how to use LOG LOG SLIDE RULES

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PICKETT
THE WORLD'S MOST ACCURATE SLIDE RULES

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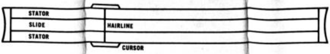
KNOW YOUR SLIDE RULE

To simplify explanations, the parts of your slide rule and their proper names are shown below. All references in these instructions will be in accordance with this identification.



HOW TO READ THE SCALES

The scale labeled C (on the slide) and the scale D (on the stator bar) are used most frequently. These two scales are exactly alike. The total length of these scales has been separated into many smaller parts by fine lines called "graduations."

Some of these lines on the D scale have large numerals (1, 2, 3, etc.) printed just below them. These lines are called *primary* graduations. On the C scale the numerals are printed above the corresponding graduations. A line labeled 1 at the left end is called the *left* index. A line labeled 1 at the right end is called the *right* index.

PRIMARY GRADUATIONS

Next notice that the distance between 1 and 2 on the D scale has been separated into 10 parts by shorter graduation lines. These are the *secondary* graduations. Each of the spaces between the larger numerals 2 and 3, between 3 and 4, and between the other primary graduations is also sub-divided into 10 parts. Numerals are not printed beside these smaller secondary graduations because it would crowd the numerals too much.

SECONDARY GRADUATIONS

When a number is to be located on the D scale, the *first* digit is located by use of the *primary* graduations. The *second* digit is located by use of the *secondary* graduations. Thus when the number 17 is located, the 1 at the left index represents the 1 in 17. The 7th secondary graduation represents the 7. When 54 is to be located, look first for primary graduation 5, and then for secondary graduation 4 in the space immediately to the right.

To find 173 on the D scale, look for primary division **1** (the left index), then for secondary division **7** (numbered) then for smaller subdivision **3** (not numbered, but found as the **3**rd very short graduation to the right of the longer graduation for **7**).

Similarly, 149 is found as the 9th small graduation mark to the right of the 4th secondary graduation mark to the right of primary graduation **1.**

To find 246, look for primary graduation **2,** then for the 4th secondary graduation after it (the 4th long line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since 3/5 = 6/10, then the third graduation, marking *three fifths,* is at the same point as *six tenths* would be.

MULTIPLICATION

Numbers that are to be multiplied are called factors. The result is called the product. Thus, in the statement 6 × 7 = 42, the numbers 6 and 7 are factors, and 42 is the product.

EXAMPLE: Multiply 2 × 3

Setting the Scales: Set the left index of the C scale on 2 of the D scale. Find 3 on the C scale, and below it read the product, 6 on the D scale.

Rule for Multiplication: **Over one of the factors on the D scale, set the index of the C scale. Locate the other factor on the C scale, and directly below it read the product on the D scale.**

EXAMPLE. Multiply 2.34 × 36.8

Estimate the result: First note that the result will be roughly the same as 2 × 40, or 80; that is, there will be two digits to the left of the decimal point. Hence, we can ignore the decimal points for the present and multiply as though the problem was 234 × 368.

Set the Scales: Set the left index of the C scale on 234 of the D scale. Find 368 on the C scale and read product 861 on the D scale.

DIVISION

In mathematics, division is the opposite or *inverse* operation of multiplication. In using a slide rule this means that the process for multiplication is reversed. To help in understanding this statement, set the rule to multiply 2×4. Notice the result 8 is found on the D scale under 4 of the C scale. Now to divide 8 by 4 these steps are reversed. First find 8 on the D scale, set 4 on the C scale over it, and read the result 2 on the D scale under the index of the C scale.

Rule for Division: **Set the *divisor* (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.**

EXAMPLES:

(a) Find $63.4 \div 3.29$. The quotient must be near 20, since $60 \div 3 = 20$. Set indicator on 63.4 of the D scale. Move the slide until 3.29 of the C scale is under the hairline. Read the result 19.27 on the D scale at the C index.

DECIMAL POINT LOCATION

In many, perhaps a majority, of the problems met in genuine applications of mathematics to practical affairs, the position of the decimal point in the result can be determined by what is sometimes called "common sense." There is usually only one place for the decimal point in which the answer is "reasonable" for the problem. Thus, if the calculated speed in miles per hour of a powerful new airplane comes out to be 4833, the decimal point clearly belongs between the 3's, since 48 m.p.h. is too small, and 4833 m.p.h. is too large for such a plane. In some cases, however, the data are such that the position of the point in the final result is not easy to get by inspection.

Another commonly used method of locating the decimal point is by estimation or approximation. For example, when the slide rule is used to find 133.4×12.4, the scale reading for the result is 1655, and the decimal point is to be determined. By rounding off the factors to 133.0×10.0, one obtains 1330 by mental arithmetic. The result would be somewhat greater than this but certainly contains four digits on the left of the decimal point. The answer, therefore, must be 1655.

PROPORTION

Problems in proportion are very easy to solve. First notice that when the index of the C scale is opposite 2 on the D scale, the ratio 1 : 2 or ½ is at the *same time* set for all other opposite graduations; that is, 2 : 4, or 3 : 6, or 2.5 : 5, or 3.2 : 6.4, etc. It is true in general that for any setting the numbers for *all pairs of opposite graduations have the same ratio.*

Rule: **Set *a* on the C scale opposite *b* on the D scale. Under *c* on the C scale read *x* on the D scale.**

EXAMPLE: Find x if $\frac{3}{4} = \frac{5}{x}$.

Set 3 on C opposite 4 on D. Under 5 on C read 6.67 on D.

Rule: **In solving proportions, keep in mind that the position of the numbers as set on the scales is the same as it is in the proportion written in the form $\frac{a}{b} = \frac{c}{d}$**

SPECIAL GRADUATIONS—CIRCULAR MEASURE

A special graduation on the C scale at $\pi/4 = 0.7854$ is especially convenient in finding the area or the diameter of a circle, and in similar problems. Since $A = (\pi/4)\,d^2$, where d is the diameter, the following rule applies.

Rule: To find the area (A) of a circle whose diameter d is known, set the right index of the C scale over d on the D scale. Move the hairline over .7854 mark on righthand section of B scale. Read area under hairline on A scale.

Rule: To find the diameter of a circle whose area A is known, set hairline over area on A scale (See rule regarding section of A scale to use). Move slide until .7854 mark on righthand section of B scale is under hairline. Read diameter on D scale below right index of C scale.

Pi (π) is shown on both C and D scales at 3.1416.

A special graduation on the C and D scales at 57.3 is indicated by a small R. Since one radian=57.3° (approximately), this R graduation is useful in changing radian measure to degree measure, and conversely.

Rule: To convert radians to degrees, set an index of the C scale to the number of radians on D. Under R on C read the number of degrees on D. To convert degrees to radians, set R on C over the number of degrees on D. At the index of C read the number of radians on D.

THE CI AND DI SCALES

The CI scale on the slide is a C scale which *increases from right to left.* It may be used for finding reciprocals. When any number is set under the hairline on the C scale its reciprocal is found under the hairline on the CI scale, and conversely.

EXAMPLES:

(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.

(b) Find 1/60.5. Set 60.5 on C. Read .0165 directly above on CI. Or, set 60.5 on CI, read .0165 directly below on C.

The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times 1/b$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule.

EXAMPLES:

(a) Find $13.6 \div 87.5$. Consider this as $13.6 \times 1/87.5$. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find $72.4 \div 1.15$. Consider this as $72.4 \times 1/1.15$. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 63.0 under the hairline on the D scale.

An important use of the CI scale occurs in problems of the following type.

EXAMPLE: Find $\frac{13.6}{4.13 \times 2.79}$ This is the same as $\frac{13.6 \times (1/2.79)}{4.13}$.

Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the settings more readily. Also, it is sometimes easier to get $a \times b$ by setting the hairline on a, pulling b on the CI scale under the hairline, and reading the result on the D scale under the index.

The DI scale (inverted D scale) below the D scale corresponds to the CI scale on the slide. Thus the D and DI scales together represent reciprocals.

THE CF/π AND DF/π SCALES

When π on the C scale is opposite the right index of the D scale, about half the slide projects beyond the rule. If this part were cut off and used to fill in the opening at the left end, the result would be a "folded" C scale, or CF scale. Such a scale is printed at the top of the slide. It begins at π and the index is near the middle of the rule. The DF scale is similarly placed. Any setting of C on D is automatically set on CF and DF. Thus if 3 on C is opposite 2 on D, then 3 on CF is also opposite 2 on DF. The CF and DF scales can be used for multiplication and division in exactly the same way as the C and D scales.

The most important use of the CF and DF scales is to avoid resetting the slide. If a setting of the indicator cannot be made on the C or D scale, it can be made on the CF or DF scale.

EXAMPLES:

(a) Find 19.2×6.38. Set left index of C on 19.2 of D. Note that 6.38 on C falls outside the D scale. Hence, move the indicator to 6.38 on the CF scale, and read the result 122.5 on the DF scale. Or set the index of CF on 19.2 of DF. Move indicator to 6.38 on CF and read 122.5 on DF.

(b) Find $\frac{8.39 \times 9.65}{5.72}$ Set 5.72 on C opposite 8.39 on D. The indicator cannot be moved to 9.65 of C, but it can be moved to this setting on CF and the result, 14.15, read on DF. Or the entire calculation may be done on the CF and DF scales.

***Rule:* If the diameter of a circle is set on D, the circumference may be read immediately on DF, and conversely.**

THE CIF SCALE

The CIF scale is a folded CI scale. Its relationship to the CF and DF scales is the same as the relation of the CI scale to the C and D scales.

EXAMPLES:

(a) Find 68.2×1.43. Set the indicator on 68.2 of the D scale. Observe that if the left index is moved to the hairline the slide will project far to the right. Hence merely move 14.3 on CI under the hairline and read the result 97.5 on D at the C index.

(b) Find $2.07 \times 8.4 \times 16.1$. Set indicator on 2.07 on C. Move slide until 8.4 on CI is under hairline. Move hairline to 16.1 on C. Read 280 on D under hairline. Or, set the index of CF on 8.4 of DF. Move indicator to 16.1 on CF, then move slide until 2.07 on CIF is under hairline. Read 280 on DF above the index of CF. Or set 16.1 on CI opposite 8.4 on D. Move indicator to 2.07 on C, and read 280 on D. Although several other methods are possible, the first method given is preferable.

THE A AND B SCALES: Square Roots and Squares

When a number is multiplied by itself the result is called the *square* of the number. Thus 25 or 5×5 is the square of 5. The factor 5 is called the *square root* of 25. Similarly, since $12.25 = 3.5 \times 3.5$, the number 12.25 is called the square of 3.5; also 3.5 is called the square root of 12.25. Squares and square roots are easily found on a slide rule.

Square Roots: To find square roots the A and D scales or the B and C scales are used.

***Rule:* The square root of any number located on the A scale is found below it on the D scale.**

Also, the square root of any number located on the B scale (on the slide) is found on the C scale (on the reverse side of the slide).

EXAMPLES: Find the $\sqrt{4}$. Place the hairline of the indicator over 4 on the left end of the A scale. The square root, 2, is read below on the D scale. Similarly the square root of 9 (or $\sqrt{9}$) is 3, found on the D scale below the 9 on the left end of the A scale.

Reading the Scales: The A scale is a contraction of the D scale itself. The D scale has been shrunk to half its former length and printed twice on the same line. To find the square root of a number between 0 and 10 the left half of the A scale is used (as in the examples above). To find the square root of a number between 10 and 100 the right half of the A scale is used. For example, if the hairline is set over 16 on the right half of the A scale (near the middle of the rule), the square root of 16, or 4, is found below it on the D scale.

The table below shows the number of digits or zeros in the number N and its square root, and also whether right or left half of the A scale should be used.

	ZEROS				or	DIGITS				
	L R	L R	L R	L	R	L R	L R	L R	L R	
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004} = 0.0632$, and $\sqrt{0.0004} = 0.02$.

Examples:

(a) Find $\sqrt{248}$. This number has 3 (an *odd* number) digits. Set the hairline on 248 of the left A scale. Therefore the result on D has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. The number has 6 (an *even* number) digits. Set the hairline on 563 of the right A scale. Read the figures of the square root on the D scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. The number of *zeros* is 4 (an *even* number.) Set the hairline on 1362 of the right half of the A scale. Read the figures 369 on the D Scale. The result has 2 zeros, and is .00369.

If the number is written in standard form, the following rule may be used. If the exponent of 10 is an even number, use the left half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the original. If the exponent of 10 is an odd number, move the decimal point one place to the right and decrease the exponent of 10 by one, then use the right half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the reduced exponent. This rule applies to either positive or negative exponents of 10.

Examples:

(1) Find the square root of 3.56×10^4. Place hairline of indicator on 3.56 on the left half of the A scale and read 1.887 on the D scale. Then the square root is $1.887 \times 10^2 = 188.7$.

(2) Find the square root of 7.43×10^{-5}. Express the number as 74.3×10^{-6} Now place the hairline of the indicator over 74.3 on the right half of the A scale and read 8.62 on the D scale. Then the desired square root is 8.62×10^{-3}.

All the above rules and discussion can be applied to the B and C scales if it is more convenient to have the square root on the slide rather than on the body of the rule.

***Squares:* *To find the square of a number, reverse the process* for finding the square root. Set the indicator over the number on the D scale and read the square of that number on the A scale; or set the indicator over the number on the C scale and read the square on the B scale.**

THE K SCALE: Cube Roots and Cubes

Just below the D scale on the back of the rule is a scale marked with the letter K; this scale may be used in finding the cube or cube root of any number.

***Rule:* The cube root of any number located on the K scale is found directly above on the D scale.**

Example: Find the $\sqrt[3]{8}$. Place the hairline of the indicator over the 8 at the left end of the K scale. The cube root, 2, is read directly above on the D scale.

Reading the scales: The cube root scale is directly below the D scale and is a contraction of the D scale itself. The D scale has been shrunk to one third its former length and printed three times on the same line. To find the cube root of any number between 0 and 10 the left third of the K scale is used. To find the cube root of a number between 10 and 100 the middle third is used. To find the cube root of a number between 100 and 1000 the right third of the K scale is used to locate the number.

In general to decide which part of the K scale to use in locating a number, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, the left third of the K scale is used; if there are two digits in the left group, the middle third of the K scale is used; if there are three digits, the right third of the K scale is used. In other words, numbers containing 1, 4, 7, . . .digits are located on the left third; numbers containing 2, 5, 8, ··· digits are located on the middle third; and numbers containing 3, 6, 9, ··· digits are located on the right third of the K scale. The corresponding number of digits or zeros in the cube roots are shown in the table below and whether the left, center or right section of the K scale should be used.

	ZEROS				or	DIGITS			
	L C R	L C R	L C R	L C	R	L C R	L C R	L C R	L C R
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

Examples:

(a) Find $\sqrt[3]{6.4}$. Set hairline over 64 on the left most third of the K scale. Read 1.857 on the D scale.

(b) Find $\sqrt[3]{64}$. Set hairline over 64 on the middle third of the K scale. Read 4 on the D scale.

(c) Find $\sqrt[3]{640}$. Set hairline over 64 on the right most third of the K scale. Read 8.62 on the D scale.

(d) Find $\sqrt[3]{6,400}$. Set hairline over 64 on the left third of the K scale. Read 18.57 on the D scale.

(e) Find $\sqrt[3]{64,000}$. Set hairline over 64 on the middle third of the K scale. Read 40 on the D scale.

(f) Find $\sqrt[3]{0.0064}$. Use the left third of the K scale, since the first group of three, or 0.006, has only one non-zero digit. The D scale reading is then 0.1857.

(g) Find $\sqrt[3]{0.064}$. Use the middle third of the K scale, reading 0.4 on D.

If the number is expressed in standard form it can either be written in ordinary form or the cube root can be found by the following rule.

Rule: Make the exponent of 10 a multiple of three, and locate the number on the proper third of the K scale. Read the result on the D scale and multiply this result by 10 to an exponent which is ⅓ the former exponent of 10.

Cubes: To find the cube of a number, reverse the process for finding cube root. Locate the number on the D scale and read the cube of that number on the K scale.

The L Scale: LOGARITHMS

The L scale represents logarithms to the base 10, or common logarithms. The logarithm of a number is the exponent to which a given base must be raised to produce the number. For example, Log 10^2 = 2.00; Log 10^3 = 3.00, etc. A logarithm consists of two parts. The *characteristic* is the part on the left of the decimal point. The *mantissa* is the decimal fraction part on the right of the decimal point. The L scale is used for finding the mantissa of the logarithm (to the base 10) of any number. The mantissa of the logarithm is the same for any series of digits regardless of the location of the decimal point.

The position of the decimal point in the given number determines the characteristic of the logarithm, and conversely. The following rules apply in determining the characteristic.

1. For 1, and all numbers greater than 1, the characteristic is one less than the number of places to the left of the decimal point in the given number.
2. For numbers smaller than 1, that is for decimal fractions, the characteristic is negative. Its numerical value is one more than the number of zeros between the decimal point and the first significant figure in the given number.

The application of these rules is illustrated by the following chart:

Digits to Left of Decimal Point								
Digits in Number	1	2	3	4	5	6	7	8
Characteristic	0	1	2	3	4	5	6	7

Zeros to Right of Decimal Point*									
Zeros in Number	0	1	2	3	4	5	6	7	8
Characteristic	-1	-2	-3	-4	-5	-6	-7	-8	-9

*Note: Count only zeros between decimal point and first significant figure.

Rule: Locate the number on the D scale (when L scale is on top or bottom stator), and read the mantissa of its logarithm (to the base 10) on the L scale. Determine the characteristic. If the L scale is on the slide, use the C scale instead of the D scale.

Example: Find the logarithm of 425.

Set the hairline over 425 on the D scale. Read the mantissa of the logarithm (.628) on the L scale. Since the number 425 has 3 digits, the characteristic is 2 and the logarithm is 2.628.

If the logarithm of a number is known, the number may be found by reversing the above process. The characteristic is ignored until the decimal point is to be placed in the number.

Example: Find x, if log x = 3.248.

Set the hairline over 248 on the L scale. Above it read the number 177 on the D scale. Since the characteristic is 3, there are 4 digits in the number, x = 1770.

Example: Find the logarithm of .000627

Opposite 627 on the D scale find .797 on the L scale. Since the number has 3 zeros, the characteristic is –4 and the logarithm is –4 + .797 and is usually written 6.797–10.

Note that the mantissa of a logarithm is always positive but the characteristic may be either positive or negative. In computations, negative characteristics are troublesome and frequently are a source of error. It is customary to handle the difficulty by not actually combining the negative characteristic and positive mantissa. For example, if the characteristic is –4 and the mantissa is .797, the logarithm may be written 0.797–4. This same number may also be written 6.797–10, or 5.797–9. and in other ways as convenient. In each of these forms if the integral parts are combined, the result is –4. Thus 0–4 = –4; 6. –10 = –4; 5. –9 = –4. The form which shows that the number 10 is to be subtracted is the most common.

Examples:

Log 1 = 10.000–10
Log .4 = 9.602–10
Log .0004 = 6.602–10

The S. T. ST Scales: TRIGONOMETRY

The commonest use of trigonometric function values is in connection with the measurement of right triangles. In this application the number x is the measure of one of the acute angles. The *unit* of measure may be either the *radian* or the angular *degree*. In practical work the degree is used much more frequently than the radian. Slide rule scales are therefore graduated in *degrees* and decimal fractions of degrees.

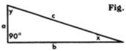


Fig. 14

In Table 1 the numeral 0.5 expresses radians. The corresponding number of degrees is $0.5 \times 180/\pi = 28.6$. The function value is the same as before, and it is customary to write sin 28.6° = 0.479. For right triangles described by the notation used in Fig. 14, the value sin x is equal to the quotient obtained by dividing the length a by the length c; that is, $\sin x = a/c$. If any two of the three numbers a, c, and $\sin x$ are known, the third can be found by a simple slide rule computation. The list of basic relations for right triangles is as follows.

$\sin x = a/c$, $\quad \operatorname{cosec} x = c/a$,
$\cos x = b/c$, $\quad \sec x = c/b$,
$\tan x = a/b$, $\quad \cot x = b/a$.

THE S SCALE: Sines and Cosines

The scale marked S is used in finding the approximate sine or cosine of any angle between 5.7 degrees and 90 degrees, Since $\sin x = \cos(90-x)$, the same graduations serve for both sines and cosines. Thus sin 6° = cos (90°–6°) = cos 84°. The numbers printed at the right of the longer graduations are read when sines are to be found. Those printed at the left are used when cosines are to be found. On the slide rule, angles are divided decimally instead of into minutes and seconds. Thus sin 12.7° is represented by the 7th small graduation to the right of the graduation marked 78|12.

Sines (or cosines) of all angles on the S scale have no digits or zeros—the decimal point is at the left of figures read from the C (or D) scale.

***Rule:* To find the sine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read sines from left to right and the numbers to the right of the graduation are for sines). Read the sine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the sine can also be read on the D scale, and the mantissa of the logarithm of the sine (log sin) may be read on the L scale.**

EXAMPLE:

(1) Find sin 15° 30′ and log sin 15° 30′. Set left index of C scale over left index of D scale. Set hairline on 15.5° (i.e., 15°30′) on S scale. Read sin 15.5° = .267 on C or D scale. Read mantissa of log sin 15.5° = .427 on L scale. According to the rule for characteristics of logarithms, this would be 9.427 − 10.

Rule: To find the cosine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read cosines from right to left and the numbers to the left of the graduation **are** for cosines). Read the cosine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the cosine can also be read on the D scale, and the mantissa of the cosine (log cos) may be read on the L scale.

EXAMPLE:

(1) Find cos 42° 15′ and log cos 42° 15′. Set left index of C scale over left index of D scale. Set hairline on 42.25° (i.e., 42° 15′) on S scale. Read cos 42.25° = .740 on C or D scale. Read mantissa of log cos 42.25° = .869 on L scale. According to rule for characteristics of logarithms, this would be 9.869 − 10.

Finding the Angle

If the value of trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C scale, and the angle itself read on the S scale.

THE T SCALE: Tangents and Cotangents

The T scale, together with the C or CI scales, is used to find the value of the tangent or cotangent of angles between 5.7° and 84.3°. Since tan x = cot (90−x), the same graduations serve for both tangents and cotangents. For example, if the indicator is set on the graduation marked 60|30, the corresponding reading on the C scale is .577, the value of tan 30°. This is also the value of cot 60°, since tan 30° = cot (90° − 30°) = cot 60°. Moreover, tan x = 1/cot x; in other words, the tangent and cotangent of the same angle are reciprocals. Thus for the same setting, the reciprocal of cot 60°, or 1/.577, may be read on the CI scale as 1.732. This is the value of tan 60. These relations lead to the following rule.

Rule: Set the angle value on the T scale and read

(i) tangents of angles from 5.7° to 45° on C,
(ii) tangents of angles from 45° to 84.3° on CI,
(iii) cotangents of angles from 45° to 84.3° on C
(iv) cotangents of angles from 5.7° to 45° on CI.

If the slide is set so that the C and D scales coincide, these values may also be read on the D scale. Care must be taken to note that the T scale readings for angles between 45° and 84.3° *increase from right to left.*

In case (i) above, the tangent ratios are all between 0.1 and 1.0; that is, the decimal point is at the left of the number as read from the C scale.

In case (ii), the tangents are greater than 1.0, and the decimal point is placed to the right of the first digit as read from the CI scale. For the cotangent ratios in cases (iii) and (iv) the situation is *reversed.* Cotangents for angles between 45° and 84.3° have the decimal point at the left of the number read from the C scale. For angles between 5.7° and 45° the cotangent is greater than 1 and the decimal point is to the right of the first digit read on the CI scale. These facts may be summarized as follows.

Rule: If the tangent or cotangent ratio is read from the C scale, the decimal point is at the *left* of the first digit read. If the value is read from the CI scale, it is at the *right* of the first digit read.

EXAMPLES:

(a) Find tan x and cot x when x = 9°50′. First note that 50′ = $\frac{50}{60}$ of 1 degree = .83°, approximately. Hence 9° 50′ = 9.83°. Locate x = 9.83° on the T scale. Read tan x = .173 on the C scale, and read cot x = 5.77 on the CI scale.

(b) Find tan x and cot x when x = 68.6°. Locate x = 68.6° on the T scale reading from right to left. Read 255 on the CI scale. Since all angles greater than 45° have tangents greater than 1 (that is, have one digit as defined above), tan x = 2.55. Read cot 68.6° = .392 on the C scale.

Finding the Angle

If the value of the trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C or CI scale, and the angle itself read on the T scale.

THE ST SCALE: Small Angles

The sine and the tangent of angles of less than about 5.7° are so nearly equal that a single scale, marked ST, may be used for both. The graduation for 1° is marked with the degree symbol (°). To the left of it the primary graduations represent tenths of a degree. The graduation for 2° is just about in the center of the slide. The graduations for 1.5° and 2.5° are also numbered.

Rule: **For small angles, set the indicator over the graduation for the angle on the ST scale, then read the value of the sine or tangent on the C scale. Sines or tangents of angles on the ST scale have one zero.**

EXAMPLES:

(a) Find sin 2° and tan 2°. Set the indicator on the graduation for 2° on the ST scale. Read sin 2° = .0349 on the C scale. This is also the value of tan 2° correct to three digits.

(b) Find sin 0.94° and tan 0.94°. Set the indicator on 0.94 of ST. Read sin 0.94° = tan 0.94° = .0164 on the C scale.

Since cot $x = 1/\tan x$, the cotangents of small angles may be read on the CI scale. Moreover, tangents of angles between 84.3° and 89.42° can be found by use of the relation $\tan x = \cot (90 - x)$. Thus cot 2° = 1/tan 2° = 28.6, and tan 88° = cot 2° = 28.6. Finally, it may be noted that $\csc x = 1/\sin x$, and $\sec x = 1/\cos x$. Hence the value of these ratios may be readily found if they are needed. Functions of angles greater than 90° may be converted to equivalent (except for sign) functions in the first quadrant.

EXAMPLES:

(a) Find cot 1.41° and tan 88.59°. Set indicator at 1.41° on ST. Read cot 1.41° = tan 88.59° = 40.7 on CI.

(b) Find csc 21.8° and sec 21.8°. Set indicator on 21.8° of the S scale. Read csc 21.8° = 1/sin 21.8° = 2.69 on CI. Set indicator on 68.2° of the S scale (or 21.8 reading from right to left), and read sec 21.8° = 1.077 on the CI scale.

When the angle is less than 0.57° the approximate value of the sine or tangent can be obtained directly from the C scale by the following procedure. Read the ST scale as though the decimal point were at the left of the numbers printed, and read the C scale (or D, CI, etc.) with the decimal point one place to the left of where it would normally be. Thus sin 0.2° = 0.00349; tan 0.16° = 0.00279, read on the C scale.

Two seldom used special graduations are also placed on the ST scale. One is indicated by a longer graduation found just to the left of the graduation for 2° at about 1.97°. When this graduation is set opposite any number of minutes on the D scale, the sine (or the tangent) of an angle of that many minutes may be read on the D scale under the C index.

Sin 0° = 0, and sin 1′ = .00029, and for small angles the sine increases by .00029 for each increase of 1′ in the angle. Thus sin 2′ = .00058; sin 3.44′ = .00100, and the sines of all angles between 3.44′ and 34.4′ have two zeros. Sines of angles between 34.4′ and 344′ (or 5.73°) have one zero. The tangents of these small angles are very nearly equal to the sines.

EXAMPLE: Find sin 6′. With the hairline set the "minute graduation" opposite 6 located on the D scale. Read 175 on the D scale under the C index. Then sin 6′ = .00175.

The second special graduation is also indicated by a longer graduation located at about 1.18°. It is used in exactly the same way as the graduation

for minutes. Sin 1″ = .0000048, approximately, and the sine increases by this amount for each increase of 1″ in the angle, reaching .00029 for sin 60″ or sin 1′ = .00029.

USE OF LOG LOG SCALES

To find the value of 1.3^7, $5.6^{3.21}$, $\sqrt[5]{38}$, $\sqrt[3.5]{84}$, and many other types of expressions, Log Log scales are used. The method of computing such expressions will be explained in later sections. First the Log Log scales will be described.

The Log Log scale has two main parts. One part is used for numbers greater than 1. The other part is used for numbers between 0 and 1; that is, for proper fractions expressed in decimal form.

READING THE SCALES

Numbers greater than 1:

On an ordinary logarithmic scale, such as the D scale, any particular graduation represents many different numbers. Thus the graduation labeled 2 represents not only 2, but also 20, 200, .2, .02, etc. In contrast, any graduation on a Log Log scale represents only one number. The principal graduations are labeled with a number in which the decimal point is shown.

The scales marked LL1+, LL2+, and LL3+ are the first, second, and third parts of one continuous scale. It begins at the left end of the scale marked LL1+. The index graduation is marked 1.01. Set the indicator on this mark and move the hairline slowly to the right, noting the graduations marked 1.02, 1.03, etc. to 1.10. The scale now continues on LL2+ through 1.11, 1.15, etc., and ends with e=2.718, the base of the Napierian system of logarithms. The LL3+ scale begins at e, continues through 3, 4, 5, etc., up to about 22,026.

There is no difficulty in reading the principal graduations since they are labeled and the decimal point is shown. Between the principal graduations the intervals are subdivided in several different ways. Thus the graduations between the numbers shown do not have the same meaning on all sections of the scale.

(1) To locate a number, look first for the nearest smaller number that appears on the scale.

(2) Second, observe the major subdivisions between the nearest smaller number and the one following it. Sometimes there are 10, at other times 5, and at still other times only 2 or 3 major parts of the interval.

The general idea used in reading the scales may be stated informally as follows: Starting with the smaller printed number, decide how you must "count" the major graduation marks to come out correctly at the larger printed number.

(3) Third, in most cases there are still other or minor subdivisions between the major ones. These minor subdivisions divide the major intervals into 10 sub-parts, 5 sub-parts, or 2 sub-parts. Use the slide rule and check the location of the numbers in the table below.

NUMBER		SCALE
1.01278	is between 1.01 and 1.015 on	LL1+
1.173	is between 1.15 and 1.2 on	LL2+
4.78	is between 4 and 5 on	LL3+
1.054	is between 1.05 and 1.06 on	LL1+
1.862	is between 1.8 and 1.9 on	LL2+
25.6	is between 20 and 30 on	LL3+

The scales marked LL1–, LL2–, and LL3– are the first, second, and third sections of one continuous scale. Note that these scales *decrease* from left to right and hence *increase from right to left*. The ranges of these scales are approximately as follows:

SCALE	LEFT INDEX	RIGHT INDEX
LL1–	.990	.905
LL2–	.905	.370
LL3–	.370	.00005

The methods of subdividing these scales are the same as those used for numbers greater than 1. The methods of reading the scales are also the same. Use the slide rule to check the location of the numbers in the table below. As before, look first for the nearest smaller number at a principal graduation mark.

NUMBER	SCALE
.984 between .98 and .985 on	LL1 -
.813 between .80 and .82 on	LL2 -
.231 between .20 and .25 on	LL3 -
.026 between .01 and .05 on	LL3 -

The value of log10N may be read directly on the D scale. The symbols D., .D, .OD and .OOD at the left end of the Log Log scales show how to place the decimal point for any number set on the corresponding scale. With this scale arrangement, it is easy, if not easier, to find logarithms to base 10 as to base e.

The Log Log scales may be used to find any power of any base. Since roots may be expressed by exponents that are fractions in decimal form, the Log Log scales may also be used to find any root of a positive number.

The problem is to compute $N=b^m$ when b and m are known numbers. The general method is given by the following rule.

Rule: To find b^m, when m>O set the index of an ordinary loggarithmic scale on the slide (C, CF,) opposite b on a Log Log scale. Move the indicator to m of the ordinary logarithmic scale, and read b^m under the hairline on the Log Log scale.