SUPPLEMENT TO MANUAL M-14

how to use MODEL 3 POWERLOG SLIDE RULES

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PICKETT
THE WORLD'S MOST ACCURATE SLIDE RULES

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SUPPLEMENT FOR MODEL 3

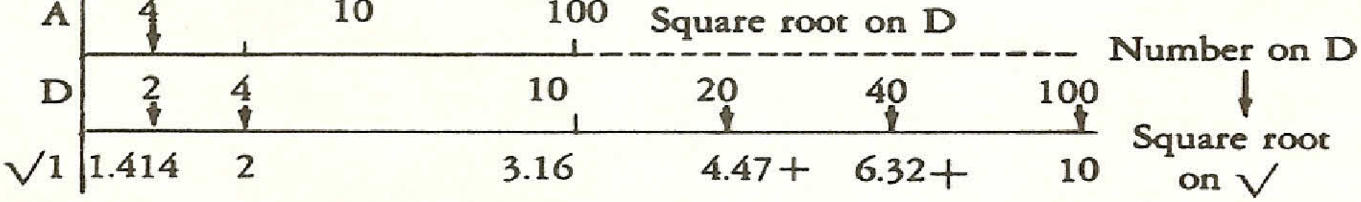
The Pickett Model 3 slide rule has scales not discussed in the manual for for Model 803 and similar slide rules. These scales are the following:

1. A "double length," or 20 inch scale for squares and square roots. This scale is labelled √, and is placed on the top of the stator on the "trig" side of the slide rule. The "right-hand portion" of this scale is placed under and "back-to-back" with the left-hand portion.
2. A "triple length," or 30-inch scale for cubes and cube roots. These scales are labelled ∛, and are placed on the bottom of the stator on the "trig" side of the slide rule.
3. A "double length," or 20-inch T scale for tangents of angles. This is placed on the slide with the other "trig" scales.
4. An extra 10 inch length of Log Log scale for which the range is 1.001 to 1.01. This scale is labelled LL0+, and it is placed on the top of Log Log side of the slide rule.
5. An extra 10 inch length of Log Log scale, for which the range is 0.999 to 0.99. This scale is labelled LL0—, and it is placed under and "back-to-back" with the LL0+ scale.

These scales provide an increase in the range, accuracy and convenience of the Model 803 in comparison with other slide rules which do not have these scales. The sections that follow tell why this is so and explain how to use these scales.

THE √SCALE; SQUARE ROOTS AND SQUARES

The relation between the D scale and the square root (or √ scale) is the same as the relation between the A scale and the D scale. For example, if the hairline is set on a number (say 4) on the A scale, the square root of 4 (or 2) is under the hairline on the D scale. Similarly, if the hairline is set on a number (say 4) on the D scale, the square root of 4 is under the hairline on the √ scale. (See figure).



Slide rules have had A and B scales for hundreds of years. On a "10 inch" slide rule the A scale is about 5 inches long. It ends in the middle of the rule. There is room to put another section, just like the first, on the right. In finding square roots, if the left index of the A scale is read as 1, the middle index is read as 10 and the right index is read as 100. The D scale is twice as long as the A scale. On a "10 inch" slide rule, it is about 10 inches long.

Now observe that the $\sqrt{\ }$scale is twice as long as the D scale. On a "10 inch" slide rule, it is about 20 inches long. The right hand half of the $\sqrt{\ }$scale is placed *under* the left hand half. This is not a "new" scale — it is also old historically.

To find square roots or squares, when a slide rule has a $\sqrt{\ }$scale, the D scale can be used *instead of the A scale,* and then the $\sqrt{\ }$scale is used *instead of the D scale.* Because these scales are longer than the A scale, greater accuracy is possible.

Rule: The square root of any number located on the D scale is found directly opposite it on the $\sqrt{\ }$ scale.

Reading the Scales. The square root scale on the top of the stator is an enlargement of the D scale itself. The D scale has been "stretched" to double its former length. Because of this the square root scale seems to be cut off or to end with the square root of 10, which is about 3.16. To find the square root of numbers greater than 10 the lower $\sqrt{\ }$scale is used. This is really the rest of the stretched D scale. The small figure 2 near the left end is placed beside the mark for 3.2, and the number 4 is found nearly two inches farther to the right. In fact, if 16 is located on the D scale, the square root of 16, or 4, is directly above it on the lower $\sqrt{\ }$scale.

In general, the square root of a number between 1 and 10 is found on the upper square root scale. The square root of a number between 10 and 100 is found on the lower square root scale. If the number has an odd number of digits or zeros (1, 3, 5, 7, . . .), the upper $\sqrt{\ }$ scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8, . . .), the lower $\sqrt{\ }$ scale is used. The first three (or in some cases even four) figures of a number may be set on the D scale, and the first three (or four) figures of the square root are read directly from the proper square root scale.

The table below shows the number of digits or zeros in the number N and its square root.

	ZEROS				or	DIGITS				
	U L	U L	U L	U	L	U L	U L	U L	U L	
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004} = 0.0632$, and $\sqrt{0.0004} = 0.02$.

EXAMPLES:

(a) Find $\sqrt{248.}$ Set the hairline on 248 of the D scale. This number has 3 (an *odd* number) digits. Therefore the figures in the square root are read from the upper $\sqrt{}$ scale as 1575. The result has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000.}$ Set the hairline on 563 of the D scale. The number has 6 (an *even* number) digits. Read the figures of the square root on the lower scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. Set the hairline on 1362 of the D scale. The number of *zeros* is 4 (an *even* number). Read the figures 369 on the lower scale. The result has 2 zeros, and is .00369.

***Squaring* is the opposite of finding the square root. Locate the number on the proper $\sqrt{}$ scale and with the aid of the hairline read the square on the D scale.**

EXAMPLES:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73 on the $\sqrt{}$ scale. On the D scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the $\sqrt{}$ scale. Find 394 above it on the D scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 628 was located on the lower of the $\sqrt{}$ scales, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the D scale read 645 above the 254 of the $\sqrt{}$ scale. The number has 3 zeros. Since 254 was located on the upper of the $\sqrt{}$ scales, the square has the odd number of digits, or 7. The result is 0.0000000645.

PROBLEMS	ANSWERS
1. $\sqrt{7.3}$	2.7
2. $\sqrt{73}$	8.54
3. $\sqrt{841}$	29
4. $\sqrt{0.062}$	0.249
5. $\sqrt{0.00000094}$	0.00097
6. $(3.95)^2$	15.6
7. $(48.2)^2$	2320
8. $(0.087)^2$	0.00757
9. $(0.00284)^2$	0.00000807
10 $(635000)^2$	4.03×10^{11}

THE $\sqrt[3]{}$ SCALES: Cube Roots and Cubes

At the top of the rule there is a cube root scale marked $\sqrt[3]{}$. It is a D scale which has been stretched to three times its former length, and then cut into three parts which are printed one below the other.

Rule. **The cube root of any number on the D scale is found directly above it on the $\sqrt[3]{}$ scales.**

Example: Find the $\sqrt[3]{8}$. Place the hairline of the indicator over the 8 on the D scale. The cube root, 2, is read above on the upper $\sqrt[3]{}$ scale.

Reading the scale: To find the cube root of any number between 0 and 10 the upper $\sqrt[3]{}$ scale is used. To find the cube root of a number between 10 and 100 the middle $\sqrt[3]{}$ scale is used. To find the cube root of a number between 100 to 1000 the lower $\sqrt[3]{}$ scale is used.

In general to decide which part of the $\sqrt[3]{}$ scale to use in locating the root, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, the upper $\sqrt[3]{}$ scale is used; if there are two digits in the left group, the middle $\sqrt[3]{}$ scale is used; if there are three digits, the lower $\sqrt[3]{}$ scale is used. Thus, the roots of numbers containing 1, 4, 7,... digits are located on the upper $\sqrt[3]{}$ scale; numbers containing 2, 5, 8,... digits are located on the middle $\sqrt[3]{}$ scale; and numbers containing 3, 6, 9,... digits are located on the lower $\sqrt[3]{}$ scale. The corresponding number of digits or zeros in the cube roots are shown in the table below and whether the upper, middle or lower section of the $\sqrt[3]{}$ scale should be used.

	ZEROS				or	DIGITS			
	U M L	U M L	U M L	U M	L	U M L	U M L	U M L	U M L
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

EXAMPLES:

(a) Find $\sqrt[3]{6.4}$. Set hairline over 64 on the D scale. Read 1.857 on the upper $\sqrt[3]{}$ scale.

(b) Find $\sqrt[3]{64}$. Set hairline over 64 on the D scale. Read 4 on the middle $\sqrt[3]{}$ scale.

(c) Find $\sqrt[3]{640}$. Set hairline over 64 on the D scale. Read 8.62 on the lower $\sqrt[3]{}$ scale.

(d) Find $\sqrt[3]{6{,}400}$. Set hairline over 64 on the D scale. Read 18.57 on the upper $\sqrt[3]{}$ scale.

(e) Find $\sqrt[3]{64{,}000}$. Set hairline over 64 on the D scale. Read 40 on the middle $\sqrt[3]{}$ scale.

(f) Find $\sqrt[3]{0.0064}$. Set hairline over 64 on the D scale. Read 0.1857 on the upper $\sqrt[3]{}$ scale.

(g) Find $\sqrt[3]{0.064}$. Set hairline over 64 on the D scale. Read 0.4 on the middle $\sqrt[3]{}$ scale.

If the number is expressed in standard form it can either be written in ordinary form or the cube root can be found by the following rule.

Rule: **Make the exponent of 10 a multiple of three, and locate the number on the D scale. Read the result on the $\sqrt[3]{}$ scale and multiply this result by 10 to an exponent which is 1/3 the former exponent of 10.**

EXAMPLES: Find the cube root of 6.9×10^{3}. Place the hairline over 6.9 on the D scale and read 1.904 on the upper $\sqrt[3]{}$ scale. Thus the desired cube root is 1.904×10^{1}. Find the cube root of 4.85×10^{7}. Express the number as 48.5×10^{6} and place the hairline of the indicator over 48.5 on the D scale. Read 3.65 on the middle $\sqrt[3]{}$ scale. Thus the desired cube root is 3.65×10^{2} or 365. Find the cube root of 1.33×10^{-4}. Express the number as 133×10^{-6} and place the hairline over 133 on the D scale. Read 5.10 on the lower $\sqrt[3]{}$ scale. The required cube root is 5.10×10^{-2}.

Cubes: To find the cube of a number, reverse the process for finding the cube root. Locate the number on the $\sqrt[3]{}$ scale and read the cube of that number on the D scale.

EXAMPLES:

(a) Find $(1.37)^3$. Set the indicator on 1.37 of the $\sqrt[3]{}$ scale. Read 2.57 on the D scale.

(b) Find $(13.7)^3$. The setting is the same as in example (a), but the D scale reading is 2570, or 1000 times the former reading.

(c) Find $(2.9)^3$ and $(29)^3$. When the indicator is on 2.9 of the $\sqrt[3]{}$ scale, the D scale reading is 24.4. The result for 29^3 is therefore 24,400.

(d) Find $(6.3)^3$. When the indicator is on 6.3 of the $\sqrt[3]{}$ scale, the D scale reading is 250.

PROBLEMS:	ANSWERS:
1. 2.45^3	14.7
2. 56.1^3	176,600
3. $.738^3$	.402
4. 164.5^3	4,451,000
5. $.0933^3$	.000812
6. $\sqrt[3]{5.3}$	1.744
7. $\sqrt[3]{71}$	4.14
8. $\sqrt[3]{815}$	9.34
9. $\sqrt[3]{.0315}$	.316
10. $\sqrt[3]{525,000}$	80.7
11. $\sqrt[3]{.156}$	.538

THE T SCALE: Tangents and Cotangents

Your Model No. 3 Slide Rule has a *double length* T scale. This simplifies computation involving tangents of angles, as you will see by studying the following instructions. The explanation and rule given in the Trig Manual is for a single length of T scale and may be ignored.

The T scale, together with the C or CI scales, is used to find the value of the tangent or cotangent of angles between 5.7° and 84.3°. Since $\tan x = \cot (90 - x)$, the same graduations serve for both tangents and cotangents. For example, if the indicator is set on the graduation marked 30, the corresponding reading on the C scale is .577, the value of tan 30°. This is also the value of cot 60°, since $\tan 30° = \cot (90° - 30°) = \cot 60°$. Moreover, $\tan x = 1/\cot x$; in other words, the tangent and cotangent of the same angle are reciprocals. Thus for the same setting, the reciprocal of cot 60°, or 1/.577, may be read on the CI scale as 1.732. This is the value of tan 60.

For double T scale

Rule. **Set the angle x on the T scale: (i) above the line if $5.7 \leq x \leq 45°$, and (ii) below the line if $45° \leq x \leq 84.3°$, and read the value of the tangent on the C scale, and cotangent on the CI scale.**

In case (i), the decimal point of the tangent is at the left of the first digit read on C. In case (ii), the decimal point of the tangent is at right of the first digit read on C. In case (i), the decimal point of the cotangent is at the right of the first digit read on CI. In case (ii), the decimal point is at the left of the first digit read on CI.

EXAMPLES:

(a) Find tan 14.7° and cot 14.7°. Set indicator over 14.7 on upper T scale. Read tan 14.7° = 0.262 on C, and cot 14.7° = 3.81 on CI.

(b) Find tan 72.3° and cot 72.3°. Set indicator over 72.3 on lower T scale Read tan 72.3° = 3.13 on C and cot 72.3 = 0.319 on CI.

PROBLEMS:	ANSWERS:
1. tan 18.6°	.337
2. tan 66.4°	2.29
3. cot 31.7°	1.619
4. cot 83.85°	.1078
5. $\tan \theta = 1.173$	$\theta = 49.55°$
6. $\cot \theta = .387$	$\theta = 68.84°$

THE EXTRA SECTIONS OF THE LOG LOG SCALE

Model 3 provides an extra section of Log Log scales. This section is used in exactly the same way as the rest of the LL scales (LL1, LL2, LL3) and requires no additional explanation. Note that it is possible to set numbers near 1 to great accuracy; thus 1.00333, which is a six figure number, is easily set. An example which illustrates the added convenience of having this scale is given below.

Example 1. Find $1.0261^{0.342}$. Set the left index of the C scale opposite 1.0261 of LL1+. Move the hairline over 342 of C, and read the result on the LL0+ scale as 1.00886.

Note that if the LL0+ scale were not on the slide rule, the *answer could not be read directly*. In that case (that is, with slide rules in which the lower bound of the range of the LL scales is 1.01 instead of 1.001), the result may be computed by using logarithms, but with less accuracy.

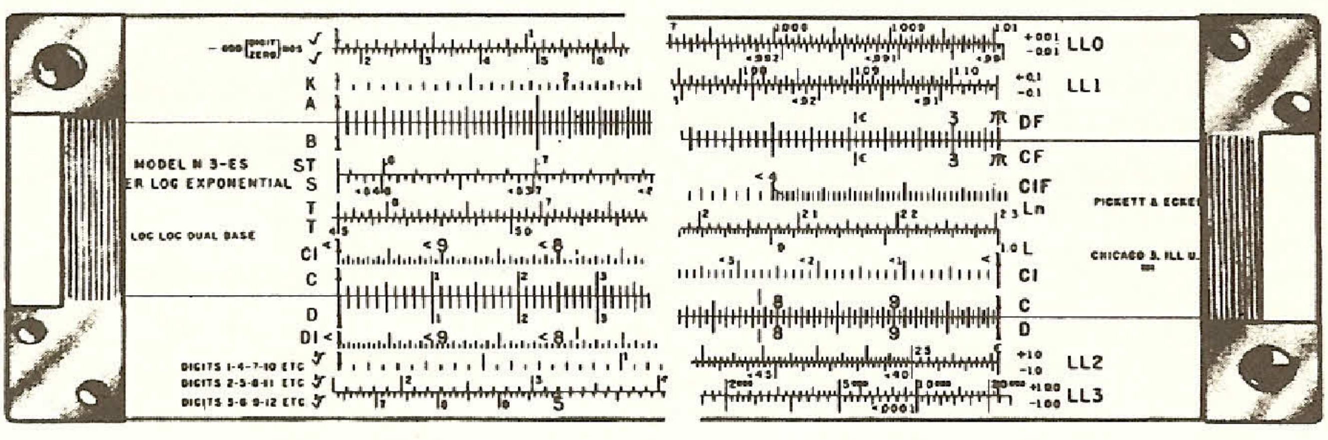
Thus, set the hairline over 1.0261 of the LL1+ scale, and the left index of the C scale under the hairline. The reading on the D scale gives $\log_e 1.0261$, which is 0.0258. Move the hairline over 0.342 of C (which multiplies 0.0258 by 0.342), and read 0.0088 on the D scale. This is $\log_e 1.261^{0.342}$. Now $\log_e (1 + X) = X$, approximately, if X is sufficiently small. In this example, let $X = 0.0088$ (which is small), and then the number, or $1 + X$, is 1.0088.

With Model 3 it is unnecessary to resort to this longer procedure.

The readings on the LL0– scale are reciprocals of the readings on the LL0+ scale. This scale extends the LL1– scale from 0.99 to 0.999 which is, of course, much closer to 1. This scale is used in the same way that the LL1–, LL2–, and LL3 scales are used. No additional explanation beyond that given in the manual for these scales should be needed.

Model 3 POWERLOG EXPONENTIAL LOG LOG *Dual-Base* SPEED RULE

FRONT BACK



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