Dale Eccles

INSTRUCTIONS FOR USING TRIG and LOG LOG SLIDE RULES

PUBLISHED BY
ACU-RULE MANUFACTURING CO.
ST. LOUIS, MO. • MT. OLIVE, ILL.

ENGRAVED SLIDE RULE

CONGRATULATIONS!

Your good judgement in selecting an Acu-Math slide rule will be confirmed in day by day use of your instrument. Because your rule is a precision instrument reasonable care should be taken to insure long and satisfactory service.

- **Keep your rule in its protective case when not in use.**
- **Keep rule clean by washing surface with a non-abrasive soap and water.**
- **To lubricate slide use a small amount of white petroleum jelly, liquid soap or silicone grease.**

HOW TO ADJUST YOUR SLIDE RULE
(For rules with adjustable end plates and cursors)

Each rule is accurately adjusted before it leaves the factory. However, handling during shipment may loosen the adjustment screws and throw the scales out of alignment. Follow this procedure for adjustment of scales and cursor.

1. Loosen end plate adjustment screws. Align C (or S) index with D. Then align A (or DF) index with B (or CF) index.
2. With rule properly aligned loosen all cursor adjustment screws and line up hairline with the left index scales on face side and tighten screws.
3. Turn rule over carefully to avoid moving cursor and repeat procedure.

SLIDE RULE PARTS LIST

ALL PARTS PRICES PREPAID
ACU-RULE MFG. CO.
BOX 142, MT. OLIVE, ILL.

CURSORS for ACU•MATH RULES

	Each
Cursor No. 1240C (For Acu-Math rules No. 1240 and No. 1240T)	$ .25
Cursor No. 1200C (For Acu-Math rules No. 1200 and No. 1200T, and No. 1211)	$ .40
Cursor Springs for No. 1240C and No. 1200C cursors	$ .05
Cursor No. 400C (For Acu-Math rules No. 400; No. 400T, No. 400B, No. 400BT	$ .50
Cursor No. 500C (For Acu-Math rules No. 500 and No. 500T)	$ .60
Cursor Springs for No. 400C and No. 500C cursors	$ .05
Cursor No. 550C (For Acu-Math rules No. 550 and No. 550T)	$ .60
Cursor Springs for No. 550C cursors	$ .05
Cursor No. 600C (Adjustable, For Acu-Math rules No. 511, No. 600 and No. 600T	$1.00
Cursor No. 900 C (Adjustable, For Acu-Math rules No. 900, No. 900T, No. 1300, No. 1300T, No. 1500 and No. 1500T)	$1.25
Cursor No. 1311C (Adjustable, For Acu-Math rule No. 1311)	$1.25
Cursor Springs for No. 600C, No. 900C and No. 1311C cursors	$ .05
Window for cursors (For Acu-Math rules No. 511, No. 600, No. 600T, No. 1500, No. 1500T, No. 1300, No. 1300T, and No. 1311	$ .35
Top or Bottom Cursor Bar (For Acu-Math rules No. 511, No. 600, No. 600T, No. 1500, No. 1500T, No. 1300, No. 1300T and No. 1311	$ .25

SCREWS FOR ACU•MATH RULES

	Each
End bracket Screws and nuts for Model No. 1211 rule (1 pair)	$.10
End bracket Screws and nuts for Model No. 550 and 550T rule (1 pair)	$.10
End bracket Screws and nuts for Model No. 500, No. 500T, No. 600, No. 600T No. 900, No. 900T, No. 1300, No. 1300T, No. 1500, No. 1500T, No. 1311, Complete set (1 pair)	$.10
Cursor Adjustment Screws	4 for $.10

INSTRUCTION MANUALS

	Each
No. 44P Manual for 5" Pocket Mannheim Rules	$.15
No. 44 Manual for 10" Mannheim Rules	$.15
No. 13 Manual for Log Log Decimal Trig Rules	$.25

ACU-MATH GUARANTEE

Congratulations on your choice of an Acu-Math slide rule.

Your rule is constructed of the finest materials available and assembled by unexcelled American craftsmen insuring a lifetime of troublefree use.

If returned to us by registered owner and found defective in material, workmanship, accuracy or stability it will be promptly serviced or replaced.

Please fill in the attached form and mail it back to us. It will be kept in our files for future reference.

ACU-RULE MFG. CO.

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ACU-MATH SLIDE RULE GUARANTEE REGISTRATION CARD

Please register my Model Number, Date of Purchase and my name as noted below, under the terms of your Guarantee. Thank you.

MODEL NUMBER ____ DATE OF PURCHASE ____
DEALER'S NAME ____
DEALER'S ADDRESS ____
DEALER'S CITY ____ ZONE ____ STATE ____

YOUR NAME ____
YOUR COMPANY OR SCHOOL ____
YOUR ADDRESS ____
YOUR CITY ____ ZONE ____ STATE ____

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Prepared by

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Washington University

CARE & ADJUSTMENT OF YOUR ACU-MATH SLIDE RULE

Your slide rule should be treated in the same manner as any precision instrument, with a minimum of care it will give you years of trouble-free service.

Keep it in its protective sheath when not in use and avoid severe shock that might upset precision alignment. Avoid exposure to extremes in temperature.

Should your rule become soiled any one of the mild liquid soaps will restore its lustre and improve slide action.

To adjust your rule loosen the two large screws on the end plates, register (line up) the scales on the left hand index and tighten the screws.

Four screws on the cursor permit proper alignment with scales.

Instructions for

LOG LOG SLIDE RULES

The following is a brief description of the various scales of the log log rule:

1. **C and D scales.** These scales, which are exactly alike, are the fundamental scales of any slide rule. They are used for multiplication and division, and are also used with the other scales in various operations.

2. **CF and DF scales.** These are C and D scales "folded" at π. The DF scale could be made by cutting a D scale at π ($= 3.1416$) and interchanging the two parts. This puts π at the ends and 1 about in the middle. These scales are used with C and D in multiplication and division in order to decrease the number of operations. They are also useful in problems requiring multiplication by π.

3. **CI scale.** This is an inverted C scale. The graduations run from *right* to *left* instead of from left to right. In order to avoid confusion in reading this scale its numbers are sometimes printed in red. It is used for reading directly the reciprocal of a number.

4. **CIF scale.** This is a CI scale folded at π. It bears the same relation to CF and DF that CI bears to C and D.

5. **S and ST scales.** These scales constitute a 20-inch scale of sines. They give both the sines and cosines of angles.

6. **T scale.** This is a tangent scale which enables one to read tangents and cotangents of angles.

7. **A and B scales.** These scales are alike, one being on the slide and the other on the stock. Either of them consists of two half-size C scales placed end to end. They are used in finding squares and square roots, and in other operations.

8. **K scale.** This scale consists of three one-third size C scales placed end to end. It is used in finding cubes and cube roots.

9. **L scale.** This scale, operating with C, enables one to read directly the mantissa of the common logarithm of a number.

10. **LL1-2-3 scales.** These scales constitute three sections of one long scale. It is used with C and D in evaluating expressions such as $(2.86)^{1.42}$ and $(1.24)^{0.63}$. It also gives directly the values of the

function $e^x (e = 2.718)$ for values of x from 0.01 to 10, and is used in reading the natural (base e) logarithms of numbers.

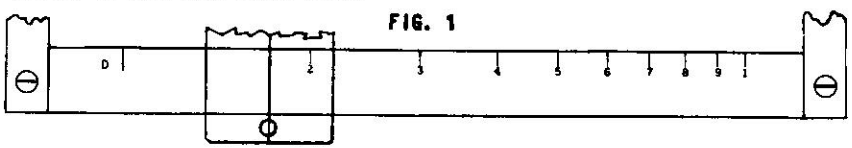
11. LLO1-02-03 scales. These are three sections of one long scale. It is used with C and D in finding powers of decimal fractions—such as $(0.54)^{3.0}$ or $(0.86)^{0.78}$. It also gives directly the values of the function e^x for negative values of x (from -0.01 to -10).

On rule models No. 550, No. 550T, No. 1500 and No. 1500T all numerals appear in black ink only.

LOCATING NUMBERS ON THE SCALES

10. Reading a slide rule scale. Anyone who knows how to read the scale on an ordinary ruler or yardstick can learn to read a slide rule scale. The only essential difference lies in the fact that the calibration marks on a slide rule scale are not uniformly spaced (except in the case of the L scale). Fig. 1, which shows only the primary divisions of the D scale, illustrates this point. It is much farther, for example, from 1 to 2 than it is from 8 to 9. The spacing is called "logarithmic" and it is based on the theory of logarithms. The student does not need to understand this in order to use the slide rule.

FIG. 1



The part of the D scale from 1 to 2 is divided into 10 secondary divisions, each representing 1/10 or 0.1; they are numbered with small numbers from 1 to 9. Each of these secondary divisions is subdivided into 10 parts, and consequently each smallest division represents 1/10 of 1/10 or 0.01.

FIG. 2

Between 2 and 4 each primary division is again divided into 10 secondary parts but the small numbers are omitted because of a lack of sufficient space. Each of these secondary divisions is further divided into 5 parts, so each smallest division represents 1/5 of 1/10 or 0.02.

Between 4 and 10 each major division is again divided into 10 parts, but each of these is subdivided into only 2 parts; each smallest division then represents ½ of 1/10 or 0.05.

In order to locate a given number on the scale one disregards the decimal point entirely. Thus the same spot on the scale serves for 1.64, 16.4, 164, and 0.0164. To locate this number one may regard the scale as running from 1 to 10, the right-hand 1 standing for 10. Then he may think of the number as 1.64 regardless of the actual position of the decimal point.

Since the first digit is 1, the number is located between the main divisions 1 and 2. Since the next digit is 6, it is between the 6th and 7th secondary calibration marks. Since each smallest division on this part of the scale represents 0.01, the number is at the fourth one of these. See Fig. 2. Several other numbers are

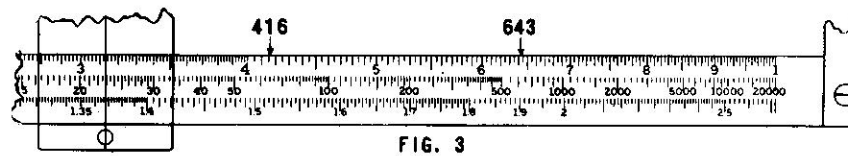


FIG. 3

located in Figs. 2 and 3. When one has learned to locate numbers on one scale he can easily do this on any of the scales.

SLIDE RULE OPERATIONS

13. Multiplication using C and D. In what follows, the left-hand 1 of a scale is called its *LEFT INDEX;* the right-hand 1 is called the *RIGHT INDEX.*

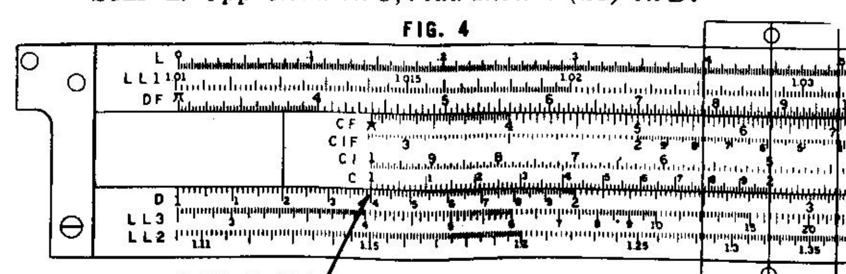
We multiply two numbers as shown by the following two examples:

Example 1. Multiply 14 × 2.

STEP 1. *Opposite 14 on D, set LEFT index of C.*

STEP 2. *Opposite 2 on C, read answer (28) on D.*

FIG. 4



STEP 1. OPPOSITE 14 ON D SET LEFT INDEX OF C

STEP 2. OPPOSITE 2 ON C READ 28 ON D

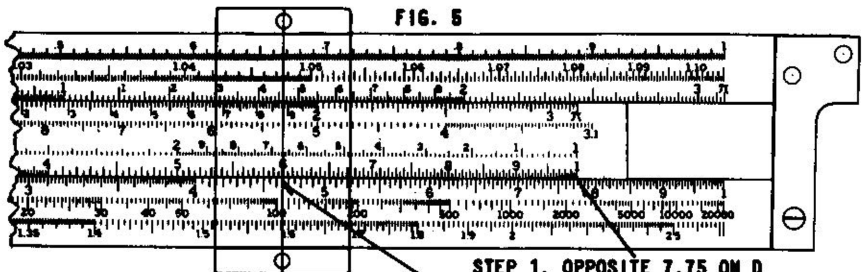
Example 2. Multiply 7.75 × 0.06.

STEP 1. *Opposite 775 on D, set RIGHT index of C.*

STEP 2. *Opposite 6 on C, read 465 on D.*

The decimal points have been disregarded in this operation. Rough mental calculation shows that the answer must be 0.465. Note in this case that the reading would have been "off scale" if the left index had been used.

FIG. 5



These examples illustrate the general rule for multiplication, namely:

STEP 1. *Locate one of the factors on the D scale and set the right or left index of C over it.*

STEP 2. *Opposite the other factor on C, read the product on D.*

14. Division using C and D. This operation is the inverse of multiplication. The division of 28 by 2 is shown in Fig. 4. The steps are:

STEP 1. *Opposite 28 on D, set 2 on C.*

STEP 2. *Opposite the index of C, read 14 on D.*

15. Use of CF and DF. As mentioned previously, these are simply C and D scales folded at π. This puts π at both ends and 1 about in the middle of the scale. These scales can often be used in problems of multiplication in order to avoid resetting when the product runs off scale:

Example. Multiply 2.6 × 2; 2.6 × 3; 2.6 × 4.

STEP 1. *Opposite 6.2 on D, set left index of C.*

STEP 2. *Opposite 2 on C, read 5.2 on D.*

STEP 3. *Opposite 3 on C, read 7.8 on D.*

STEP 4. *The 4 on C is off scale. To avoid resetting, however, opposite 4 on CF, read 10.4 on DF. (Fig. 6.)*

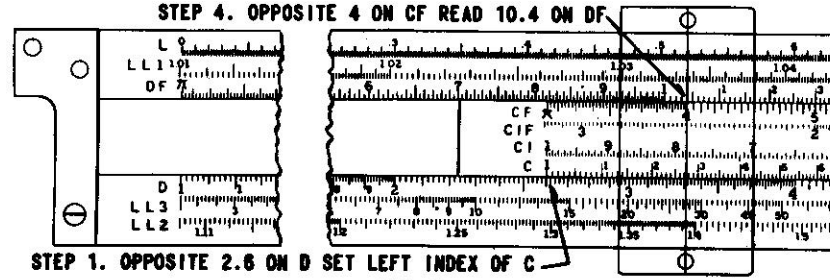


FIG. 6

The above example illustrates the fundamental property of CF and DF, namely: *When the slide is in any position, with a number x on D appearing opposite a number y on C, then this same number x appears also on DF opposite y on CF.* If the reading is off scale on C-D it may be found on CF-DF.

Another use of these folded scales is in problems requiring multiplication by π (= 3.142 approximately): *Opposite any number on D, read π times this number on DF.* Thus, opposite 2.5 on D, read 2.5 π = 7.85 on DF. If the diameter of a circle is 2.5″ its circumference is 7.85″.

16. The number of digits in a number. If a number is greater than 1, the number of digits in it is defined to be the number of figures to the left of the decimal point. If a (positive) number is less than 1 the number of digits in it is defined to be a negative number equal numerically to the number of zeros between the decimal point and the first significant figure.

Examples: 746.22 has 3 digits. 0.43 has 0 digits.

3.06 has 1 digit. 0.004 has — 2 digits.

Rules can be given for keeping track of the decimal point in multiplication and division in terms of the numbers of digits in the numbers but these will not be stressed here. An example is: When two numbers are multiplied using C and D as described in section 13, the number of digits in the product is equal to the sum of the numbers of digits in the factors if the slide projects to the left—and one less than this if it projects to the right.

In the following sections (17-21) we show how to find the squares and square roots of numbers, the cubes and cube roots of numbers, and the reciprocal of a number. (By the reciprocal of a number x we mean the number 1/x). We show also how to read off the sine, cosine, or tangent of a given angle. All of these operations are carried out with the rule "closed", that is, with the left and right indices of the C and D scales in alignment. One does not move the slide at all; he merely locates the given number on a certain scale and moves the cursor to this spot — so that the hairline is over the given number. His answer is then the corresponding number on one of the other scales — the number that is at the hairline.

17. Squares and Square roots. *Opposite any number on C or D, read its square on A or B.* Thus,

Opposite 2.47 on D, read 6.1 on A.

Opposite 0.498 on D, read 0.248 on A.

The decimal point may be fixed by making a rough mental calculation.

Conversely, *opposite any number on A, read its square root on D.* Use the LEFT half of A if the number has an ODD number of digits—such as 1, 3, 5, — 1, — 3, etc. Use the RIGHT half of A if the number has an EVEN number of digits—0, 2, 4, — 2, etc. Thus,

Opposite 4.58 on A (left), read 2.14 on D.

Opposite 56.7 on A (right), read 7.53 on D.

18. Cubes and Cube roots. *Opposite any number on D, read its cube on K.* Thus,

Opposite 4.2 on D, read 74 on K.

Opposite 0.665 on D, read 0.294 on K.

The decimal point may be fixed by making a rough mental calculation.

Conversely *opposite a number on K, read its cube root on D.* Use the right third of K if the number of digits in the number is a multiple of 3 (— 3, 0, 3, 6, etc.) ; use the middle third if the number of digits is *one less* than a multiple of 3 (— 1, 2, 5, 8, etc.) ; use the left third if the number of digits is *two less* than a multiple of 3 (— 2, 1, 4, 7, etc.).

Examples:

Opposite 2 on K (left), read 1.26 on D.

Opposite 64 on K (middle), read 4 on D.

Opposite 125 on K (right), read 5 on D.

19. Reciprocals. *Opposite any number on C, read its reciprocal on CI.* Thus,

Opposite 2 on C, read ½ = 0.5 on CI.

Opposite 38.4 on C, read 1/38.4 = 0.026 on CI.

The decimal point is fixed by the rule that if a number which is not a power of 10 has x digits, its reciprocal has (1 — x) digits. Thus 38.4 has 2 digits and its reciprocal has (1 — 2) = — 1 digits.

20. The sine of an angle. If an angle is between 5.74° and 90°, its sine is between 0.1 and 1. The S scale gives the sines of angles in this range:

Opposite the angle on S (black numbers) *read its sine on D. Put the decimal point before the first figure.* Thus,

Opposite 12° on S (black), read 0.208 on D.

Opposite 27.2° on S (black), read 0.457 on D.

Opposite 54.5° on S (black), read 0.814 on D.

If an angle is between 0.57° and 5.74°, its sine is between 0.01 and 0.1. The ST scale gives the sines of angles in this range:

Opposite the angle on ST, read its sine on D. Put one zero between the decimal point and the first significant figure. Thus,

Opposite 1.3° on ST, read 0.0227 on D.

Opposite 3.5° on ST, read 0.0610 on D.

On some slide rules the S and T scales are graduated in degrees and minutes instead of in degrees and decimal fractions of a degree. This makes no difference as far as reading off the value of the sine is concerned. Thus, for example, opposite 27° 40′ on S one would read sin 27° 40′ on D. If one wanted sin 27.3° he would note that .3° = 3/10 x 60 = 18 minutes so that 27.3° = 27° 18′.

21. The cosine of an angle. We find the cosine of an angle A by reading the sine of its complement, 99° — A. Thus cos 40° = sin 50°, etc. In order to eliminate the necessity for subtracting the given angle from 90°, the complement of each angle on S(black) is given by the *red* number. Thus the mark that is numbered 40° in black is also numbered 50° in red. Hence: *Opposite an angle on S(red), read its cosine on C.* Thus.

Opposite 58° on S(red), read cos 58° = 0.53 on D.

22. The sine law. The sine law states that in any triangle the sides are proportional to the sines of the opposite angles; that is, in Fig. 7,

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}.$$

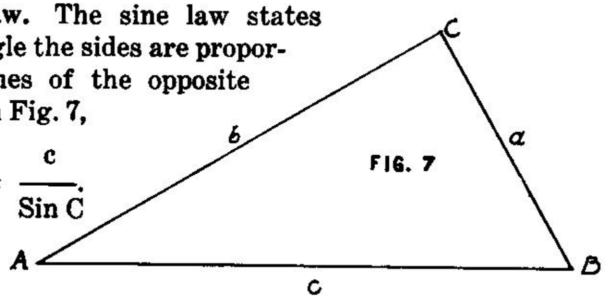


FIG. 7

This law can be used to obtain the unknown parts in a triangle (right or oblique) with a *single setting* of the slide when two angles and a side are known.

Example. Solve the triangle shown in Fig. 8.

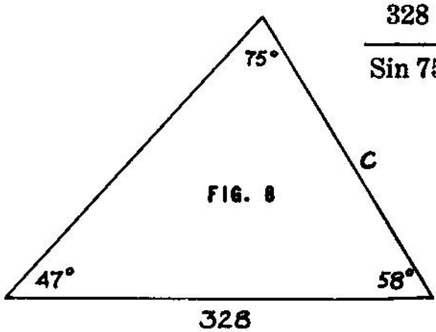


FIG. 8

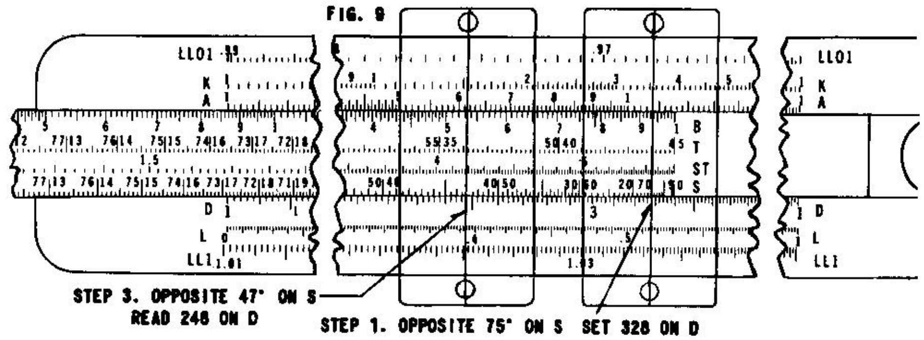
$$\frac{328}{\text{Sin } 75°} = \frac{b}{\text{Sin } 58°} = \frac{c}{\text{Sin } 47°}.$$

STEP 1. *Opposite 328 on D, set 75° on S.*

STEP 2. *Opposite 58° on S, read 288 on D. Thus b = 288.*

STEP 3. *Opposite 47° on S, read 248 on D. Thus c = 248.*

FIG. 9



23. The tangent of an angle. The T scale gives the tangents and cotangents of angles between 5.71° and 84.29°.

For an angle between 5.71° and 45°, the tangent is between 0.1 and 1. *Opposite the angle on T, read its tangent on D.* Put the decimal point before the first figure. Thus,

Opposite 18° on T, read 0.325 on D.

Opposite 31.4° on T, read 0.610 on D.

If an angle is between 45° and 84.29° its tangent is between 1 and 10. In this range use the *red* numbers on T and read on *CI* (rule closed) instead of on D. Put the decimal point after the first figure. Thus,

Opposite 61.5° on T(red), read 1.84 on CI.

Note that in reading the tangent of an angle one always reads from *black to black* or from *red to red.*

The cotangent of an angle is the reciprocal of its tangent. Hence, when we have the tangent of an angle on C, we at the same time may read its cotangent on CI, and vice versa. Thus,

Opposite 12° on T, read cot 12° = 4.70 on CI.

In reading cotangents one thus always reads *from black to red* or from *red to black.*

24. Logarithms. Only the mantissa or decimal part of the common logarithm of a number is read from the slide rule. The characteristic is supplied by rules with which the reader is assumed to be familiar. The scales used are D and L.

Opposite any number on D, read the mantissa of its common logarithm on L. Thus,

Opposite 15 on D. read .176 on L.

Opposite 278 on D, read .444 on L.

Opposite 628 on D, read .798 on L.

Then log 15 = 1.176; log 278 = 2.444; log 628 = 2.798.

25. Raising a number to a power without using LL scales. To find the value of N^x we must take log N, multiply by x, and then find the number having this last result for its logarithm.

Example 1. Evaluate $(17.5)^{1.42}$.

STEP 1. *Opposite 175 on D, read .243 on L. Then log 17.5 = 1.243.*

STEP 2. *Multiply 1.42 × 1.243 = 1.765.*

STEP 3. *Find the number whose logarithm is 1.765. Opposite .765 on L, read 582 on D.* Since the characteristic is 1, *the required number is 58.2.*

Example 2. Evaluate $(0.673)^{0.54}$.

STEP 1. *Opposite 0.673 on D, read .828 on L. Then log 0.673 = 9.828 — 10 = — .172.*

STEP 2. *Multiply 0.54 × (— .172) = — 0.0929. Rewrite this result as 9.9071 — 10.*

STEP 3. *Find the number whose logarithm is 9.9071 — 10. Opposite .907 on L, read 808 on D.* Since the characteristic is 9 — 10 or — 1, *the required number is 0.808.*

26. Natural logarithms without using LL scales. Logarithms to the base e, where e = 2.71828, are called *natural* logarithms. We may denote the natural logarithm of a number N by the symbol **ln N** in order to distinguish it from the common logarithm or logarithm to the base 10, which we denote by log N. The relation between these logarithms is

$$\ln N = 2.303 \log N.$$

Thus we find the natural logarithm of N by multiplying its common logarithm by 2.303.

Example. Find ln 41.4.

STEP 1. *Opposite 414 on D, read .617 on L. Then log 41.4 = 1.617.*

STEP 2. *Multiply 2.303 × 1.617 = 3.72. Then ln 41.4 = 3.72.*

27. The LL1-2-3 scales. As mentioned previously these are three sections of one long scale running from $e^{0.01}$ up to e^{10} as follows:

LL1 runs from $e^{0.01}$ = 1.010 to $e^{0.1}$ = 1.105
LL2 runs from $e^{0.1}$ = 1.105 to e = 2.718
LL3 runs from e = 2.718 to e^{10} = 22,026.

A major difficulty in reading the LL scales results from the fact that the amount represented by a smallest division changes rapidly. The whole LL1 scale covers only the range from 1.010 to 1.105; at its left end the smallest division represents only 0.0001. The LL3 scale covers the range from 2.718 to 22,026; between 10,000 and 20,000 each smallest division represents 1000.

A direct use of these scales is to give the values of e^x for values of x from 0.01 to 10:

Opposite x on D, read e^x on
LL1 if x is between 0.01 and 0.1;
LL2 if x is between 0.1 and 1;
LL3 if x is between 1 and 10.

Example. Opposite 25 on D, read $e^{2.5}$ = 12.2 on LL3;
Opposite 54 on D, read $e^{0.54}$ = 1.716 on LL2;
Opposite 7 on D, read $e^{0.07}$ = 1.0725 on LL1.

Conversely, if we read from LL1-2-3 to D, we get the natural logarithms of numbers between 1.010 and 22,026:

Opposite N on LL1, 2, or 3, read $\log_e N$ or ln N on D. If N is on LL1, ln N is between 0.01 and 0.1; if N is on LL2, ln N is between 0.1 and 1; if N is on LL3, ln N is between 1 and 10. This fixes the decimal point.

Examples:

Opposite 8.5 on LL3, read ln 8.5 = 2.14 on D;

Opposite 64 on LL3, read ln 64 = 4.16 on D;

Opposite 1.6 on LL2, read ln 1.6 = 0.47 on D;

Opposite 1.08 on LL1, read ln 1.08 = 0.077 on D.

28. The LLO1-02-03 scales. These are three sections of one long scale running from $e^{-0.01}$ to e^{-10} as follows:

LLO1 runs from $e^{-0.01} = 0.990$ to $e^{-0.1} = 0.905$;

LLO2 runs from $e^{-0.1} = 0.905$ to $e^{-1} = 0.368$;

LLO3 runs from $e^{-1} = 0.368$ to $e^{-10} = 0.00005$.

It gives directly the values of e^{-x} for x from 0.01 to 10 as follows:

If x is between 0.01 and 0.1: Opposite x on D read e^{-x} on LLO1.

If x is between 0.1 and 1: Opposite x on D read e^{-x} on LLO2.

If x is between 1 and 10: Opposite x on D read e^{-x} on LLO3.

Examples:

Opposite 4 on D, read $e^{-0.04} = 0.9608$ on LLO1.

Opposite 4 on D, read $e^{-0.4} = 0.670$ on LLO2.

Opposite 4 on D, read $e^{-4} = 0.0183$ on LLO3.

Opposite 193 on D, read $e^{-1.93} = 0.145$ on LLO3.

By reading from LLO1-02-03 to D one finds, conversely, the natural logarithms of numbers. Thus we found above that $e^{-1.93} = 0.145$. It follows that ln 0.145 = —1.93. Of course we actually read only the numbers 193 on the slide rule and supply the negative sign and decimal point. The decimal point is fixed as follows:

If N is on LLO1 then ln N is between —.01 and —.1 so there is one zero between the decimal point and the first significant digit.

Example: Opposite .9608 on LLO1 read 4 on D. Then ln .9608 = —.04.

If N is on LLO2 then ln N is between —.1 and —1 so the decimal point immediately precedes the first significant digit.

Example: Opposite .74 on LLO2 read 301 on D. Then ln .74 = — .301.

If N is on LLO3 then ln N is between —1 and —10 so the decimal point follows the first significant digit.

Example: Opposite .145 on LLO3 read 193 on D. Then ln .145 = —1.93.

29. The Hyperbolic Functions. The hyperbolic functions sinh x and cosh x are defined as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2}; \qquad \cosh x = \frac{e^x + e^{-x}}{2}.$$

They are thus just new names given to half the sum and half the difference of e^x and e^{-x}. In order to find the value of sinh x or cosh x for a given value of x one simply reads off the values of e^x and e^{-x} and then takes half the sum or half the difference of these numbers.

Example: To evaluate sinh x and cosh x for x = 1.5 we proceed as follows:

Opposite 1.5 on D, read $e^{1.5} = 4.48$ on LL3 and $e^{-1.5} = 0.223$ on LLO3. Then,

$$\sinh 1.5 = \tfrac{1}{2}(4.48 - 0.22) = 2.13;$$

$$\cosh 1.5 = \tfrac{1}{2}(4.48 + 0.22) = 2.35.$$

Note that in the final calculation we "rounded off" the reading 0.223 to 0.22 because the other number involved (4.48) could be read only to two decimal places.

30. Raising a number to a power using the LL scales. We have already seen that an expression of the form b^x can be evaluated by multiplying the common logarithm of b by x and then reading the antilogarithm. A more convenient method employs the LL scales as follows: If b between 1.01 and 22,026, and if b^x lies also in this range, then,

STEP 1. *Opposite b on LL1, 2, or 3 set an index of C.*

STEP 2. *Opposite x on C, read b^x on LL1, 2, or 3.*

The LL1-2-3 group is here regarded as one long scale, LL1 running from 1.010 to 1.105, LL2 from 1.105 to 2.718, and LL3 from 2.718 to 22,026.

Example 1. Evaluate $(3.84)^{2.28}$.

STEP 1. *Opposite 3.84 on LL3, set left index of C.*

STEP 2. *Opposite 2.28 on C, read 21.5 on LL3.*

In this case both the number 3.84 and the answer 21.5 are in the range covered by LL3.

Example 2. Evaluate $(2.14)^{3.36}$.

STEP 1. *Opposite 2.14 on LL2, set right index of C.*

STEP 2. *Opposite 3.36 on C, read 12.9 on LL3.*

In this case the number 2.14 is in the range covered by LL2 but the answer (12.9) is in that covered by LL3.

Example 3. Evaluate $(68)^{0.0388}$.

STEP 1. *Opposite 68 on LL3, set right index of C.*

STEP 2. *Opposite 388 on C, read 1.178 on LL2.*

Observe that the value of $(68)^{0.388} = 5.14$ would be read on LL3 with this same setting.

The LLO1-02-03 combination is used with C in the same manner to find powers of decimal fractions between 0.00005 and 0.990:

Example. Evaluate $(0.85)^{2.1}$.

STEP 1. *Opposite 0.85 on LLO2, set the left index of C.*

STEP 2. *Opposite 21 on C, read 0.71 on LLO2. Thus $(0.85)^{2.1} = 0.71$.*

Observe that the value of $(0.85)^{21} = 0.033$ can be read on LLO3 opposite 21 on C with the same setting—and that $(0.85)^{.21} = 0.996$ can be read on LLO1.

* * * * * *

DECIMAL EQUIVALENTS

1/64 .015625	1/16-.0625	1/8-.125	3/16-.1875
1/32 .03125	5/64 .078125	9/64 .140625	13/64 .203125
3/64 .046875	3/32-.09375	5/32-.15625	7/32-.21875
	7/64 .109375	11/64 .171875	15/64 .234375
1/4-.25	5/16-.3125	3/8-.375	7/16-.4375
17/64 .265625	21/64 .328125	25/64 .390625	29/64 .453125
9/32-.28125	11/32-.34375	13/32-.40625	15/32-.46875
19/64 .296875	23/64 .359375	27/64 .421875	31/64 .484375
1/2-.5	9/16-.5625	5/8-.625	11/16-.6875
33/64 .515625	37/64 .578125	41/64 .640625	45/64 .703125
17/32-.53125	19/32-.59375	21/32-.65625	23/32-.71875
35/64 .546875	39/64 .609375	43/64 .671875	47/64 .734375
3/4-.75	13/16-.8125	7/8-.875	15/16-.9375
49/64 .765625	53/64 .828125	57/64 .890625	61/64 .953125
25/32-.78125	27/32-.84375	29/32-.90625	31/32-.96875
51/64 .796875	55/64 .859375	59/64 .921875	63/64 .984375

PROPERTIES OF CIRCLES

Circumference = diameter × 3.1416.
Area = square of radius × 3.1416.
= square of diameter × 0.7854.
Side of inscribed square = diameter × 0.7071.
Side of inscribed Hexagon = radius of circle.
Length of arc = number of degrees in angle × diameter × 0.008727.
Length of chord = diameter of circle × sine of ½ included angle.
Area of sector = length of arc × ½ of radius.
Area of segment = area of sector minus area of triangle.

FORMULAS FOR AREA

Rectangle—base × altitude.
Parallelogram—base × altitude.
Triangle—½ base × altitude.
Trapezoid—½ sum of parallel sides × altitude.
Parabola—⅔ base × altitude.
Ellipse—product of major and minor diameters × 0.7854.
Regular polygon—½ sum of sides × perpendicular distance from center to sides.
Lateral area of right cylinder = perimeter of base × altitude.
Total area = lateral area + areas of ends.
Lateral area of right pyramid or cone = ½ perimeter of base × slant height.
Total area = lateral area + area of base.
Lateral area of frustum of a regular right pyramid or cone = ½ sum of peri meters of bases × slant height.
Surface area of sphere = square of diameter × 3.1416.

FORMULAS FOR VOLUME

Right or oblique prism—area of base × altitude.
Cylinder—area of base × altitude.
Pyramid or cone—⅓ area of base × altitude.
Sphere—cube of diameter × 0.5236.
Frustum of pyramid or cone—add the areas of the two bases and add to this the square root of the product of the areas of the bases; multiply by ⅓ of the height: $V = \frac{1}{3}h(B + b + \sqrt{B \times b})$.

IMPORTANT CONSTANTS

$\pi = 3.1416$. $\pi^2 = 9.8696$.
$\sqrt{\pi} = 1.7724$. $1 \div \pi = 0.3183$.
Base of natural logarithms = e = 2.71828.
$M = \log_{10} e = 0.43429$.
$1 \div M = \log_e 10 = 2.3026$.
$\log_e N = 2.3026 \times \log_{10} N$.
Number of degrees in 1 radian = 180 ÷ π = 57.2958.
Number of radians in 1 degree = π ÷ 180 = 0.01745.

WEIGHTS AND MEASURES

Avoirdupois Weight

27 11/32 grs. = 1 dram
16 drams = 1 ounce
16 ounces = 1 pound
25 pounds = 1 quarter
4 quarters = 1 cwt.
2,000 lbs. = 1 short ton
2,240 lbs. = 1 long ton

Mariners' Measure

6 feet = 1 fathom
120 fathoms = 1 cable length
7½ cable lengths = 1 mile
5,280 ft. = 1 stat. mile
6,085 ft. = 1 naut. mile

Troy Weight

24 grains = 1 pwt.
20 pwt. = 1 ounce
12 ounces = 1 pound
Used for weighing gold, silver and jewels.

Apothecaries' Weight*

20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound
*The ounces and pound in this are the same as in Troy weight.

Long Measure

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
40 rods = 1 furlong
8 furlongs = 1 stat. mile
3 miles = 1 league

Square Measure

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
30¼ sq. yds. = 1 sq. rod
40 sq. rods = 1 rood
4 roods = 1 acre
640 acres = 1 sq. mile.

Cubic Measure

1,728 cu. in. = 1 cu. ft.
128 cu. ft. = 1 cord wood
27 cu. ft. = 1 cu. yd.
40 cu. ft. = 1 ton (shpg.)
2,150.42 cu. inches = 1 standard bushel
231 cubic inches = 1 standard gallon
1 cubic foot = about four-fifths of a bushel.

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel
36 bushels = 1 chaldron

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel
2 barrels = 1 hogshead

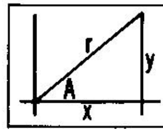
Surveyors' Measure

7.92 inches = 1 link
25 links = 1 rod
4 rods = 1 chain
10 sq. chains or 160 sq. rods = 1 acre
640 acres = 1 sq. mile
36 sq. miles (6 miles sq.) = 1 township

Miscellaneous

3 inches = 1 palm
4 inches = 1 hand
6 inches = 1 span
18 inches = 1 cubit
21.8 = 1 Bible cubit
2½ ft. = 1 military pace

FORMULAE



$\sin A = y/r$
$\cos A = x/r$
$\tan A = y/x$

$\sin(90° + A) = \cos A$
$\cos(90° + A) = -\sin A$
$\tan(90° + A) = -\cot A$
$\cot(90° + A) = -\tan A$

$\sin^2 A + \cos^2 A = 1$
$1 + \tan^2 A = \sec^2 A$
$1 + \cot^2 A = \csc^2 A$
$\frac{\sin A}{\cos A} = \tan A$

$\sin 2A = 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

sphere $V = 4/3\,\pi r^3$ $S = 4\pi r^2$
circle $A = \pi r^2$ $C = 2\pi r$
cyl. or prism $V = Bh$
cone or pyramid $V = \frac{1}{3}Bh$

1 radian = 57.296°
1° = 0.01745 radian
e = 2.71828
$\log_e N = 2.3026 \log_{10} N$

$\sin^2 \frac{1}{2}A = \frac{1}{2}(1 - \cos A)$
$\cos^2 \frac{1}{2}A = \frac{1}{2}(1 + \cos A)$
$\tan \frac{1}{2}A = \frac{1 - \cos A}{\sin A}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
$c^2 = a^2 + b^2 - 2ab \cos C$