PROPERTIES OF CIRCLES

Circumference = diameter × 3.1416.
Area = square of radius × 3.1416.
= square of diameter × 0.7854.
Side of inscribed square = diameter × 0.7071.
Side of inscribed Hexagon = radius of circle.
Length of arc = number of degrees in angle × diameter × 0.008727.
Length of chord = diameter of circle × sine of ½ included angle.
Area of sector = length of arc × ½ of radius.
Area of segment = area of sector minus area of triangle.

FORMULAS FOR AREA

Rectangle—base × altitude.
Parallelogram—base × altitude.
Triangle—½ base × altitude.
Trapezoid—½ sum of parallel sides × altitude.
Parabola—⅔ base × altitude.
Ellipse—product of major and minor diameters × 0.7854.
Regular polygon—½ sum of sides × perpendicular distance from center to sides.
Lateral area of right cylinder = perimeter of base × altitude.
Total area = lateral area + areas of ends.
Lateral area of right pyramid or cone = ½ perimeter of base × slant height.
Total area = lateral area + area of base.
Lateral area of frustum of a regular right pyramid or cone = ½ sum of perimeters of bases × slant height.
Surface area of sphere = square of diameter × 3.1416.

FORMULAS FOR VOLUME

Right or oblique prism—area of base × altitude.
Cylinder—area of base × altitude.
Pyramid or cone—⅓ area of base × altitude.
Sphere—cube of diameter × 0.5236.
Frustum of pyramid or cone—add the areas of the two bases and add to this the square root of the product of the areas of the bases; multiply by ⅓ of the height: $V = \frac{1}{3} h (B + b + \sqrt{B \times b})$.

IMPORTANT CONSTANTS

$\pi = 3.1416$. $\pi^2 = 9.8696$.
$\sqrt{\pi} = 1.7724$. $1 \div \pi = 0.3183$.
Base of natural logarithms = $e = 2.71828$
$M = \log_{10} e = 0.43429$.
$1 \div M = \log_e 10 = 2.3026$.
$\log_e N = 2.3026 \times \log_{10} N$.
Number of degrees in 1 radian = $180 \div \pi = 57.2958$.
Number of radians in 1 degree = $\pi \div 180 = 0.01745$.

WEIGHTS AND MEASURES

Avoirdupois Weight

27 ¹¹⁄₃₂ grs. = 1 dram
16 drams = 1 ounce
16 ounces = 1 pound
25 pounds = 1 quarter
4 quarters = 1 cwt.
2,000 lbs. = 1 short ton
2,240 lbs. = 1 long ton

Mariners' Measure

6 feet = 1 fathom
120 fathoms = 1 cable length
7½ cable lengths = 1 mile
5,280 ft. = 1 stat. mile
6,085 ft. = 1 naut. mile

Troy Weight

24 grains = 1 pwt.
20 pwt. = 1 ounce
12 ounces = 1 pound
Used for weighing gold, silver and jewels.

Apothecaries' Weight*

20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound
*The ounces and pound in this are the same as in Troy weight.

Long Measure

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
40 rods = 1 furlong
8 furlongs = 1 stat. mile
3 miles = 1 league

Square Measure

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
30¼ sq. yds. = 1 sq. rod
40 sq. rods = 1 rood
4 roods = 1 acre
640 acres = 1 sq. mile.

Cubic Measure

1,728 cu. in. = 1 cu. ft.
128 cu. ft. = 1 cord wood
27 cu. ft. = 1 cu. yd.
40 cu. ft. = 1 ton (shpg.)
2,150.42 cu. inches = 1 standard bushel
231 cubic inches = 1 standard gallon
1 cubic foot = about four-fifths of a bushel.

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel
36 bushels = 1 chaldron

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel
2 barrels = 1 hogshead

Surveyors' Measure

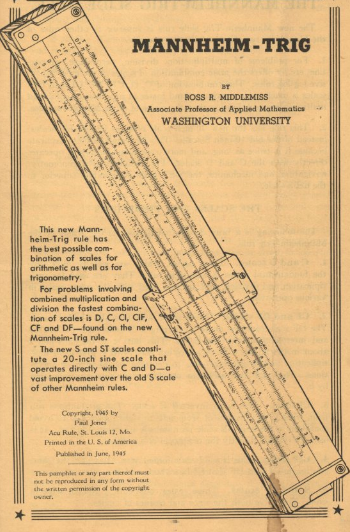
7.92 inches = 1 link
25 links = 1 rod
4 rods = 1 chain
10 sq. chains or 160 sq. rods = 1 acre
640 acres = 1 sq. mile
36 sq. miles (6 miles sq.) = 1 township

Miscellaneous

3 inches = 1 palm
4 inches = 1 hand
6 inches = 1 span
18 inches = 1 cubit
21.8 = 1 Bible cubit
2½ ft. = 1 military pace

MANNHEIM-TRIG

BY
ROSS R. MIDDLEMISS
Associate Professor of Applied Mathematics
WASHINGTON UNIVERSITY



This new Mannheim-Trig rule has the best possible combination of scales for arithmetic as well as for trigonometry.

For problems involving combined multiplication and division the fastest combination of scales is D, C, CI, CIF, CF and DF—found on the new Mannheim-Trig rule.

The new S and ST scales constitute a 20-inch sine scale that operates directly with C and D—a vast improvement over the old S scale of other Mannheim rules.

Acu Rule, St. Louis 12, Mo.
Printed in the U. S. of America
Published in June, 1945

THE MANNHEIM-TRIG SLIDE RULE

The new Mannheim-Trig slide rule is superior to other rules of the Mannheim type for the following reasons:

1. For problems of multiplication, division, percentage, proportion, etc., it offers the *same* combination of scales as the most expensive log log rules. This combination of C, D, CF, DF, CI, and CIF scales is much more convenient and faster in problems of combined multiplication and division than the scales of other Mannheim rules.

2. This rule has the new 20-inch scale of sines (S and ST scales) instead of the old 10-inch S-scale. This new scale is more accurate because it is twice as long, and more convenient because it operates directly with the C and D scales. For problems of trigonometry, navigation, and mechanics, the S-ST scale is definitely superior to the old S scale.

THE SCALES AND THEIR USES

The following is a brief description of the various scales of the Mannheim-Trig rule.

1. **C and D scales.** These scales, which are exactly alike, are the fundamental scales of any slide rule. They are used for multiplication and division, and are also used with the other scales in various operations.

2. **CF and DF scales.** These are C and D scales "folded" at π. The DF scale could be made by cutting a D scale at π ($=3.1416$) and interchanging the two parts. This puts π at the ends and 1 about in the middle. These scales are used with C and D in multiplication and division in order to decrease the number of operations. They are also useful in problems requiring multiplication by π.

3. **CI scale.** This is an inverted C scale. The graduations run from *right* to *left* instead of from left to right. In order to avoid confusion in reading this scale its numbers are printed in red. It is used for reading directly the reciprocal of a number.

4. **CIF scale.** This is a CI scale folded at π. It bears the same relation to CF and DF that CI bears to C and D.

5. **S and ST scales.** These scales constitute a 20-inch scale of sines. They give both the sines and cosines of angles.

6. **T scale.** This is a tangent scale which enables one to read tangents and cotangents of angles.

7. **B scale.** This scale consists of two half-size C scales placed end to end. It is used in finding squares and square roots.

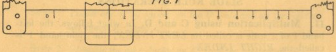
8. **K scale.** This scale consists of three one-third size C scales placed end to end. It is used in finding cubes and cube roots.

9. **L scale.** This scale, operating with C, enables one to read directly the mantissa of the common logarithm of a number.

LOCATING NUMBERS ON THE SCALES

10. **Reading a slide rule scale.** Anyone who knows how to read the scale on an ordinary ruler or yardstick can learn to read a slide rule scale. The only essential difference lies in the fact that the calibration marks on a slide rule scale are not uniformly spaced (except in the case of the L scale). Fig. 1, which shows only the primary divisions of the D scale, illustrates this point. It is much farther, for example, from 1 to 2 than it is from 8 to 9. The spacing is called "logarithmic" and it is based on the theory of logarithms. The student does not need to understand this in order to use the slide rule.

Fig. 1



The part of the D scale from 1 to 2 is divided into 10 secondary divisions, each representing 1/10 or 0.1; they are numbered with small numbers from 1 to 9. Each of these secondary divisions is subdivided into 10 parts, and consequently each smallest division represents 1/10 of 1/10 or 0.01.

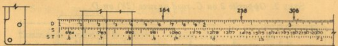


FIG 2

Between 2 and 4 each primary division is again divided into 10 secondary parts but the small numbers are omitted because of a lack of sufficient space. Each of these secondary divisions is further divided into 5 parts, so each smallest division represents 1/5 of 1/10 or 0.02.

Example. Multiply 2.6 × 2; 2.6 × 3; 2.6 × 4.

STEP 1. *Opposite 2.6 on D, set left index of C.*

STEP 2. *Opposite 2 on C, read 5.2 on D.*

STEP 3. *Opposite 3 on C, read 7.8 on D.*

STEP 4. *The 4 on C is off scale. To avoid resetting, however, opposite 4 on CF, read 10.4 on DF.* (*Fig. 6.*)

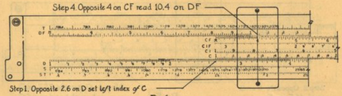


Fig. 6

The above example illustrates the fundamental property of CF and DF, namely: *When the slide is in any position, with a number x on D appearing opposite a number y on C, then this same number x appears also on DF opposite y on CF.* If the reading is off scale on C-D it may be found on CF-DF.

Another use of these folded scales is in problems requiring multiplication by π (= 3.142 approximately): *Opposite any number on D, read π times this number on DF.* Thus, opposite 2.5 on D, read 2.5 π = 7.85 on DF. If the diameter of a circle is 2.5″ its circumference is 7.85″.

14. The number of digits in a number. If a number is greater than 1, the number of digits in it is defined to be the number of figures to the left of the decimal point. If a (positive) number is less than 1 the number of digits in it is defined to be a negative number equal numerically to the number of zeros between the decimal point and the first significant figure.

Examples: 746.22 has 3 digits. 0.43 has 0 digits.

3.06 has 1 digit. 0.004 has − 2 digits.

Rules can be given for keeping track of the decimal point in multiplication and division in terms of the numbers of digits in the numbers but these will not be stressed here. An example is: When two numbers are multiplied using C and D as described in section 11, the number of digits in the product is equal to the sum of the numbers of digits in the factors if the slide projects to the left—and one less than this if it projects to the right.

15. Squares and Square roots. *Opposite any number on C* (back face of rule), *read its square on B.* Thus,

Opposite 2.47 on C, read 6.1 on B.

Opposite 0.498 on C, read 0.248 on B.

The decimal point may be fixed by making a rough mental calculation.

Conversely, *opposite any number on B, read its square root on C.* Use the LEFT half of B if the number has an ODD number of digits —such as 1, 3, 5, − 1, − 3, etc. Use the RIGHT half of B if the number has an EVEN number of digits—0, 2, 4, − 2, etc. Thus,

Opposite 4.58 on B (left), read 2.14 on C.

Opposite 56.7 on B (right), read 7.53 on C.

16. Cubes and Cube roots. *Opposite any number on C, read its cube on K.* Thus,

Opposite 4.2 on C, read 74 on K.

Opposite 0.665 on C, read 0.294 on K.

The decimal point may be fixed by making a rough mental calculation.

Conversely, *opposite a number on K, read its cube root on C.* Use the right third of K if the number of digits in the number is a multiple of 3 (−3, 0, 3, 6, etc.); use the middle third if the number of digits is *one less* than a multiple of 3 (−1, 2, 5, 8, etc.); use the left third if the number of digits is *two less* than a multiple of 3 (−2, 1, 4, 7, etc.).

Between 4 and 10 each major division is again divided into 10 parts, but each of these is subdivided into only 2 parts; each smallest division then represents ½ of 1/10 or 0.05.

In order to locate a given number on the scale one disregards the decimal point entirely. Thus the same spot on the scale serves for 1.64, 16.4, 164, and 0.0164. To locate this number one may regard the scale as running from 1 to 10, the right-hand 1 standing for 10. Then he may think of the number as 1.64 regardless of the actual position of the decimal point.

Since the first digit is 1, the number is located between the main divisions 1 and 2. Since the next digit is 6, it is between the 6th and 7th secondary calibration marks. Since each smallest division on this part of the scale represents 0.01, the number is at the fourth one of these. See Fig. 2. Several other numbers are located in

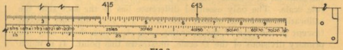


FIG 3

Figs. 2 and 3. When one has learned to locate numbers on one scale he can easily do this on any of the scales.

SLIDE RULE OPERATIONS

11. Multiplication using C and D. In what follows, the left-hand 1 of a scale is called its *LEFT INDEX;* the right-hand 1 is called the *RIGHT INDEX.*

We multiply two numbers as shown by the following two examples:

Example 1. Multiply 14 × 2.

STEP 1. *Opposite 14 on D, set LEFT index of C.*

STEP 2. *Opposite 2 on C, read answer (28) on D.*

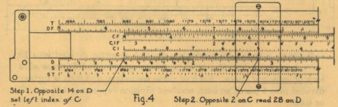


Fig.4

Example 2. Multiply 7.75 × 0.06.

STEP 1. *Opposite 775 on D, set RIGHT index of C.*

STEP 2. *Opposite 6 on C, read 465 on D.*

The decimal points have been disregarded in this operation. Rough mental calculation shows that the answer must be 0.465. Note in this case that the reading would have been "off scale" if the left index had been used.

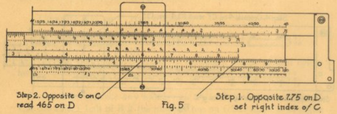


Fig. 5

These examples illustrate the general rule for multiplication, namely:

STEP 1. *Locate one of the factors on the D scale and set the right or left index of C over it.*

STEP 2. *Opposite the other factor on C, read the product on D.*

12. Division using C and D. This operation is the inverse of multiplication. The division of 28 by 2 is shown in Fig. 4. The steps are:

STEP 1. *Opposite 28 on D, set 2 on C.*

STEP 2. *Opposite the index of C, read 14 on D.*

13. Use of CF and DF. As mentioned previously, these are simply C and D scales folded at π. This puts π at both ends and 1 about in the middle of the scale. These scales can often be used in problems of multiplication in order to avoid resetting when the product runs off scale:

Examples:

Opposite 2 on K (left), read 1.26 on C.

Opposite 64 on K (middle), read 4 on C.

Opposite 125 on K (right), read 5 on C.

17. Reciprocals. *Opposite any number on C,* (or D if rule is closed) *read its reciprocal on CI.* Thus,

Opposite 2 on C, read ½ = 0.5 on CI.

Opposite 38.4 on C, read 1/38.4 = 0.026 on CI.

The decimal point is fixed by the rule that if a number which is not a power of 10 has x digits, its reciprocal has (1 − x) digits. Thus 38.4 has 2 digits and its reciprocal has (1 − 2) = − 1 digits.

18. The sine of an angle. If an angle is between 5.74° and 90°, its sine is between 0.1 and 1. The S scale gives the sines of angles in this range:

Opposite the angle on S(black numbers) *read its sine on D. Put the decimal point before the first figure.* Thus,

Opposite 12° on S(black), read 0.208 on D.

Opposite 27.2° on S(black), read 0.457 on D.

Opposite 54.5° on S(black), read 0.814 on D.

If an angle is between 0.57° and 5.74°, its sine is between 0.01 and 0.1. The ST scale gives the sine of angles in this range:

Opposite the angle on ST, read its sine on D. Put one zero between the decimal point and the first significant figure. Thus,

Opposite 1.3° on ST, read 0.0227 on D.

Opposite 3.5° on ST, read 0.0610 on D.

19. The cosine of an angle. We find the cosine of an angle A by reading the sine of its complement, 90° − A. Thus cos 40° = sin 50°, etc. In order to eliminate the necessity for subtracting the given angle from 90°, the complement of each angle on S(black) is given by the *red* number. Thus the mark that is numbered 40° in black is also numbered 50° in red. Hence: *Opposite an angle on S(red), read its cosine on D.* Thus,

Opposite 58° on S(red), read cos 58° = 0.53 on D.

20. The sine law. The sine law states that in any triangle the sides are proportional to the sines of the opposite angles; that is, in Fig. 7,

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}.$$

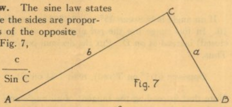


Fig. 7

This law can be used to obtain the unknown parts in a triangle (right or oblique) with a *single setting* of the slide when two angles and a side are known.

Example. Solve the triangle shown in Fig. 8.

$$\frac{328}{\text{Sin } 75°} = \frac{b}{\text{Sin } 58°} = \frac{c}{\text{Sin } 47°}.$$

Step 1. *Opposite 75° on S, set 328 on C.*

Step 2. *Opposite 58° on S, read b = 288 on C.*

Step 3. *Opposite 47° on S, read c = 248 on C.*

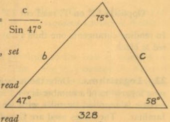
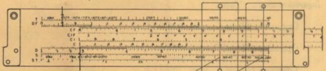


Fig. 8



Step 3. Opposite 47° on S read 248 on C. Fig. 9. Step 1. Opposite 75° on S set 328 on C

21. The tangent of an angle. The T scale gives the tangents and cotangents of angles between 5.71° and 84.29°.

For an angle between 5.71° and 45°, the tangent is between 0.1 and 1. *Opposite the angle on T, read its tangent on D.* Put the decimal point before the first figure. Thus,

Opposite 18° on T, read 0.325 on D.

Opposite 31.4° on T, read 0.610 on D.

If an angle is between 45° and 84.29° its tangent is between 1 and 10. In this range use the *red* numbers on T and read on *CI* (rule closed) instead of on D. Put the decimal point after the first figure. Thus,

Opposite 61.5° on T(red), read 1.84 on CI.

Note that in reading the tangent of an angle one always reads from *black to black* or from *red to red.*

The cotangent of an angle is the reciprocal of its tangent. Hence, when we have the tangent of an angle on C, we at the same time may read its cotangent on CI, and vice versa. Thus,

Opposite 12° on T, read cot 12° = 4.70 on CI.

In reading cotangents one thus always reads from *black to red* or from *red to black.*

22. Logarithms. Only the mantissa or decimal part of the common logarithm of a number is read from the slide rule. The characteristic is supplied by rules with which the reader is assumed to be familiar. The scales used are C and L.

Opposite any number on C, read the mantissa of its common logarithm on L. Thus,

Opposite 15 on C, read .176 on L.

Opposite 278 on C, read .444 on L.

Opposite 628 on C, read .798 on L.

Then log 15 = 1.176; log 278 = 2.444; log 628 = 2.798.

23. Raising a number to a power. To find the value of N^x we must take log N, multiply by x, and then find the number having this last result for its logarithm.

Example 1. Evaluate $(17.5)^{1.42}$.

STEP 1. *Opposite 175 on C, read .243 on L. Then log 17.5 = 1.243.*

STEP 2. *Multiply 1.42 × 1.243 = 1.765.*

STEP 3. *Find the number whose logarithm is 1.765.*

Opposite .765 on L, read 582 on C. Since the characteristic is 1, *the required number is 58.2.*

Example 2. Evaluate $(0.673)^{0.54}$.

STEP 1. *Opposite 0.673 on C, read .828 on L. Then log 0.673 = 9.828 −10 = −.172.*

STEP 2. *Multiply 0.54 × (−.172) = −0.0929. Rewrite this result as 9.9071 −10.*

STEP 3. *Find the number whose logarithm is 9.9071 −10.*

Opposite .907 on L, read 808 on C. Since the characteristic is 9 −10 or −1, *the required number is 0.808.*

24. Natural logarithms. Logarithms to the base e, where e = 2.71828, are called *natural* logarithms. We may denote the natural logarithm of a number N by the symbol **ln N** in order to distinguish it from the common logarithm or logarithm to the base 10, which we denote by log N. The relation between these logarithms is

ln N = 2.303 log N.

Thus we find the natural logarithm of N by multiplying its common logarithm by 2.303.

Example. Find ln 41.4.

STEP 1. *Opposite 414 on C, read .617 on L. Then log 41.4 = 1.617.*

STEP 2. *Multiply 2.303 × 1.617 = 3.72. Then ln 41.4 = 3.72.*