\\
1/7 \times 0{\cdot}8685890 \times \left(\frac{h}{2a+h}\right)^7 &= 0{\cdot}0000016\\
1/9 \times 0{\cdot}8685890 \times \left(\frac{h}{2a+h}\right)^9 &= 0{\cdot}0000000\\
\hline
\log. (a+h) = \log. 1{\cdot}5 &= 0{\cdot}1760912
\end{aligned}$$

(the lines for 1/5 and 1/7: slide rule!)

(The exact value is 0·1760913 = log. 3 — log. 2). The unimportant difference results from the rounding up, which we cannot avoid.

Remark: In order to prove, that the proceeding as shown, can be practically employed, we have chosen a very complicated example. We should have done better to set for $1{\cdot}5 = 2 - 0{\cdot}5$. Then we note $\frac{h}{2a+h} = \frac{-0{\cdot}5}{4-0{\cdot}5} = -\frac{0{\cdot}5}{3{\cdot}5} = -\frac{1}{7}$.

Thus, log. a	$= 0{\cdot}3010300$	
$0{\cdot}8685890 \times \frac{h}{2a+h}$	$= -0{\cdot}1240841$	
$1/3 \times 0{\cdot}8685890 \left(\frac{h}{2a+h}\right)^3$	$= -0{\cdot}0008441$	
$1/5 \times 0{\cdot}8685890 \left(\frac{h}{2a+h}\right)^5$	$= -0{\cdot}0000103$	slide rule!
$1/7 \times 0{\cdot}8685890 \left(\frac{h}{2a+h}\right)^7$	$= -0{\cdot}0000002$	
log. $(a+h)$ = log. 1·5	$= 0{\cdot}1760913$	

Remark: Logarithms of seven places are very seldom used in practical calculations. If we execute the above given calculations for ace logarithms the development will be the following: (0·86859 means k)

log. a	0		0·30103	
$1/3\ k \left(\frac{h}{2a+h}\right)^3$	0·17372		—0·12408	
$1/5\ k \left(\frac{h}{2a+h}\right)^5$	0·00232	slide rule!	— 0·00084	slide rule!
$1/7\ k \left(\frac{h}{2a+h}\right)^7$	0·00006		— 0·00001	
	0·17610		0·17610	

Example No. 51. Determine the log. of the factor of interest of $q = 1{\cdot}03$; $= 1{\cdot}035$; $1{\cdot}04$ etc. with 7 places.

Solution: In the first case we have $a = 1$, $h = 0{\cdot}03$; $\frac{h}{2a+h} = \frac{0{\cdot}03}{2{\cdot}03} =$ $= 0{\cdot}0147783$; log. $q = 0 + 0{\cdot}0128363 + 0{\cdot}0000009 = 0{\cdot}0128372$; for the others we get successively 0·0149403; 0·0170333; 0·0191163; 0·0211893 etc.

Example No. 52. Determine the logarithms of 6·1; 6·2; 6·3, etc. etc.

Solution: 0·7853298; 0·7923917; 0·7993405; 0·8061800; 0·8129134.

Increasing of the Precision when Determining the Numeri.

If log. $a = b$, $a = 10^b = 1 + \frac{mb}{1} + \frac{(mb)^2}{1 \times 2} + \frac{(mb)^3}{1 \times 2 \times 3} + \frac{(mb)^4}{1 \times 2 \times 3 \times 4} + \ldots,$ m being $= 2{\cdot}3025851$ ($= ln\ 10$). We proceed more rapidly, when considering the relation $10^{b+h} = 10^b \times 10^h$ and making use of the following table

b	0	0·1	0·2	0·3	0·4	0·5
10^b	1	1·258925	1·584893	1·995262	2·511886	3·162278
b	0·6	0·7	0·8	0·9	1·0	
10^b	3·981072	5·011872	6·309573	7·943282	10	

Example No. 53. To which amount increase $ 1000.— in 20 years at 3%?

Solution: $x = 1000 \times 1{\cdot}03^{20}$. log. $1{\cdot}03 = 0{\cdot}0128372$. $20 \times$ log. $1{\cdot}03 = 0{\cdot}256744$. $1{\cdot}03^{20} = 10^{0{\cdot}2} \cdot 10^{0{\cdot}056744}$. $b = 0{\cdot}2$; $h = 0{\cdot}056744$; $mh = 0{\cdot}1306579$.

We make the calculation with different precision:

	4 places		5 places		6 places		7 places	
1	1		1		1		1	
mh	0·1307	slide rule!	0·13066		0·130658		0·1306579	
$\frac{(mh)^2}{2}$	0·0085		0·00854	slide rule!	0·008536		0·0085357	
$\frac{(mh)^3}{1 \times 2 \times 3}$	0·0004		0·00037		0·000372	slide rule!	0·0003718	
$\frac{(mh)^4}{1 \times 2 \times 3 \times 4}$	0·0000		0·00001		0·000012		0·0000121	slide rule!
$\frac{(mh)^5}{1 \times 2 \times 3 \times 4 \times 5}$			0·00000		0·000000		0·0000003	
10^h	1·1396		1·13958		1·139578		1·1395778	
$10^{0{\cdot}2} \times 10^h$	1·806		1·8061		1·80611		1·806109	

The value of the final capital is found by multiplying the last number by 1000. The reader will find, that for most of the values employed in this calculation the slide rule can be employed with advantage.

Example No. 54. Calculate the same problem with $3^1/_2$%, 4%, $4^1/_2$%, 5%, etc.

Solution (with 7 places) 1989.784; 2190.119; 2411.715; 2653.298.

§ 21. Squares and Square Roots.

SQUARES.

The procedure of determining squares and square roots and their number of places has been fully explained on page 11, and we think that a few examples will suffice to show the use of the respective scales for practical purposes.

Example No. 55. The base of Cheops Pyramid is a square of which the base is measuring 255 yards. Determine the surface. Setting the cursor line to 255 on *D* gives the result equally under the cursor line on scale *A*, i. e. 65000 yards (exactly 65025 yards).

Example No. 56. Determine the weight of ingot iron bars if the sides of the section measure 5; 6; 7; 8; 9 ..... mm. The specific weight of ingot-iron is 7·85.

Solution: As the weight is to be found in kilos, the measures must be given in dms. The length is supposed to be 1 meter. Thus, we have for the first section the formula: $0{\cdot}05^2 \times 10 \times 7{\cdot}85 = 0{\cdot}0025 \times 7{\cdot}85 = 0{\cdot}196$ kilos, and in order to obtain a whole table of all the possible values, we have only to set the final index of scale *B* to 785 of scale *A* and to set cursor line successively on the different measures read on scale *C*. The results then can be read on scale *A* as shown by the following table:

d	5	6	7	8	9	10	mm.
G	0·196	0·283	0·385	0·502	0·636	0·785	kg.

Example No. 57. A copper wire has the resistance of $w = 0{\cdot}207$ ohm, the intensity of current first is $i = 10$ ampères, increasing successively to 12, 14, 16 .... ampères. Determine the calories produced in each of the mentioned cases during one second.

Calories $Q = 0{\cdot}24\ i^2\ w$ gramcalories, thus, $0{\cdot}0497 \times i^2$ and the following table:

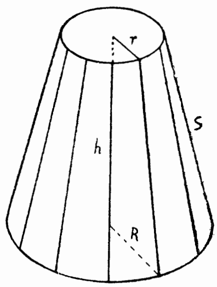
i	10	12	14	16	18	20	amp.
Q	4·97	7·15	9·74	12·72	16·1	19·9	gr.-cal.

Setting the end or initial line of *B* to 497 of a *A* gives the results on *A* opposite the different values of intensity set on *C*.

SQUARE ROOTS.

Example No. 57. The frustum of a cone has the radii *R* and *r*, the height *h*, the bases $R^2\pi$ and $r^2\pi$ or $D^2 \times \frac{\pi}{4}$ and $d^2 \times \frac{\pi}{4}$ and the cubic contents $\frac{\pi h}{3}$ $(R^2 + Rr + r^2)$.

fig. 5.



Determine the contents of a tub of the following dimensions: $D = 33''$, $d'' = 23$, $h = 40''$.

Contents $v = \frac{\pi}{3} \times 40\ (16{\cdot}5^2 + 16{\cdot}5 \times 11{\cdot}5 + 11{\cdot}5^2) =$ 24900 cubic inches.

Example No. 58. An aerostat floats 1200 m. above the terrestrial surface. One of the navigators drops a bottle from this height. Determine the velocity, with which it will hit the earth. The metric formula for the determination of the acceleration of the free fall is $\sqrt{2\,g\,h}$, *g* being the value of 9·81; *h* the height, *v* the acceleration. The result here will be 153·4 m./sec.

Example No. 59. A gasometer consists of a cylinder, covered by a spherical calotte, figure 9. The gasometer is constructed of iron plate of which the weight is 50 lbs. per square yard.

The height of the cylinder is $20^1/_2$ yards, the diameter $d = 31$ yards. The calotte has the radius of curvature of $r = 32$ yards.

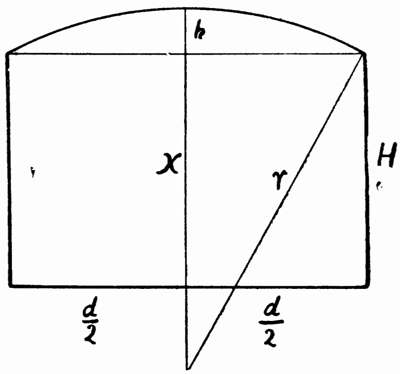
Determine the cubic contents and the weight of the construction.

First find piece x according to the theorem of pythagoras $x = \sqrt{r^2 - \left(\frac{d}{2}\right)^2} =$ $= \sqrt{1024 - 240} = \sqrt{784} = 28{\cdot}0$. Then $h =$ $= r - x = 4$. The contents of the cylinder will be $\frac{\pi}{4} \times d^2 \times H = \frac{\pi}{4} \times 31^2 \times 20{\cdot}5 = 15470$, that of the calotte $\frac{\pi h^2}{3} (3r - h) = \frac{\pi \times 16}{3} \times$ $\times (96 - 4) = 1540$ and the total cubic contents 17010 cubic yards.

The convex surface of the cylinder is $\pi \times d \times h = \pi \times 31 \times 20{\cdot}5 = 1997$ square yards and the surface of the calotte $2 r \pi h =$ $= 2 \times 32 \times \pi \times 4 = 804$ square yards, total 2801 square yards. The total weight, therefore, will be $2801 \times 50 = 140000$ lbs.

After having learned how to use the different scales, the attentive reader will be in a position to solve also this problem without difficulty whatsoever.

fig. 6.



Square Roots of Compound Expressions.

If we have to calculate expressions as $\sqrt{a \times b}$, $\sqrt{\frac{a}{b}}$, $\sqrt{a\,b\,c}$, $\sqrt{\frac{ab}{c}}$ we first determine the values under the radical sign and then the root, combining scales A and B and D. Care must be taken in setting properly as explained on page 12.

Example No. 60. The area F of a circle being $F = \frac{\pi}{4} \times d^2$, $d \sqrt{F : \left(\frac{\pi}{4}\right)}$. If, for instance, the area of the circle be 3000 square inches, the diameter will be $\sqrt{3000 : \left(\frac{\pi}{4}\right)} = 61^2/_3$ inches. Setting the gauge point of $\frac{\pi}{4} = 0{\cdot}785$ on scale B opposite 3 of A (right unit), gives the result sought at the final index of C on D.

Increasing the Degree of Precision of Square Roots.

Example No. 61. Calculate exactly $\sqrt{15251}$ (approximate value 123·5). For the exact value we set $123{\cdot}5 + h$. Then $(123{\cdot}5 + h)^2$ must equal 15251, thus, $15252{\cdot}25 + 247\,h + h^2 = 15251$. As h^2 is a very little value, we can omit it, determining only; $247\,h = 15251 - 15252{\cdot}25$; $h = -\frac{1{\cdot}25}{247} = -0{\cdot}00506$; $x = 123{\cdot}5 -$ $- 0{\cdot}00506 = 123{\cdot}49494$. If the precision of the result should not yet seem sufficient, we could still calculate h^2.

Increasing the Degree of Precision of Squares.

Considering squaring as a multiplication of two factors, we can refer for increasing the degree of precision to the explanations given on page 26. Further, we recommend using the formula $(a + b)$ or $(a + b + c)$, as it will always be easy to make selection of the members of the expressions in a manner which allows the determination of absolutely accurate results with the slide rule. Of course, the definite result will then also be exact.

Example No. 62. Determine the area of a square parcel of ground of which one side measures 26 1/3 yard.

Solution: $a = 26$; $b = 0{\cdot}3$

$a^2 =$	676
$2ab =$	15·6
$b^2 =$	0·09
$F =$	691·69 yards.

The direct reading on the slide rule gives 692 yards.

Example No. 63. In a table we find $\sqrt{3} = 1{\cdot}73205$. Determine the difference between the square of this number and 3.

Solution: $a = 1{\cdot}7$; $b = 0{\cdot}032$; $c = 0{\cdot}00005$ we find $1{\cdot}73205^2 = 2{\cdot}9999972025$, thus, the difference will be 0·0000027975.

§ 22. Cubes and Cube Roots.

The procedure for determining cubes and cube roots has already been explained on pages 12 etc., and hereafter we are giving a few practical examples.

Example No. 64. One inch equals 2·54 cm., one foot 3·05 dm., one yard 0·914 m. Determine the corresponding cubic measures. Setting on scale *D* the cursor line to the corresponding values, gives the results on scale *K*, i. e. 1 cubic inch = 16·4 cm.^{3}; 1 cubic foot = 28·4 dcm.^{3}; 1 cubic yard = = 0·763 m.^{3}.

Example No. 65. Determine the cubic measure of three balloons, having the diameter of 12·5; 13; 14·5 yards. The cubic contents are calculated after the formula $V = \frac{\pi}{6} \times d^3$. We first determine the cubes of the diameter, setting the cursor line of the corresponding values on scale *D* and reading the results on scale *K*. Then we set the initial index of scale to the value of $\frac{\pi}{6} = 0{\cdot}5236$ on scale *D* and get the final results opposite the value read on *K* scale and then transposed to scale *C*. We shall get the values of 1023; 1150; and 1596 cubic yards respectively.

Example No. 66. On the Dresden exhibition 1928, "The Technical City", there was a house of spherical form 32 m. high. Determine the cubic contents thereof The formula explained above gives us as result 17160 m.^{3}.

Example No. 67. Determine the length of edge of cubic tin boxes which shall contain 1 gallon = 4·54 liters. Setting cursor line to 4·54 of the first logarithmic unit of scale *K* gives us the result sought in cm.

Example No. 68. Determine the diameter of spheres, having the cubic measures of 1; 2; 3; 4; 5 yards respectively. The formula for the determination of the diameter of spheres being the following: $d = \sqrt[3]{\frac{6V}{\pi}}$, we get the following table:

Volume	1	2	3	4	5
d^3	1·91	3·82	5·73	7,64	9·55
d	1·24	1.56	1·79	1·97	2,12 yards.

Determining of $6V:\pi$ on scales *C* and *D*, transposing of the values found on *K* scale and finally reading of the third root on *D* scales will give the results as per table above.

For the calculation of spheres, we have to consider that $\frac{\pi}{6} = \frac{11}{21} - 0{\cdot}00021075 = \frac{1}{2} + \frac{1}{42} + 0{\cdot}00021075$. If a sphere has the diameter of 36 inches, we derive the following calculation:

$\frac{1}{2} d^3 = 23328{\cdot}00$; $\frac{1}{42} d^3 = 1110{\cdot}86$; $-0{\cdot}00021075\ d^3 = -9{\cdot}84$; $v = 24429{\cdot}02$.
With the table of logs. of 7 places we get 24429·03.

Increasing of the Precision of Cubes.

Setting $a^3 = a \times a \times a$, we can refer for increasing of the precision of results to the explanation given page 26 for the corresponding procedure for multiplications. Further, we can make use of the formula $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3ab(a + b) + b^3$. For instance, we have for $36^3 = (30 + 6)^3 = 27\,000 + 540 \times 36 + 216 = 46\,656$. Each of the terms can be determined exactly with the slide-rule. Other examples can be easily added by the reader and results checked by calculations made in the usual way.

Increasing of the Precision of Cube Roots.

Example No. 69. If we wish to calculate exactly $x = \sqrt[3]{9}$, we first find the approximate value $x_0 = 2{\cdot}08$, and the exact value must be $2{\cdot}08 + h$. This latter value in comparison with x must be a small one. We develop now the following formula $(x_0 + h)^3 = 9$, thus, $x_0^3 + 3x_0^2h + 3x_0h^2 + h^3 = 9$. Omitting the terms h^2 and h^3, we obtain $3x_0^2h = 9 - x_0^3 = 9 - 8{\cdot}998912$ (to be calculated exactly) $= 0{\cdot}001088$ $h = \frac{0{\cdot}001088}{3{\cdot}208^2} = 0{\cdot}0000837$ (to calculate with the slide rule), thus, $x = 2{\cdot}0800837$. The exact value calculated up to 9 places is 2·08008382.

Example No. 70. $x = \sqrt[3]{11{\cdot}2}$ can be calculated likewise. The appproximation here is $x_0 = 2{\cdot}24$. This root exactly calculated gives 11·239424.

$x = \sqrt[3]{11{\cdot}2} = \sqrt[3]{11{\cdot}239424 - 0{\cdot}039424} = \sqrt[3]{11{\cdot}239424\ (1 - 0{\cdot}00351} = 2{\cdot}24\ (1 - 0{\cdot}00351)^{1/3}$. The binomal term is $(1 - h)^{1/3} = 1 - {}^1/_3h - {}^1/_9h^2 \ldots$, thus, $x = 2{\cdot}24\ (1 - 0{\cdot}00117 - 0{\cdot}00000136) = 2{\cdot}24 - 0{\cdot}00262 - 0{\cdot}000003 = 2{\cdot}23738$, more exactly (2·23737788).

§ 23. Circumference and Area of the Circle.

For the determination of the circumference we have the formula $\pi \times D$. Thus, setting of the middle index of scale *CF* to the initial or end mark of *DF* gives us the circumferences to any diameter set on scale *CF* on *DF*.

Example No. 71. Diameter $5\,^1/_4$ yard in decimals 5·25. Circumference 16·48 yards or, rounded up, $16\,^1/_2$ yards or 16 yards $1\,^1/_2$ foot.

For determining the area of circles we make use of the following procedure: On scales *A* and *B* we have the gauge points $0{\cdot}7854 = \frac{\pi}{4}$. Setting the end line of *B* to this gauge point on *A*, we can read the areas to any diameter set under the cursor line on scale *C* equally under the cursor line on scale *A*.

Example No. 72. Diameter 3 yards $1\,^1/_2$ foot. Area 9·6 square yards.

Example No. 73. Examine the accuracy of the following table:

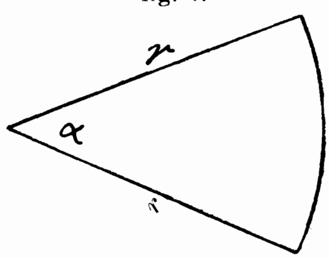
d	1	2	3	4	5	6	7	8	9
$\frac{\pi d^2}{4}$	0·7854	3·142	7·069	12·57	19·63	28·27	38·48	50·27	63·62

d	5	5·1	5·2	5·3	5·4	5·5	5·6	5·7	5·8	5·9
$\frac{\pi d^2}{4}$	19·63	20·43	21·24	22·06	22·90	23·76	24·63	25·52	26·42	27·34

Example No. 74. The cupola of an observatory has the outer diameter of 6 yards 9″. Determine its surface. The formula for determining the surface of the sphere being πD^2, we have here for the semi-sphere $\frac{1}{2}\pi \times 6{\cdot}25^2$. Transforming it into $2 \frac{\pi \times 6{\cdot}25^2}{4}$ we can set as follows: Final index of B to gauge point 0·7854, cursor line to 625 on scale C, final index of B under the cursor line, result opposite 2 of B on $A = 61{\cdot}3$ yards = 61 yards 1 foot.

§ 24. Calculations of Sectors, Measure of the Arc.

fig. 7.



Tracing from the center of a circle two radii as shown by figure 7, we get arc b and the sector limited by r, r and b. The arc between the radii represents the measure of the arc of angle α, its length determines the angle as precisely as do the indications of degrees, minutes and seconds. See fig. 7. Supposing the length of the radius to be 1 (dm., inch, foot etc.), the arc of the semi-circle will measure π units. For a circle of the radius 1 and, consequently, the diameter 2 we, therefore, have the following values:

Degrees	$360°$	$180°$	$90° \ldots$	$1°$	$1'$	$1''$
Measure of the Arc	2π	π	$\frac{1}{2}\pi$	$\frac{\pi}{180}$	$\frac{\pi}{10800}$	$\frac{\pi}{648000}$

In order to calculate rapidly, we make use of the reciprocals of these quotients for the degrees 57·3, for the minutes 3438 and for the seconds 206265. In the formulae degrees are designed as $\varrho°$, minutes as ϱ' and seconds as ϱ''.

Example No. 75. Determine $\alpha° = 134$ in the measure of the arc:

$$\text{a) } \alpha = \frac{134°}{\varrho°} = 2{\cdot}34;\ \text{b) } \alpha = \frac{8040}{\varrho'} = 2{\cdot}34;\ \text{c) } \alpha = \frac{482400}{\varrho''} = 2{\cdot}34.$$

Example No. 76. Convert $\alpha = 33°\ 46'\ 55''$ in the measure of the arc.

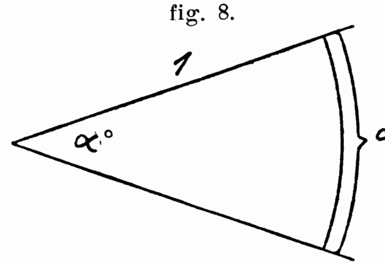
Solution: $\text{a) } \alpha = \frac{33°}{\varrho°} + \frac{46'}{\varrho'} + \frac{55''}{\varrho''} = 0{\cdot}576 + 0{\cdot}01338 + 0{\cdot}000267 = 0{\cdot}590.$

Example No. 77. An angle in the measure of the arc has 0·735. Determine its value in degrees.

Solution: a) $\alpha° = 0{\cdot}735 \times \varrho° = 42{\cdot}1°\ (= 42°\ 6')$; b) $\alpha' = 0{\cdot}735 \times \varrho' = 2526'\ (= 42°\ 6')$; c) $\alpha'' = 0{\cdot}735 \times \varrho'' = 151600'' = 42°\ 6'$.

The exact value will be $\alpha'' = 151605'' = 42°\ 6'\ 45''$.

fig. 8.



The measure of the Arc is used especially for calculating the arc and the area F of circular sector according to the formulae:

$b = \frac{2 r \pi \alpha°}{360}$, thus, in the measure of the arc

$$b = r\alpha = \frac{d}{2} \times \alpha;$$

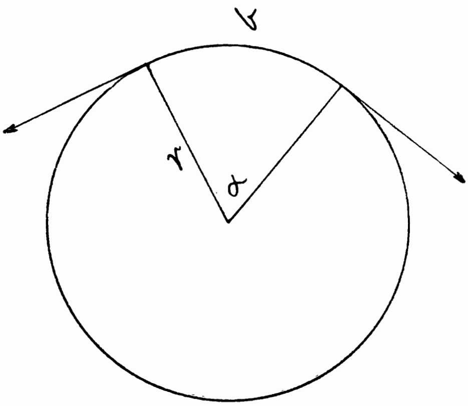
$$F = \frac{r^2 \pi \alpha°}{360} = \frac{r^2}{2} \times \alpha = \frac{d^2}{8} \times \alpha.$$

Example No. 78. A railway line describes a curve of which the radius of curvature is 430 yards. Lines traced from the initial and the final point of the curve form the angle $\alpha = 37\frac{1}{2}°$. Derive from these data the length of the curve.

Solution: $\alpha = \frac{37{\cdot}50°}{\varrho°} = \frac{2250'}{\varrho'} = \frac{135000''}{\varrho''} = 0{\cdot}654.$ This intermediate result need not be read, we can multiply it directly with $r = 430$ yards, thus, getting $b = 281{\cdot}4$ yards.

Example No. 79. The direction of a power shall be changed by a pulley of the diameter of 435 mm. by 65°. Determine the length of the transmission rope coming in contact with the pulley. See fig. 9.

fig. 9.



Solution: 247 mm. The procedure does not need any further explanation.

Example No. 80. For making funnels, a manufacturer employs met l sheets in form of circular sectors. Derive the weight of the metal pieces, supposing that a square meter of metal weighs 2·5 kilos.

r	125	310	25	150 mm.
α	160°	140°	220°	240°

r	225	555 mm.
α	180°	170°

Solution: The area will be $A = \frac{r^2}{2} \times \alpha$ and the weight $G = A \times 2{\cdot}5$. The lengths must be given in m., the areas in m.² and then we get the weight in kilos:

α	2·79	2·44	3·84	4·19	3·142	2·97	
A	0·0218	0·1173	0·0012	0·0472	0·0795	0·456	m.²
G	0·0545	0·293	0·0030	0·118	0·199	1·14	kg.

§ 25. Expressions of the Form $\frac{a}{b^2}$, $\frac{a^2}{b}$.

Example No. 81. An arc lamp has the intensity of $a = 1800$ candle powers. Determine the intensity of light, falling on a surface in the distance of $b = 1$ m.; 3·7 m.; 8·5 m.; 10 m.; 11 m.; 12 m. etc. The intensity is $\frac{a}{b^2}$. Setting b on C opposite a on A, gives the result sought at the initial or final index of C on A, thus, 1800; 131; 24·9; 18; 14·9; 12·5 M.

Example No. 82. In a circle we have a chord of $2a = 154$ mm. By its center pass various others, and the sections of them which fall within the sector be 70; 65; 60; 55 mm. Derive from these data the length of the other pieces of the chord in the other sector.

According to the theorem of the chords we have $a \times a = b \times x$ and $x = \frac{a^2}{b}$. Setting a on D with the cursor line, we get on A a^2. Bringing into coincidence with this value b on B, gives us the results sought at the initial or final index of B on A.

§ 26. Multiple Powers. The Horner Scheme.

The use of the log. log. scales has been fully explained on pages 16 etc., and for the determination of multiple powers and roots with the mantissae scale we have reference to the explanations given on page 16. As multiple powers are often made use of for the development of serial powers as shown in the following example, we shall give hereafter a practical device called Horner Scheme. Supposing that x is given in the measure of the arc sin. $x = x - \frac{1}{1 \times 2 \times 3} x^3 + \frac{1}{1 \times 2 \times 3 \times 4 \times 5} x^5 \ldots$ In the usual way we calculate, for instance, for $x = 50^\circ = 0{\cdot}873$; $\frac{x^3}{6} = \frac{0{\cdot}665}{6} = 0{\cdot}111$; $\frac{x^5}{120} = \frac{0{\cdot}506}{120} = 0{\cdot}004$; $\frac{x^7}{5040}$ can be omitted for the precision required in the present example.

With the Horner Scheme the calculation can be executed more rapidly in the following way. We set $y = ax + b$ and write the indices a and b in alignment. Multiplying a with x and setting the result under b, will give as result of the addition of these values $y = ax + b$.

$$\begin{array}{ccc} a & & b \\ & & ax \\ \hline a & \longrightarrow & \underline{|\, y} \end{array}$$

If we have $y = ax^2 + bx + c$, we procede in the same way. We set x on scale D as constant factor, multiply this value with a (result ax) and add it to b (result b_1), then multiplying b_1 by x, we get b_1x, which added to c gives y.

$$\begin{array}{ccccc} a & & b & & c \\ & & ax & & b_1x \\ \hline a & \longrightarrow & b_1 & \longrightarrow & \underline{|\, y} \end{array}$$

Proof $b_1 = ax + b$; $b_1x = ax^2 + bx$; $b_1x + c = ax^2 + bx + c$.

For $y = ax^3 + bx^2 + cx + d$ we have the scheme:

$$\begin{array}{ccccccc} a & & b & & c & & d \\ & & ax & & bx & & cx. \\ \hline a & \longrightarrow & b_1 & \longrightarrow & c_1 & \longrightarrow & \underline{|\, y} \end{array}$$

Rough estimation shows us for the above given example, that for $x = 0{\cdot}873$ the term $\frac{x^7}{5040}$ can be ignored.

$$y = \frac{1}{120}x^5 - \frac{1}{6}x^3 + x = 0{\cdot}00833\,x^5 + 0 \times x^4 - 0{\cdot}1667\,x^3 + 0 \times x^2 + 1 \times x + 0,$$

	0·00833	0	—0·1667	0	1	0
thus		0·00728	0·00636	—0·1400	—0·1222	0·766
	0·00833	0·00728	—0·1603	—0·1400	0·8778	0·766

Example No. 83. Calculate the values of the following angles. The results must be those, given opposite each angle.

x	10°	20°	30°	40°	50°	60°	70°	80°	90°
sin. x	0·1736	0·342	0·500	0·643	0·766	0·866	0·940	0·985	1·000

Example No. 84.

$$\cos. x = 1 - \frac{x^2}{1 \times 2} + \frac{x^4}{1 \times 2 \times 3 \times 4} - \frac{x^6}{1 \times 2 \times 3 \times 4 \times 5 \times 6} + \dots.$$

Calculate for the same angles the corresponding cosine values:

x	10°	20°	30°	40°	50°	60°	70°	80°
cos. x	0·985	0·940	0·866	0·766	0·643	0·500	0·342	0·1736

Proof a) $\cos. x^\circ = \sin. (90^\circ - x^\circ)$; b) $(\sin. x)^2 + (\cos. x)^2 = 1$.

Example No. 85. $e = 2{\cdot}718\,..$, then $e^x = 1 + x + \frac{x^2}{1 \times 2} + \frac{x^3}{1 \times 2 \times 3} + \frac{x^4}{1 \times 2 \times 3 \times 4} + \dots.$

Examine the following table:

x	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
e^x	1·105	1·221	1·350	1·492	1·649	1·822	2·014	2·226	2·460

Practical Examples for Using the Scales *LL 1, 2, 3.*

Example No. 86. In order to find the frictional resistance of transmision ropes, belts and brakes, we make use of the expression $e^{\mu\alpha}$. μ being a factor of friction, α the angle in the measure of the arc, $e = 2{\cdot}718$. In a handbook we find for $\mu = 0{\cdot}3$ the following table:

α°	36°	72°	108°	144°	153°	162°	171°	180°
α	0·628	1·257	1·885	2·51	2·67	2·83	2·98	3·14
$\mu\alpha$	0·188	0·377	0·566	0·754	0·801	0·849	0·896	0·943
$e^{\mu\alpha}$	1·21	1·46	1·76	2·12	2·23	2·34	2·45	2·57

Examine its precision.

For such problems, the slide rule does better services than any other calculating device. We bring C in alignment with $LL\ 3$, set cursor line to the different values $\mu\alpha$ read on scale C. According to the definition of the LL scales given on page 16, the results are not found on scales $LL\,3$ but on $LL\ 2$, as we have as exponent a decimal fraction.

Example No. 87. We design as hyperbolic function, expressions of the form of sine $x = \frac{e^x - e^{-x}}{2}$ and cos. hyp $x = \frac{e^x + e^{-x}}{2}$, e being as previously 2·718.

Also, for this example, we set C in alignment with $LL\ 3$, and as we have to determine the decimal fraction powers of e, we read the results not on scale $LL\ 3$, but on $LL\ 2$. The e^{-x} values are obtained by bringing the e^x values set on scale CI into coincidence with those of C, $e^{-x} = \frac{1}{e^x}$. Forming the differences d and the sums s cannot make any difficulty whatsoever, neither determining of the final results of sine x and cosine x.

x	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
e^x	1·1052	1·221	1·350	1·492	1·649	1·822	2·014	2·226	2·460	2·718
e^{-x}	0·9048	0·819	0·741	0·670	0·607	0·549	0·497	0·449	0·407	0·368
d	0·2004	0·402	0·609	0·822	1·042	1·273	1·517	1·777	2·053	2·350
s	2·0100	2·040	2·091	2·162	2·256	2·371	2·511	2·675	2·867	3·086
Sin. x	0·1002	0·201	0·304$_5$	0·411	0·521	0·636$_5$	0·758$_5$	0·888$_5$	1·026$_5$	1·175
Cos. x	1·0050	1·020	1·045$_5$	1·081	1·128	1·185$_5$	1·255$_5$	1·337$_5$	1·433$_5$	1·543

Example No. 88. The ratio of the contents of the barrel of an airpump to its recipient is 1 : 20. If the pressure in the interior of the recipient is one atmosphere, it will be $\frac{20}{21}$ atmospheres after one stroke and after n strokes $\frac{(20)^n}{21} =$ $= 0{\cdot}9524^n$ abstracting of the noxious space. We shall find the following values:

Strokes of barrel	20	40	60	80	100	120	140
Rarefaction of air	0·377	0·143	0·0535	0·0202	0·00760	0·00286	0·00108
Millimeters of mercury	286	108	40·7	15·3	5·78	2·18	0·821

160	180	200
0·00041	0·00015	?
0·31	0·11	?

Only the values of rarefaction for 20, 40 and 60 strokes can be read, setting the final index of scale B to ·9524 on $LL\,0$, the results can be read opposite 2, 4 and 6 of the right half of scale B. The further values must be calculated by means of the L scale as shown on page 16. The height of mercury in mm. results from the multiplication of the amounts of rarefaction by 760.

Example No. 89. Calculate the compound interest for $ 782.— at 4.75% for $n = 12$ years.

Formula $q = 1{\cdot}0475$, $n = 12$, $K =$ capital, $K \times q^n = E$ capital at the end of the period. We first have to determine the value of q^n, setting the initial index of C to 1·0475 on $LL\ 1$, as we have to raise to the power of 12, the result must be read not on scale $LL\ 1$ but on the scale above and will read as 1·745. This value must be set on scale D with the final index of C, and the result will appear at 782 of C on D, and is $ 1365.—.

Example No. 90. Determine the final capital for the same amount, same rate of interest and same number of years, but with charging of the interest semi-annually. Here we have to calculate the double number of periods, but only half the rate of interest. The result will be $ 1374.—, thus, some more odd dollars than for calculating with yearly interest.

Example No. 91. A person pays at the beginning of each year 15 times the amount of $ 430.—. How much will he possess including compound interest, if the bankers allow 4½%? The formula for similar calculations being $E = Vq\,\frac{q^n - 1}{q - 1}$, E being the final capital, V the amount paid each time, q the factor of interest, here 1·045. We find q^n setting the initial index of C opposite

1·045 of $LL\,1$, then the result can be read on $LL\,2$ 1·935. The result of the successive operations will be ƒ 9340.—.

Example No. 92. What annuity will be paid on a capital of ƒ 35000, the rate of interest being $4^1/_4$%, and the number of years 25? This example calculates after the formula $\frac{V \times (q^n - 1)}{q - 1}$ if the respective amounts are paid at the end of the year, and not at the beginning as in the preceding example. After having determined q^n as 2·831, we have the following formula $V =$
$= \frac{35000 \times 2{\cdot}831 \times 0{\cdot}0425}{1{\cdot}831} =$ ƒ 2300.—.

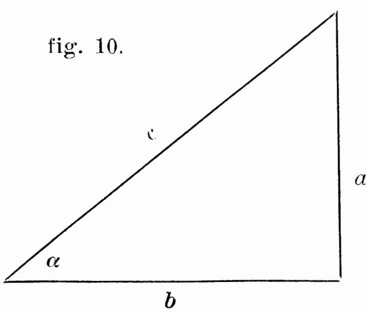
It will now be quite clear how to determine q^n, and the other operations can also not be doubtful in any way.

§ 27. The Use of the Slide-Rule for Trigonometrical Calculations

together with a brief explanation of trigonometrical functions.

In the rectangular triangles, the sides which enclose the right angle (fig. 10) are called cathetus, the opposite side is called hypotenuse (*c*). If to the cathetus a is opposed the acute angle α, we have the following determinations (see fig. 10).

fig. 10.



$$\sin.\ \alpha = \frac{\text{opposite cathetus}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos.\ \alpha = \frac{\text{adjacent cathetus}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{tg.}\ \alpha = \frac{\text{opposite cathetus}}{\text{adjacent cathetus}} = \frac{a}{b}$$

$$\text{ctg.}\ \alpha = \frac{\text{adjacent cathetus}}{\text{opposite cathetus}} = \frac{b}{a}$$

$$\sec.\ \alpha = \frac{\text{hypotenuse}}{\text{adjacent cathetus}} = \frac{c}{b}$$

$$\text{cosec.}\ \alpha = \frac{\text{hypotenuse}}{\text{opposite cathetus}} = \frac{c}{a}$$

An instruction for reading of the trigonometrical scales has been already given in the first part of this booklet, and we can limit ourselves in giving a range of examples.

Example No. 93. Determine the values of 5°, 10°, 15°, 20° up to 90°, then comparing results with the following table:

α	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
sin.	0·0872	0·1736	0·259	0·342	0·423	0·5	0·574	0·643	0·707	0·766	0·819	0·866

65°	70°	75°	80°	85°	90°
0·906	0·940	0·966	0·985	0·996	1

Example No. 94. Determine the angles for the sine values 0·1; 0·2; 0·3; and up to 1. Compare the values found with the table:

sine α	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
angle α	5°44′	11° 32′	17° 27′	23° 35′	30°	36° 50′	44° 30′	53° 10′	64° 10′	90°

Multiplications and Divisions of the Sines.

For the determination of the product $a \times$ sine α, if a and the angle α are given, we have the following method:

Set a on upper slide scale A and the initial or end line of sine scale into coincidence. The result of the multiplication then can be found opposite the value of sine α on the scale A.

Example No. 95. A workman has to pull up a barrel of oil on a board in inclined position. The weight of the barrel is $P = 75$ lb. Which will be the energy to be employed for the different inclinations of 5°, 10°, 15° up to 45°? The energy R will be : $P \times$ sine α, thus for our example:

α	5°	10°	15°	20°	25°	30°	35°	40°	45°	
R	6·54	13·03	19·42	25·7	31·7	37·5	43·0	48·2	53·0	lbs.

Example No. 96. As shown by Foucauld, a free oscillating pendulum deviates from the original direction each hour by $A = 15° \times$ sine φ, this latter value being the geographical latitude.

What will be the amount for a) San Salvador ($\varphi = 13°\ 42'$); b) Mexico ($\varphi = 19°\ 26'$); c) Charlottesville ($\varphi = 38°\ 2'$); d) Philadelphia ($\varphi = 39°\ 57'$); e) Montreal ($\varphi = 45°\ 30'$); f) for the most northern part of America ($\varphi = 73°\ 54'$)? The solution of this problem with the slide rule will be quite clear, the results will be the following:

Latitudes as above	a	b	c	d	e	f
Deviation A in decimals	3.55°	4.99°	9.24°	9.63°	10.70°	14.41°
Deviation A in degrees and min.	3° 33′	4° 59′	9° 14′	9° 38′	10° 42′	14° 25′

The Cosine.

We may presume that the formula for the determination of the cosines "cos. α = sine $(90° - \alpha)$" be known to every reader. If the angle α is given and its cosine sought, we first determine the complementary angle $90° - \alpha$ and then derive of this the sine. For angles of greater amplitude than 90° we have the formula:

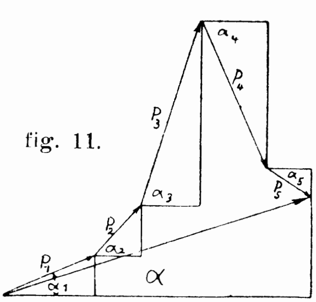
cos. $(180° - \alpha) = -$cos. α, cos. $(180° + \alpha) = -$cos α. cos. $(360° - \alpha) =$ $= +$ cos. α. Further cos. $(90° + \alpha) = -$sin. α, cos. $(270° + \alpha) = +$ sin. α, cos. $(-\alpha) = +$ cos α.

Example No. 97. Determine the value of cosines from 5°, 10°, 15° up to 90°. We shall get the same table as for example 94 page 44, only in inverse direction, as cos. 5° = sin. 85° = 0·996 etc.

Example No. 98. Determine cos. 117° 50′; cos. 192°; cos. 177°; cos. 290° 30′. The answer will be: cos. 117° 50′ = —sin. 27° 50′ = —0·467; cos. 192° = —cos. 12° = —sin. 78° = 0·978; cos. 290° 30′ = + sin. 20° 30′ = + 0·350; cos. 177° = = —sin. 87° = 0·999.

Example No. 99. A point is influenced by different forces as follows: $P_1 = 3{\cdot}52$ lbs.; $P_2 = 2{\cdot}34$; $P_3 = 6{\cdot}89$; $P_4 = 5{\cdot}66$; $P_5 = 1{\cdot}85$ lbs. Their direction gives with the x — axis the following angles: $\alpha_1 = 23°\ 10'$; $\alpha_2 = 48°$; $\alpha_3 = 73°$; $\alpha_4 = -65°$; $\alpha_5 = -32°$. Determine the energy which is composed of these different forces by the parallelogram of forces, and which is the angle it forms with the axis and what will be the respective projections on same?

fig. 11.



Solution: According to fig. 11 the horizontal projections are: P_1, cos. α_1 etc. its sum $x = P_1$ cos. $\alpha_1 + P_2$ cos. $\alpha_2 + P_3$ cos. $\alpha_3 + P_4$ cos. $\alpha_4 + P_5$ cos. $\alpha_5 = 3{\cdot}24 + 1{\cdot}57 + 2{\cdot}01 + 2{\cdot}39 + 1{\cdot}57 =$ $= 10{\cdot}78$ lbs. The vertical projections are: P_1 sin. α_1 etc., its sum $1{\cdot}38 + 1{\cdot}74 + 6{\cdot}59 - 5{\cdot}13 - 0{\cdot}98 =$ $= 3{\cdot}60$ lbs. The value of the resultant be: $R =$ $= \sqrt{x^2 + y^2} = \sqrt{116{\cdot}2 + 12{\cdot}96} = \sqrt{129{\cdot}6} = 11{\cdot}36$ lbs. The angle α which it forms with the x — axis, results from the proportion: cos. $\alpha = \frac{x}{R} = \frac{10{\cdot}78}{11{\cdot}36} = 0{\cdot}949$; thus, $\alpha = 9 -°072° = 18°$. For the angle β between R and the Y — axis we have

$\cos. \beta = \frac{y}{R} = \frac{3 \cdot 60}{11 \cdot 36} = 0 \cdot 317.$ β $90° - 18° 30' = 71° 30'$: The total amount of the angles α and β should be exactly 90°. The difference results from the inaccuracy in the determination of α. Whilst the determination of small angles with the sines gives precise results, this is not the case with the cosines, and results found in this way are not quite as precise.

The Functions of Cosecans and of Secans.

It results from the summary given on page 44 that the function of cosecans is the reciprocal value of the sine, secans the reciprocal value of the cosine. From this we get the following method of the determination of the cosecans:

Set α on scale S under the end line of A, and find opposite the initial line of S the value of cosec. α. Let the reader bear in mind, that cos. α 0° 34′ 23″ = 100 and that the values of the cosecans diminish with the increasing angle. Thus we have for cos. $\alpha = 5° 44'$ the value of 10 and for cos. α 90° = = the value of 1. The determination of the secans is made in the following way: If α is given, cos. α = sine $(90 - \alpha)$ and consequently sec. α = cosec. $(90° - \alpha)$.

Example No. 100. Establish a table of the functions sec. α and cosec. α to begin with 2° and to increase by 2° up to 10°. From 10° upwards, the intervals should be 10°. The table thus obtained will be the following:

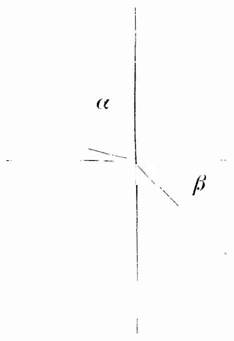
α	2°	4°	6°	8°	10°	20°	30°	40°	50°
cos. α	28·7	14·34	9·57	7·19	5·76	2·92	2·—	1·556	1·305
sec. α	1·001	1·002	1·006	1·010	1·015	1·064	1·155	1·305	1·556

α	60°	70°	80°	90°
cosec. α	1·155	1 064	1·015	1·000
sec. α	2·00	2·92	5·76	∞

The difficulties that may arise for the determination of the angles near to 0° and near to 90° will be discussed on page 48.

Example No. 101. (Fig. 12). A beam of light undergoes a certain refraction when entering into water. If we design the angle, the nonrefracted beam forms with the vertical line as α, the angle of the refracted beam as β, we get the proportion: $\frac{\sin. \alpha}{\sin. \beta} = n$. The value of n is for water 1·333, for other transparent mediums we have other values, as shown by the following table:

fig. 12.



	a.	b.	c.
Medium	Ether	Benzol	Carbonic disulphide
n	1·354	1·501	1·628

d.	e.
Flint-glass	Diamond
1·608	2·417

The above given formula is valid also in the case of the beam entering from the medium of greater optical density into the air. If angle β is too great, the beam cannot enter into the air and we have a total reflection. The critical angle β_o must have the aperture, that $\alpha = 90°$ and consequently sin. $\alpha = 1$. Calculate the critical angles for the media above given.

Solution: $\frac{1}{\sin. \beta_o} = n$; consequently β_o must be determined thus, that cos. $\beta_o = n$. This will give the following table:

Medium	a.	b.	c.	d.	e.
β_o	47° 40′	41° 50′	38°	38° 30′	24° 30′

For water $\beta_o = 48° 40'$. If for glass, we should have the same value or one a little more than 45°, the construction of total reflecting binocles should be impossible.

Tangent and Cotangent.

Between tg. α and ctg. α we have the relation:

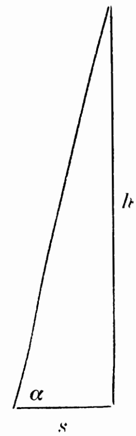
$$\text{tg. } \alpha \ \frac{1}{\text{ctg. } \alpha}, \quad \text{ctg } \alpha = \frac{1}{\text{tg. } \alpha}$$

Further we have the relations:

$$\begin{aligned}
\text{tg. } (90^\circ - \alpha) &= \text{ctg. } \alpha; & \text{ctg. } (90^\circ - \alpha) &= \text{tg } \alpha \\
\text{tg. } (180^\circ - \alpha) &= -\text{tg. } \alpha; & \text{ctg. } (180 - \alpha) &= -\text{ctg } \alpha \\
\text{tg. } (180^\circ + \alpha) &= \text{tg. } \alpha; & \text{ctg. } (180^\circ + \alpha) &= \text{ctg. } \alpha \\
\text{tg. } (360^\circ - \alpha) &= -\text{tg. } \alpha; & \text{ctg } (360^\circ - \alpha) &= -\text{ctg. } \alpha.
\end{aligned}$$

Examples, how to read the values of tangents, have been given already on page 15. Cotangential values are determined as follows: Cotangent α is brought into coincidence with the end line of scale D, the value of ctg. α then can be read on scale D opposite the initial line of T. The corresponding values must be brought into coincidence by means of the hair line of the double cursor.

fig. 13.



Determination of tg. α and ctg. α if α greater than 45°.

If α greater than 45°, $90^\circ - \alpha$ less than 45°. Thus tg. α = ctg. $(90^\circ - \alpha)$ and ctg. α = tg. $(90^\circ - \alpha)$. These values can be determined by means of the slide rule as explained before.

Example No. 102. Establish a table of tangential and of cotangential functions, progressing from 10 to 10°.

α	10°	20°	30°	40°	50°	60°	70°	80°
tg α	0·1763	0·364	0·577	0·839	1·192	1·732	2·75	5·67
ctg. α	5·67	2·75	1·732	1·192	0·839	0·577	0·364	0·1763

Example No. 103. An inhabitant of New York ascertains that on the date of June 22nd some trees in Central Park cast a shadow of the following length $s_1 = 7{\cdot}28$; $s_2 = 14{\cdot}24$; $s_3 = 9{\cdot}28$; $s_4 = 25{\cdot}2$; $s_5 = 30{\cdot}1$ feet. Derive the height of the trees from the above given data.

Solution: From fig 13 it will result that tg. $\alpha = \frac{h}{s}$, thus, $h = s - \text{tg. } \alpha$. In this formula s means the length of the shadow, α the angle of the incident sun beams. $\alpha = 90 - \varphi + \delta$. For New York $\varphi = 40^\circ\, 45'$, the 22nd June $\delta = 23^\circ\, 27'$. Thus, the formula for this day and for New York will be:

$\alpha = 90^\circ - 40^\circ\, 45' + 23^\circ\, 27' = 72^\circ\, 42'$. In order to get the final result, we must multiply the tangent of this angle, 3·21 by the length of the shadow, thus getting 23·4; 45·7; 29·8; 80·9; 96·6 feet respectively.

Determination of the Sines and Tangents of small angles.

The trigonometrical scales, we make use of, go down for the sines to $\alpha = 0^\circ\, 34'\, 23''$ and for the tangents to $5^\circ\, 42'\, 38''$. Within these limits the values of the sines and tangents show so close an approximation, that it is impossible to treat them separately with the precision the slide rule allows. We, therefore, calculate the tangents α between $0^\circ\, 34'$ and $5^\circ\, 43'$ with the sine scale. For lesser angles the coincidence is still closer. We can make use of the following formulae without any perceptible difference:

a) $\sin. \alpha \backsim \text{tg. } \alpha \backsim \frac{\alpha^\circ}{57{\cdot}3}$, if α is given in degrees,

b) $\sin. \alpha \backsim \text{tg. } \alpha \backsim \frac{\alpha'}{3438}$, if α is given in minutes,

c $\sin. \alpha \backsim \text{tg. } \alpha \backsim \frac{\alpha''}{206\,300}$, if α is given in seconds.

Cosecans and Cotangent of small Angles.

As these functions are the reciprocal values to sin. α and tg. α, we have the following formulae for approximation:

$$\text{cosec. } \alpha \backsim \text{ctg. } \alpha \backsim \frac{57{\cdot}3}{\alpha^{\circ}}, \text{ if } \alpha \text{ is given in degrees,}$$

$$\text{cosec. } \alpha \backsim \text{ctg. } \alpha \backsim \frac{3438}{\alpha'}, \text{ if } \alpha \text{ is given in minutes,}$$

$$\text{cosec. } \alpha \backsim \text{ctg. } \alpha \backsim \frac{206\,300}{\alpha''}, \text{ if } \alpha \text{ is given in seconds.}$$

Cosines and Secans for small Angles.

For the first approximation we can set cos. $\alpha \backsim$ sec. $\alpha \backsim 1$, if α is little, for instance, we read $\alpha = 5^{\circ}$ in a table of goniometric functions sec. $\alpha = 1{\cdot}003820$, cos. $\alpha = 0{\cdot}996195$. The following formulae for approximation are still more precise:

a) cos. $\alpha \backsim 1 - {}^{1}/_{2}$ sin.$^{2}\alpha$

b) sec. $\alpha \backsim 1 + {}^{1}/_{2}$ sin.$^{2}\alpha$.

Example No. 104. On the side of a railway we see a poster with the inscription inclined upwards 1:384; 1:275; 1:65; whilst on another with the inscription inclined downwards we read 1:305; 1:720; 1:145. What will be the incline of the railway line upwards and downwards?

Solution: For the first case we have tg. $\alpha = 1:384 = 0{\cdot}00260$. $\alpha = 206300 : 384 = 206300 \times 0{\cdot}00260 = 537'' = 8'\ 53''$. Further: $\alpha = 750'' = 12'\ 30''$; $\alpha = 3170'' = 52'\ 50''$; $\alpha = 676'' = 11'\ 16''$; $\alpha\ 286'' = 4'\ 46''$; $\alpha = 1422'' = 23'\ 42''$.

Example No. 105. The numbers 384, 275 etc. in the former example are measured horizontally. Which will be the difference of the measurement for 1 mile?

Solution: If x is the distance in miles, then we have cos. $\alpha = \frac{1}{x}$; x cos. $\alpha = 1$; $x = 1 :$ cos. $\alpha =$ sec. α. The amount sought is $y = x - 1 =$ sec. $\alpha - 1$. As sec $\alpha \backsim 1 + {}^{1}/_{2}$ sin.$^{2}\alpha$, we shall have $y = {}^{1}/_{2}$ sin.$^{2}\alpha$. Thus, we get the corresponding values of 0·00000339; 0·00000661; 0·0001184; 0·00000538; 0·000000965; 0·0000238 miles. In order to give a clear idea of the trifling amounts, we have to do with in this example, we transform them into inches, thus, getting 0·215; 0·419; 7·50; 0·341; 0·0611; 1·506 inches.