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The Pipe Line Flow Constant

0.0288

BY

FORREST M. TOWL, C.E.

NEW YORK, N. Y.

1943

PROOF EDITION

PRINTED FOR CONVENIENT

CORRECTION OF ERRORS

FOR PRIVATE DISTRIBUTION

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THE PIPE LINE FLOW CONSTANT

0.0288

f THE PIPE LINE FLOW FACTOR

THEIR RELATION TO

DENSITY GRAVITY

VELOCITY VISCOSITY

AND THE

REYNOLDS NUMBER

BY

FORREST M. TOWL, C.E.

Am. Soc. C.E., A.S.M.E., A.A.A.S.

NEW YORK

1943

By FORREST M. TOWL
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N. Y.

PROOF EDITION

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To
Cornell University
and to
The Memory of my Father

THEODORE M. TOWL

ONE OF THE PIONEERS IN THE PIPE LINE TRANSPORTATION OF PETROLEUM
AND WHO IN SEPTEMBER 1881 ENROLLED ME AS A STUDENT
AT CORNELL UNIVERSITY

CONTENTS

INTRODUCTION

The author graduated, as a Civil Engineer, from Cornell University in June, 1886, and while there studied Hydraulics under Professor Church. During the next three years, he made many tests on the pumping of Crude Petroleum through pipe lines. These tests were made at the suggestion of Professor Church who used some of the results in his 1889 edition of Fluids.

About this time, Osborne Reynolds introduced a series of numbers in his papers on the definite relation between velocity, diameter, density and viscosity. Reynolds used the Darcy experiments in demonstrating his law.

The book, "*f* The Pipe Line Flow Factor," published May, 1934, contained this Statement,—

> "The number of cubic feet of water flowing per hour through a smooth pipe one foot in diameter is one-fourth the Reynolds Number for the corresponding *f*."

The book was based on Dr. Durand's paper which used the Darcy form of the Pipe Line Flow equation and contained a table of values for *f* and the corresponding Reynolds Numbers. The statement quoted above was questioned by critics who claimed accuracy to within 5% when using Reynolds Numbers, density, viscosity, velocity and diameter to find the proper flow factor.

In September, 1934, work was started to reconcile the Darcy Flow Formula and the Reynolds Numbers with the axioms of Newton when using any one unit of length to measure The Volume and The Force.

Like the Reynolds Numbers, the Resistance coefficients are numbers. The character *f* will not be introduced until after a unit of length has been adopted.

The *f* book is being entirely re-written and is based on the Axioms of Newton and on three well established formulas.

The Circumference of a circle
The Area of a circle and
The Volume of a cylinder.

The author takes this opportunity to thank all who have assisted in the investigations undertaken at Cornell University.

Particular mention is made of Prof. C. C. Murdock and Prof. J. R. Collins, whose untiring criticism and advice have resulted in the fixing of the Pipe Line Flow constant; Prof. R. C. Gibbs, Director of the Department of Physics, particularly for the use of a laboratory in Rockerfeller Hall where most of the work was done; Dr. E. W. Schoder, for his continued assistance in the investigations; Dean S. C. Hollister, for his continued assistance and advice throughout the past six years; Director W. L. Malcolm, who has given valuable assistance in this work; Prof. Samuel L. Boothroyd; and of those graduate students who have assisted in making tests and computing results, particularly Mr. A. V. Peterson, Mr. H. V. Hawkins, and Mr. Aldus Fogelsanger.

There is not room here to mention all of those at the University who have assisted in this work.

Thanks are also due to Dr. W. F. Durand and to the late Dr. Edgar Buckingham who have assisted with many suggestions and with whom the author has consulted on a number of occasions.

The following list is from the 1934 edition of the *f* book. The first three named have rendered valued assistance in the preparation of this book.

* Daniel O. Towl, B.S., C.E.
John H. Peper, M.E.
Theodore C. Towl C.E.
Douglas S. Bushnell, M.E.
Thomas R. Weymouth, B.S.
Hubert H. Hall, A.B.
Herbert S. Austin, C.E.

* Deceased, Dec. 1941

During these investigations, the National Tube Company, Revere Copper and Brass Company and Johns-Manville Corporation furnished pipe and other material used in the tests which were made.

Registering as a graduate student, the author received his M.C.E. degree in 1935. This book is the first report on the completion of the work undertaken in 1934 and is based on the Axioms of Newton and on three well established formulas.

THIS INTRODUCTION CLOSES WITH THE DEDICATION OF THE 1934 BOOK.

To the Memory of

IRVING PORTER CHURCH, C.E.,

PROFESSOR OF APPLIED MECHANICS AND HYDRAULICS, COLLEGE OF CIVIL ENGINEERING, CORNELL UNIVERSITY,

all that is worthy in this book is gratefully dedicated.

None of the characters in the first five chapters are dimensioned, i.e., any uniform system of units may be used, but in Chapter IV One Foot is adopted as the Unit of Length and used to establish ONE Numerical value of The Pipe Line Flow Constant 0.0288.

The even numbered pages are reserved for Notes, diagrams, etc., and where not used for such purposes the space will be left blank for the reader's comments. The notes are numbered so that they can be conveniently referred to by page and number.

CHAPTER I

PURPOSE

This is written to establish, determine and definitely FIX the intimate relation that always exists among all the FACTORS when there is a STEADY FLOW of any one FIXED VOLUME of any one LIQUID through any one PIPE.

The following Statements appear in a translation of Newton's Principia in the New York Eng. Library, Case 531–N 48–AZ 2:

"AXIOMS or LAWS OF MOTION

LAW I. Every body perseveres in its state of rest or of uniform motion in a right line unless it is compelled to change that state by forces impressed thereon.

LAW II. The alteration of motion is ever proportional to the motive force imposed and is made in the direction of the right line in which that force is impressed.

LAW III. To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

These axioms and the statement that, in Steady Flow,

> THE RESISTANCE is always equal to THE FORCE and acts in an opposite direction

will be accepted and adopted as true and requiring no proof.

NOTES

Charles Lane Poor: Ph. D., Johns Hopkins, 1892:

> Assistant of and successor to Simon Newcomb as head of the Department of Astronomy, Johns Hopkins, 1892–1900. Professor of Astronomy at Columbia to 1910. Professor of Celestial Mechanics at Columbia 1910 to date. Author of many works on gravitation, on celestial mechanics, on navigation, and on the construction of racing yachts; inventor and designer of navigational devices used during the War of 1917.

The quotation, opposite, is from Reynolds paper reported in " Transactions, Royal Philosophical Society, London, 1883, p. 937, Part II." Quoting from page 977 (same volume):

> " For velocities above the critical values, the most important experiments were those of Darcy — approved by the Academy of Sciences and published 1845 — on which the formula in general use has been founded. Notwithstanding that the formula as propounded by Darcy himself could not, by any possibility, fit the results which I have obtained, it seemed possible that the experiments on which he had based his law might fit my law. A comparison was therefore undertaken."

The writer is under obligation to Stevens Institute for permission to use some of its splendid collection of reference books. The temporary loan of a copy of the " Transactions " is appreciated as it gave time for a careful study of Reynolds work.

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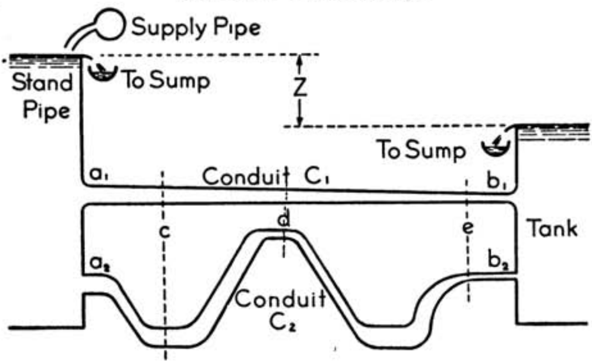
Edgar Buckingham: Ph.D., Leipsic, 1893:

Graduated, Harvard 1887. Assistant in Physics, Harvard 1888–89, 1891–92, Strassburg 1889–90; Physicist, Bureau of Standards for many years. Author of numerous scientific articles. See page 86.

Filled Conduits



1. When through *any filled Conduit* a Fixed Volume (Bbls. per day, Liters per minute, etc.) of any Liquid is regularly passing an up-stream point (a), passing on through the system and passing a down-stream point (b), Then the same Fixed Volume passes each point along the route in the same length of time. The Liquid, the material and shape of the Conduit or the roughness of its surface, do not change the condition of Steady Flow.

When there is any one Steady Flow, the Speed at which any actual Volume passes any one point in any one time is not the Velocity of any definite Volume contained in even the shortest length of any Conduit or of any Pipe at that point.

Dr. Edgar Buckingham states, Vol. 37, Transactions, A.S.M.E., page 266, —

"When a liquid flows, at a constant rate, through a smooth straight pipe, the pressure gradient G may be expected to depend on the diameter D, speed S, and density ρ and viscosity μ of the liquid."

When Any One Liquid is Flowing through Any One Rigid, Filled Conduit at Any One Steady FIXED Rate of Flow, the same Volume will Continue to Flow at such FIXED Rate until there is some change in the conditions under which the liquid is flowing.

Velocity is not a Factor in these Steady Flow problems for the reason that the same Volume regularly passes each point along the line.

Density is not a Factor as only one Liquid is present during any one Steady Flow.

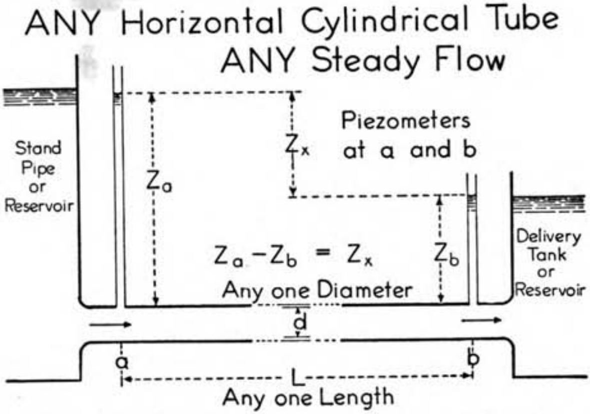
Viscosity is not a Factor as any one Liquid, at any one Temperature, during any one Steady Flow has but one Viscosity.

STEADY FLOW depends Exclusively on The Liquid, The Pipe and on a Steady Force.

The essential Factors are

THE LIQUID,
THE PIPE,
THE FORCE,
THE RESISTANCE and
THE STEADY FLOW of a FIXED VOLUME of The Liquid past each point in The Line.

2. When *any one Fixed Volume* (______ Bbls. per day, ______ Cu. Ft. per hour or any other Fixed Volume) of *any one Liquid* is passing regularly into *any one Filled Pipe* at an up-stream point (a), passing on through the Pipe and passing a down-stream point (b), Then the *same Fixed Volume* passes each point along the route in the *same length of time.* When such a condition exists, it will be considered that there is *a Steady Flow of a Fixed Volume of the Liquid through the Pipe.*



3. The Liquid has but one Density and but one Viscosity at any one temperature.

4. The Liquid, the material or the roughness of the Pipe surface, do not change this condition of Steady Flow. Such Steady Flow has no definite relation to the velocity of any part of the Liquid at any one point at any cross-section of the Pipe.

5. When the actual Conduit is a Horizontal, Cylindrical Tube it is called The Pipe. The dimensions of The Pipe are

Length L, Diameter d, Circumference πd,
Area $\pi \frac{1}{4} d^2$ and Volume $\pi \frac{1}{4} d^2 x$.
π is the constant **3.1416**.
For any one Pipe, the L does not change.

CHAPTER II

THE PIPE LINE FLOW FORMULA

When there is a STEADY FLOW of any one FIXED VOLUME of any one Liquid through any one Pipe

THE FACTORS are THE LIQUID, THE PIPE, THE FORCE, THE RESISTANCE and the STEADY FLOW of a FIXED VOLUME of THE LIQUID past each point in the line.

STEADY FLOW depends Exclusively on The Liquid, The Pipe and on a Steady FORCE produced by a Steady Pressure-Difference.

In order to avoid endless repetition, the meaning of certain words and characters is limited and defined as follows:

THE LIQUID—Any one homogeneous, practically non-compressible Fluid at any one uniform temperature.

THE PIPE—Any one horizontal, cylindrical tube constructed of rigid, homogeneous material, having a uniform interior surface, filled with The Liquid and at The Liquid temperature.

Let L be its Length between two points.

Let d be its Diameter.

Let q be the Volume flowing steadily, each second, past each point in the Line.

Let x be the length of THE PIPE that will contain q.

The x is used so that any one Volume per second may be stated in dimensions of The Pipe.

6. Pressures expressed in terms of the height of a column of The Flowing Liquid can be changed to any other pressures (lbs. per sq. in., etc.) by taking into consideration the weight of a given Volume of The Flowing Liquid.

7. The Pressures, Z_a and Z_b, must be measured at corresponding points on the cross-section of The Pipe. When measured by an open tube piezometer at the top of The Pipe, it insures a filled line.

8. The Unit of Length used does not change the actual Pressure, z_x, the Liquid, the Pipe material or the roughness of the Pipe. Φ is the Number that makes the Force equal to the Resistance.

9. A careful study of Reynolds' work shows that Volume (as obtained from the length, x, and the diameter, d) is at the base of his statements. Reynolds called this length "the velocity."

10. Any Unit of Length that may be selected and used makes no difference in the actual dimensions of the Pipe, i.e., with x, L, d, πd, $\pi \frac{1}{4} d^2$, $\pi \frac{1}{4} d^2 x$, or q_x.

11. The known Volume and the diameter establishes the length x in terms of the same unit of length that is used in measuring the Volume and The Pipe.

12. The Volume, a cubic, changes in direct proportion to the length, x.

13. When a unit of length is selected, adopted and used consistently throughout the equations to measure The Pipe and the Pressure, *then for any one Steady Flow of any one Liquid through any one Pipe*

(A) The Numerical Volume, $\pi \frac{1}{4} d^2 x$ is Fixed;

(B) The Numerical Force, $\pi \frac{1}{4} d^2 z_x$ is Fixed, and

(C) The Numerical Resistance, $L \pi d \Phi_x$ is Fixed by the Numerical Force.

ALL PRESSURES are expressed in terms of the height of a column of the Flowing Liquid.

Let Z_a be the Pressure at (a).
Let Z_b be the Pressure at (b).
Let z be $Z_a - Z_b$, the Pressure which, when multiplied by THE PIPE Area, determines the Force.

THE FORCE that causes the Steady Flow between (a) and (b) is due to the Pressure z acting throughout the area of THE PIPE.

THE RESISTANCE is due to the clinging of The Liquid to The Pipe Surface $\pi\, d\, L$.

Let $\Phi\, \pi\, d\, L$ be the Resistance.
Let ϕ be 4Φ. $(0.25\phi = \Phi)$

When there is a Steady Flow

(A) The Volume, by definition, is $\pi\, d^2\, 0.25\, x$

(B) The Force, by definition, is $\pi\, d^2\, 0.25\, z_x$

The relation between Volume and Force is established by (A) and (B) for any one Steady Flow, i.e., for every Steady Flow.

(C) The Resistance, by definition, is $\pi\, d\, L\, \Phi_x$

The relation between Volume and Resistance is established by (A) and (C) for every Steady Flow.

The Resistance is always equal to the Force and can be measured in no other way.

14. Equation (I) is the Volume of a cylinder equal to the Volume of The Liquid that passes each point between (a) and (b) in each second as long as the one Steady Flow continues.

15. To agree dimensionally with x, the Force must have but one dimension of length. The Pressure, measured by one dimension of length, remains a pressure. When the Force expression is written $d(0.7854\, z_x)^{\frac{1}{2}}$ it contains but one dimension of length. This statement does not mean that this rule can be applied either to z_x or to the Force without considering the small changes in the Resistance character ϕ_x.

16. The numerical Pressure changes as the square root of the Volume changes. This is confirmed by all reliable tests that have ever been reported, but this Rule, often called a Law, holds only for small changes in Volume and the corresponding changes in Pressure. **Such apparent variations from the Law are due to the the fact that for every change in the value of x, the values of q_x, z_x, and ϕ_x, all change at the same time, but each changes according to the formula in which it appears.**

17. Equation (II) is simply a formulated statement that the Resistance is always equal to the Force.

$$\pi\, d^2\, 0.25\, z_x = \pi\, d\, L\, 0.25\, \phi_x$$

$$d^2\, z_x = \phi_x\, L\, d \qquad d^2\, z_x\, L^{-1} = \phi_x\, d$$

For any one Steady Flow, that is, for each and for every Steady Flow

The Volume measured in dimensions of The Pipe is

$$q = \pi d^2 0.25 x \qquad (I)$$

Let q_x be the Volume $\pi d^2 0.25 x$.

Let z_x be the Pressure that corresponds with q_x.

Let Φ_x, or ϕ_x, be the corresponding Resistance character.

The Force that causes the Steady Flow q_x is $\pi d^2 0.25 z_x$. Numerically, any one q_x divided by its corresponding $\pi d^2 0.25 z_x$ must equal some one number, say N_x.

$q_x = N_x \pi d^2 0.25 z_x$. Compare this with (I)

$q_x = \pi d^2 0.25 x$ then, numerically,

$N_x z_x = x$ in which z_x is a pressure active in one direction and x is a length. This is not a proper equation and is of no use except to show that a FIXED numerical relation exists between Volume and Force in any one Steady Flow.

For any one Steady Flow, The Liquid, The Pipe, and the value of each of the characters used is FIXED for that one Steady Flow. The relation among the Factors is not apparent until a Unit of Length is adopted.

As The Resistance is always equal to The Force and can be measured in no other way

$$\pi d L \Phi_x = \pi d^2 0.25 z_x \qquad (II)$$

$$\frac{\Phi_x}{z_x} = \frac{\pi d^2 0.25}{\pi L d} = \frac{\text{Pipe Area}}{\text{Area of The Pipe surface}}$$

18. These are all Static Equations because they apply only to the one condition that can exist at any one time of Steady Flow.

19. In any one Steady Flow, the d, the L, the x and the z_x are measured in terms of the same unit of length as is used for the Volume, but the Resistance, represented by the Φ_x or the ϕ_x can not be measured in this way.

20. Except for the canceled constants, equation (III) contains all of the characters that are represented in The Force and all that are represented in the Resistance. Each character is shown in its proper relation to all of the other characters. The d^2 and the $L\,d$ represent areas. The z_x may represent any Pressure and such Pressure is always used to measure The Force and such Force is used to measure the corresponding Resistance in which the ϕ_x appears.

21. For any one Steady Flow, the Numerical value of each and every character used in (IV) is FIXED; the constants π and $\frac{1}{4}$ are either present or have been eliminated.

22. In equation $(IV)^*$, the representatives of the Volume, the Force and the Resistance are each separately maintained in their entirety and each is in its proper relation to the other two.

23. In equation $(IV)_2$, the same actual diameter appears in the Volume section as d^4, it appears in the Force section as d^2 and in the Resistance section as d.

24. Any one number may represent the diameter of the same actual pipe according to the unit of length that is selected and used. Apply such a unit to equation $(IV)_2$ in which d^4 appears in the Volume section, d^2 appears in the Force section and d appears in the Resistance section. For any one Steady Flow, the actual pressure remains the same and is the pressure used to find the numerical value of ϕ_x. The actual x remains the same.

Using $0.25\,\phi_x$ for Φ_x and canceling removes the constants, and (II) becomes $\phi_x L\, d = d^2 z_x$. The $\dfrac{d^2 z_x}{L}$ represents The Force per unit of length and the $d\phi_x$ represents The Corresponding Resistance.

$d^2 z_x$ divided by $\phi_x L\, d$ equals one.

$$\frac{d^2 z_x}{L\, d\, \phi_x} = \left(\frac{d^2 z_x}{L\, d\, \phi_x}\right)^{\frac{1}{2}} = 1^{\frac{1}{2}} = 1 \qquad (III)$$

From equation (I) $\quad \pi\, d^2\, 0.25\, x = q_x = q$

Multiplying (I) by (III),

$$q = \pi\, d^2\, 0.25\, x \left(\frac{d^2 z_x}{L\, d\, \phi_x}\right)^{\frac{1}{2}} \qquad (IV)$$

$$q = (\pi\, d^2\, 0.25\, x) \left(\frac{\pi\, 0.25\, d^2 z_x}{L\, (\pi\, d\, \Phi_x)}\right)^{\frac{1}{2}} \qquad (IV)^*$$

This q is always the ONE VOLUME under consideration and, at every such time, the actual value of each character in the equation is FIXED.

$$q = \left((\pi\, 0.25)^2\, d^4\, x^2\, \frac{d^2 z_x}{L\, d\, \phi_x}\right)^{\frac{1}{2}} \qquad (IV)_1$$

$$q = \left(0.61685\, d^4\, x^2\, \frac{d^2 z_x}{L\, d\, \phi_x}\right)^{\frac{1}{2}} \qquad (IV)_2$$

in which $(\pi\, 0.25)^2$ is replaced by the constant 0.61685 and The Volume and The Force have the same grade or standing in the equation.

25. Equation (IV) in any of its many forms can not be used as a functional equation. It is not an empirical formula. It is not based on any tests or experiments. It is only for one Steady Flow at any one time.

26. Equations (II) and $(IV)_3$ each show a relation that always exists during any one Steady Flow. Equation (V) shows the relation that always exists between ϕ_x and q. Such relation is established for every one of the many values of q_x that are in the number of C which changes when the length x changes.

27. The Force section, which includes z_x, divided by the Resistance section, which includes ϕ_x, is always equal to *1*. When the x changes, the z_x changes and the ϕ_x changes accompanying the change in Volume due to the change in the length x. As the diameter does not change in the Any One Pipe under consideration, and as the Liquid always remains the same in Any One Steady Flow, the difference in Numerical value in the three sections must be due to the difference in the d^4, d^2 and d.

28. This C contains any one value of z_x which changes when q_x and ϕ_x change. The relation in (IV) is always maintained. In the q_x, the C includes d^4. It also includes d^2 from the Force and d from the Resistance, each in its proper relation to the other.

When a unit of length is adopted and used consistently to measure The Pipe and The Pressure,

Then, for any one Liquid flowing steadily through any one Pipe, The Liquid and The Pipe do not change. The Steady Flow Volume, The Force and The Resistance all change when any one of the three changes.

Using the $(IV)_1$ form of the Equation

x^2 in The Volume, i.e., in $(q_x)^2$
z_x in The Force, and
ϕ_x in The Resistance, must all change at the same time when there is a change in The Steady Flow.

Then for different values of any one q_x Flowing Steadily

x^2 is multiplied by *0.61685* d^4,
z_x is multiplied by d^2,
ϕ_x is multiplied by $L\,d$ and the Numerical value of these multipliers does not change.

The Pressure changes as the square root of The Volume changes; The Resistance changes as the fourth root of The Volume changes, and The Force is always equal to The Resistance.

Again using form $(IV)_2$ of the equation, and using C to replace all of the characters except q and ϕ_x

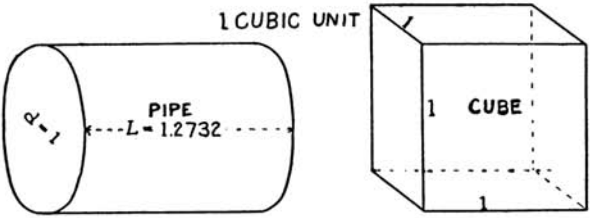
Let C be $0.61685\, d^4\, x^2 \dfrac{d^2\, z_x}{L\, d}$

Then equation (IV) becomes

$$q = \left(\frac{C}{\phi_x}\right)^{\frac{1}{2}} \qquad (IV)_3$$

29. When q is 1^3 and d is 1, then $q = \pi \frac{1}{4} d^2 x$ becomes $1^3 = (0.7854)\ 1^2 x$ and

$$x = \frac{1}{0.7854} = 1.2732.$$



30. This C has no definite numerical value until after a unit of length is adopted.

The unit of length FIXES

The Numerical Volume and The Numerical Force but always The unit of area to which The Force is applied must also be adopted because The Force is applied to all points of the area of the cross-section of The Pipe.

$$C = (q_x)^2 \left(\frac{d^2\, z_x}{L\, d}\right) = 0.61685\, d^4\, x^2 \left(\frac{d^2\, z_x}{L\, d}\right)$$

The Equation $q = \left(\frac{C}{\phi_x}\right)^{\frac{1}{2}}$ is only for one Steady Flow, i.e., for every Steady Flow when considered by itself. The Equation $q^{\frac{1}{2}} = \frac{C}{\phi_x}$ is for every steady flow. ALL of the characters in (IV) with the exception of q and ϕ_x are, by assumption, inherent in C. (See Note 25, Page 14.)

We are interested in the relation that exists between the Any One value of ϕ_x and the corresponding Volume, q_x; The Volume appears in (IV) as q and also as $\pi\, 0.25\, d^2\, x$; The same Volume appears in $(IV)_2$ as $(0.61685\, d^4\, x^2)^{\frac{1}{2}}$, and The same Volume appears in $(IV)_3$ as $q = \left(\frac{C}{\phi_x}\right)^{\frac{4}{3}}$.

It is this relation between the q and the ϕ_x, as they appear in $(IV)_3$, that interests us. Such relation depends on the $\boldsymbol{d^4}$ (in the $\boldsymbol{q_x^2}$) and on the $\boldsymbol{d}$ (in the Pipe Surface) as they appear in the definition of $\boldsymbol{C}$. This relation is

$$\boldsymbol{q^{\frac{3}{4}} = \frac{C}{\phi_x}} \qquad (V)$$

At all times When $\boldsymbol{q = 1^3}$ Then $\phi_1 = \boldsymbol{C_1}$. The subscript, $_1$, indicates that there is a Steady Flow of 1 cubic unit, each second, past each point between (a) and (b) for the duration of the one Steady Flow.

The changes in ϕ are so small that they are often disregarded and ϕ is considered to remain the same even when there is a considerable difference in the volume flowing. These differences are so small that, unless there is quite a difference in the volume flowing, the changes in ϕ are obscured by the small errors of observation.

The changes in z_x are also small for small changes in Volume, BUT

$d^2\, z_x$ always equals $L\, d\, \phi_x$
$0.7854\, d^2\, z_x$ always equals $L\, \pi\, d\, \Phi_x$
The Force always equals The Resistance.

31. When there is a Steady Flow of any one Liquid through any one Pipe, there is no change in The Pipe, in The Liquid or in the actual value of any character that appears in formula (II) or in formula (IV). There is no change in the heaviness (γ) or in the viscosity of The Liquid.

32. For many purposes the pressure is expressed in terms of pressure in pounds per square inch. Force, in terms of pounds per square inch (for the same h_v) will "vary directly with the heaviness (γ) of the liquid."

33. "Abstract number, a number considered apart from the objects enumerated." When considered as applied to such things as Density, Gravity, Velocity or Viscosity, the character f represents some "abstract number" as it is entirely "apart from" such "objects."

34. "Concrete number, a number applied to particular objects." When the character f is considered as applied to the Resistance of any one Specified Liquid when Flowing Steadily, during any one period of time, through any one Specified Pipe, it is a concrete number and depends absolutely on one fixed Force and the length, the area of such Pipe and the Volume in steady flow.

CHAPTER III

f THE PIPE LINE FLOW FACTOR

The character f is used in the Darcy type of Pipe Line Flow formulas, to represent the resistance.

"The amount of this resistance (often called *skin-friction*) for a given extent of rubbing surface is, by experiment, found—

1. To be *independent of the pressure* between the liquid and the solid;
2. To vary nearly with the *square of the relative velocity;*
3. To vary directly with the *amount of rubbing surface;*
4. To vary directly with the *heaviness* (γ, . . .) of the liquid.

"Hence for a given velocity v, a given rubbing surface of area $= S$, and a liquid of heaviness γ, we may write

$$\text{Amount of friction (force)} = f\,S\,\gamma\,\frac{v^2}{2\,g},$$

in which f is an abstract number called the *coefficient of fluid friction*, to be determined by experiment. For a given liquid, given character (roughness) of surface, and small range of velocities, it is approximately constant. The object of introducing the $2\,g$ is not only

35. For any one Steady Flow, the Darcy type Formula is

$$q = 0.7854\ d^2 \left(\frac{2\,g\,h\,d^2}{f\,L\,d}\right)^{\frac{1}{2}}$$

$q \div 0.7854d^2$ = what is called the "mean velocity." It has no connection with the relative velocity of particles of the Steady Flowing Liquid.

$q \div 0.7854d^2$ = a length which is here called v and is the length of pipe which will contain q.

$2g$ is a constant and does not change in actual value with any unit of length used. Using the foot as the measure of length and calling $(2g)^{\frac{1}{2}} = 8$, then the formula becomes

$$q = 6.2832\ d^2 \left(\frac{h\,d^2}{f\,L\,d}\right)^{\frac{1}{2}}$$

but the square root of this $2g$ is a little more than 8. The English have used 6.3 for the constant in their Darcy type formulas for more than 60 years. As long as the diameter is fixed, the only characters that can change when q changes are in $\left(\frac{h}{f\,L}\right)^{\frac{1}{2}}$. The ratio of h to L does not change with a change in the unit of length. The Force represented by the pressure h does not change for any one Steady Flow; therefore, the f for any one steady flow does not change with the unit of length.

36. Such elimination of the v is not good algebra;

$$v = \frac{q}{0.7854\ d^2} \text{ is always true;}$$

$$v_g = \left(\frac{2\,g\,h\,d^2}{f\,L\,d}\right)^{\frac{1}{2}} \text{ is for one value of } (2\,g)^{\frac{1}{2}};$$

$\left(\frac{h\,d^2}{f\,L\,d}\right)^{\frac{1}{2}}$ = **1** at all times, therefore, v_g must always be equal to the square root of **2** g.

because $\frac{v^2}{2\,g}$ is a familiar and useful function of v, but that $v^2 \div 2\,g$ is a *height*, or distance, and therefore the product of S (an area) by $v^2 \div 2\,g$ is a *volume*, and this volume multiplied by γ gives the *weight* of an ideal prism of the liquid; hence, $S\,\frac{v^2}{2\,g}\,\gamma$ is a *force* and f must be *an abstract number* and, therefore, the same in all systems of units, in any given case or experiment."

(This quotation is from Mechanics of Engineering, Church, Fluids, 1889.)

The $\frac{v^2}{2\,g}$ is a length and $S\,\frac{v^2}{2\,g}$ is a volume. The v^2 and the $2\,g$ are from the familiar formula $v^2 = 2\,g\,H$ which is used to obtain the velocity of a body falling freely from rest a distance H. In $v = (2\,g\,H)^{\frac{1}{2}}$ the v is a length.

The Distance H has absolutely no connection with the h in common use in the Darcy type of flow formula. The v in the Darcy type formula is neither a velocity nor a mean velocity but is a length. The $2\,g$ is a constant interjected with the unused, here useless, distance H.

$$v = \frac{q}{0.7854\,d^2} \qquad\qquad v = \left(\frac{2\,g\,h\,d^2}{f\,L\,d}\right)^{\frac{1}{2}}$$

Eliminating the v then (See Note 36.)

$$q = 0.7854\,d^2\left(\frac{2\,g\,h\,d^2}{f\,L\,d}\right)^{\frac{1}{2}}$$

The Darcy Type Formula is equation (IV) except that it contains the constant $2\,g$ and the v has been eliminated. **The Formula is not dimensionally true.**

37. The Darcy type of flow formula is not a functional equation. The elimination of the v is not logical reasoning.

Using the renamed characters

38. In Flow formula (IV) it is evident that neither $q = 0.7854\, d^2\, v$ nor $\left(\frac{d^2\, h_v}{L\, d\, f_v}\right)^{\frac{1}{2}} = 1$ has anything to connect it with more than the one case of Steady Flow that exists at any one time. It is only through formula (V) that the different values of h_v, as it appears in the formula $C = (0.7854\, d^2\, v)^2 \frac{d^2\, h_v}{L\, d}$, can be connected to the different values of q. The equation $q^{\frac{1}{2}} = \frac{C}{f_v}$ connects the h_v that is in the C with the Volume q and the coefficient f_v, and was used, under The Specification, to establish the Volume and f_v tables and the charts which appeared in the first edition of the Pipe Line Flow Factor book. The Specification establishes one set of numbers referring to an actual size of Pipe, of hypothetical roughness, and a hypothetical Liquid and such actual values which are close to those for Water, at some temperature, when flowing steadily through a Pipe one foot in diameter of some unspecified roughness of pipe surface.

39. A Cylindrical Pipe has a fixed area, therefore, in Steady Flow, the progress of any one given Volume is also fixed but the Velocity of particles of the Liquid is not fixed. (*Continued on page 24.*)

It is proposed to adopt the characters in common use and to rename some of them.

Let L be the Length between two points.

Let d be the pipe diameter.

Let q or V be the Volume flowing steadily, each second, past each point in the Line.

Let v be the length, x, of The Pipe that will contain V or q.

Let h_v be z_x the Pressure difference measured in terms of the height of a column of The Flowing Liquid.

Let f_v be the Resistance factor, ϕ_x.

Equation (IV) thus modified becomes

$$q = 0.7854\, d^2\, v \left(\frac{h_v\, d^2}{L\, d\, f_v}\right)^{\frac{1}{2}}$$

and Equation (V) becomes (See (V) page 17.)

$$q^{\frac{1}{2}} = \frac{C}{f_v}.$$

For every Steady Flow

$$q = 0.7854\, d^2\, v = 0.7854\, d\, (v\, d)$$

In order that $d\,v$ may be the same for pipes of different diameters, the v must change inversely as the diameter changes. The $h_v \div f_v$ does not change with the pipe diameter but v and q_v change when the diameter changes.

39 (*Continued*). When any one Pipe-Liquid system is under consideration, then for any one value of q there is but one value of q_v, one value of C and one value of f_v. For such system for each q_v there will be only one value of h_v and one value of f_v.

$$q = 0.7854\, d^2\, v \left(\frac{h_v\, d^2}{L\, d\, f_v}\right)^{0.5} \qquad (IV)$$

$$q_v = 0.7854\, d^2\, v \qquad\qquad \frac{h_v\, d^2}{L\, d\, f_v} = 1$$

$$C = (q_v)^2\, \frac{h_v\, d^2}{L\, d} \qquad\qquad q^{0.25} = \frac{C}{f_v} \qquad (V)$$

The f_v for any cross-section of a meter of the Venturi type will not change for any one q_v.

CHAPTER IV

A UNIT OF LENGTH IS ADOPTED

In this chapter the Foot as the Unit of Length and the Second as the Unit of Time are adopted BUT they are used only to establish numerical values for the Pipe Line Flow Factor *f*.

The numerical relation among the Factors and Characters can only be shown when a Unit of Length is adopted and used.

THE FACTORS are THE LIQUID, THE PIPE, THE FORCE, THE RESISTANCE and the STEADY FLOW of a FIXED VOLUME of THE LIQUID past each point in the line.

STEADY FLOW depends Exclusively on The Liquid, The Pipe and on a Steady FORCE produced by a Steady Pressure-Difference.

In order to avoid endless repetition, the meaning of certain words and characters is limited and defined as follows:

THE LIQUID—Any one homogeneous, practically non-compressible Fluid at any one uniform temperature.

40. The difference in Pressure, $\boldsymbol{h_v}$, is the only source available for furnishing the Force required to overcome the Resistance due to the Pipe surface, the Liquid, its Density or its viscosity. It makes no difference whether the resistance to the Steady Flowing stream is due to "turbulent flow" or to "parallel flow," as all of the resistance is transmitted through the Liquid to the Pipe surface.

The pressure $\boldsymbol{h_v}$ is measured by the height of a column of the Flowing Liquid. As the height of a column of the Liquid can be used to measure the density it seems unnecessary to introduce a character to represent density into the flow formula, particularly for the reason that during any one Steady Flow there is but one Liquid having a fixed density. Density is not a factor in the Steady Pipe Line Flow Formula.

THE PIPE—Any one horizontal, cylindrical tube constructed of rigid, homogeneous material, having a uniform interior surface, filled with The Liquid and at The Liquid temperature.

Let $\boldsymbol{L}$ be its Length between two points.

Let $\boldsymbol{d}$ be its Diameter.

Let $\boldsymbol{q}$ be the Volume flowing steadily, each second, past each point in the Line.

Let $\boldsymbol{v}$ be the length of THE PIPE that will contain $\boldsymbol{q}$.

ALL PRESSURES are expressed in terms of the height of a column of the Flowing Liquid.

Let Z_a be the Pressure at (a).

Let Z_b be the Pressure at (b).

Let $\boldsymbol{h}$ be $Z_a - Z_b$, the Pressure which, when multiplied by THE PIPE Area, determines the Force.

THE RESISTANCE is due to the clinging of The Liquid to The Pipe Surface $\pi \boldsymbol{d} \boldsymbol{L}$.

Let $\Phi \pi \boldsymbol{d} \boldsymbol{L}$ be the Resistance.

Let $0.25\, \boldsymbol{f}$ be Φ.

When there is a Steady Flow

(A) The Volume, by definition, is $\pi\, \mathbf{0.25}\, \boldsymbol{d}^2\, \boldsymbol{v}$.

(B) The Force, by definition, is $\pi\, \mathbf{0.25}\, \boldsymbol{d}^2\, \boldsymbol{h}_v$.

(C) The Resistance, by definition, is $\pi\, \boldsymbol{d}\, \boldsymbol{L}\, \mathbf{0.25}\, \boldsymbol{f}_v$.

The subscript $\boldsymbol{v}$ indicates that the character is for the one Volume $\boldsymbol{q}_v$.

41. The Steady Flow Volume is always

$$q = 0.7854\, d^2\, v$$

that is

$$q = 0.7854\, d\, (v\, d)$$

When the product $v\, d$ is the same, even though the pipes are of different diameters, the same value of f_v is indicated.

42. Formulas (*IV*) and (*V*) "establish, determine and definitely FIX the intimate relation that always exists among all the FACTORS when there is a STEADY FLOW of any one FIXED VOLUME of any one LIQUID through any one PIPE."

43. In any one Steady Flow, a change in the unit of length does not change the actual dimensions of The Pipe or the actual difference in Pressure between the two points (*a*) and (*b*); also, it does not change the Volume, the Force or the Resistance.

44. In such Steady Flow, the actual dimensions in (IV) FIX the actual relation that always exists at the time of such any one Steady Flow, and a change of the unit of length does not change such actual relation.

45. When the same Volume of any Liquid is Flowing Steadily through a pipe of the same diameter, the ratio of h_v to f_v does not change, but the ratio of h_v to L will change with The Liquid and The Pipe.

Equation (IV) using these dimensioned characters becomes

$$q = 0.7854\, d^2\, v \left(\frac{d^2\, h_v}{L\, d\, f_v}\right)^{\frac{1}{2}} \qquad (IV)$$

and Equation (V) becomes

$$q^{\frac{1}{2}} = \frac{C}{f_v} \qquad (V)$$

f_v is a concrete number.

The substitution of numerical values in (IV) is not adequate to establish the relation among the characters in the three Basic Formulas.

(A) Volume = $0.7854\, d^2\, v = q_v$.

(B) Force = $0.7854\, d^2\, h_v$.

(C) Resistance = $\pi\, d\, L\, \Phi_v$ or $\pi\, d\, L\, 0.25\, f_v$.

The three formulas are different in nature and can not be included in any general equation, but the Static Equation (IV) applies to any one Steady Flow when considered by itself. Each of the three formulas contains the characters π and d and also each depends on one Volume measured by the Pipe Area and v, a length of the Pipe.

(A) is the Volume, q_v, measured in Pipe dimensions.

(B) is the Force measured by the Pipe Area and the difference in the pressures at two points (a) and (b). The Pressure difference is measured in terms of a length which has no relation to the length of the Pipe between (a) and (b).

(C) is a Resistance measured by $\pi\, d$, the length L and the numerical character Φ_v which makes the Resistance equal to the Force.

46. It is self-evident that a change of the unit of length does not change

(*a*) the actual Volume Flowing;
(*b*) the actual diameter or the area of The Pipe;
(*c*) any actual length of The Pipe;
(*d*) any actual Pressure;
(*e*) neither does it change the ratio of h_v to L, nor the ratio of h_v to f_v;

therefore, both ratios are FIXED for any one Steady Flow through The Pipe but only for such Steady Flow.

47. The First Specification fixes the actual diameter at **1** ft., the Steady Flow volume at **1** cu. ft. and $\boldsymbol{v_1}$ is **1** cu. ft. divided by The Pipe Area, 0.7854 sq. ft. which is 1.2732 ft.

48. The Second Specification fixes actual values for $\boldsymbol{f_1}$, $\boldsymbol{h_1}$ and $\boldsymbol{L}$. These two Specifications and the constant 0.7854 FIX numerical values for each and every character that is in equation (IV) as Specified.

49. The Third Specification designates one square inch as the area to which the pressure is applied.

The Specified $\boldsymbol{d^2}$ (as it appears in $\boldsymbol{d^2\, h_1}$) is 144 and the $\boldsymbol{d^2\, h_1}$ represents the Force acting throughout the 10,000 ft. For each foot in length $\boldsymbol{d^2\, h_1}$ can be written, numerically, either 144×0.000720 or 0.0144×7.20.

It is noted that 0.0288 is twice 0.0144 as should be expected because the $\boldsymbol{d^2}$ is **twice as strong** as the $\boldsymbol{d}$. Such relation between The Pipe Area, due to the $\boldsymbol{d^2}$ and The Area of The Pipe Surface, due to the $\boldsymbol{d}$, can be established by simple arithmetic. Assume a 1 foot Pipe increased in diameter to 1.0001, then the corresponding $\boldsymbol{d}$ is 1.0001 and the corresponding $\boldsymbol{d^2}$ is 1.00020001 or practically twice the amount of the change in the diameter.

It is practically impossible to select and specify one particular Pipe and one particular Liquid to be the two dominant Factors in a Pipe Line Flow Formula. Therefore, it is proposed to **specify a hypothetical Pipe and hypothetical Liquid** as a Standard or Base with which to compare other Pipe-Liquid combinations.

SPECIFICATIONS

Specifications for a Standard Pipe-Liquid Combination to be used in comparing Pipes and Liquids as to their Resistance to the Steady Flow of any one fixed Volume of any one Liquid through any one Pipe:

First— Any one Pipe one foot in diameter through which one cubic foot of any one Liquid passes in Steady Flow each second.

Second—It is further specified that when one cubic foot of such a Liquid is flowing steadily through such a one foot Pipe, the numerical value of f will be **0.0288** and h will be **7.2** feet when L is **10,000** feet of level pipe.

Third— The static pressure difference **7.2** feet will be considered as acting on one square inch area.

50. Were the pressure $\boldsymbol{h}_v$, i.e., 7.2 feet, in the Second Specification, changed, the Force would be changed in direct proportion but the constant 0.0288 would not be changed.

51. Were the third Specification changed to some other basic pressure area, the constant 0.0288 would change, but the other specification values would not change.

52. If the basic pressure area was changed from one square inch to one square foot, the d^2 in $d^2\ h_1$ would be 1 sq. ft. instead of 144 sq. in. and f_1 would be 2 instead of 0.0288.

53. The square inch has been used as the base for Liquid pressures on most of our important tests. There are many reasons why this practice should continue. There is a strong trend toward using metric system units but the Pipe Line Flow Factor f is tied to a series of numbers attributed to Professor Osborne Reynolds, the English Scientist, and such numbers are tied to the pressure per square inch. The writer used Dr. W. F. Durand's Reynolds Number tables as the foundation for the first "f" book. (*See note 60, page 36.*)

54. For many years the author used $f_v = 0.024$ when v in the 1 ft. pipe was 2.5 ft. Such Basic Pipe Line Flow Factor, 0.024, has been used for pipes of various diameters when the product $v\ d$ was 2.5.

55. The value $f_1 = 0.0288$ and $f_v = 0.024$ agree with the tables and the charts in the first "f" book.

The numerical values specified are as follows:

0.7854 is the Pipe Area

d_1 is **1** Foot in Length

h_1 is **7.20** feet

v_1 is $\frac{\mathbf{1}}{\mathbf{0.7854}} = \mathbf{1.2732}$

q_1 is **1** Cubic Foot

d_1^2 is **1** Foot in Area

L is **10,000** feet

f_1 is by assumption **0.0288**

C_1 is equal to $f_1 = \mathbf{0.0288}$

The fraction $\frac{h_v d^2}{L d f_v}$ is always the number **1**

When $q = 1^3$ then $f_1 = \mathbf{0.0288}$ which is C_1.

The numerical value of the expression $\frac{h_v}{L}$ **is not adequate to fix the numerical values for and among the other characters in (IV).** The reason being that when h_v and f_v are both multiplied by the same number, the value of the expression $\frac{h_v d^2}{L d f_v}$ remains equal to the number **1**.

For the One Foot Hypothetical Pipe and the Hypothetical Liquid, Equation (*V*) becomes, numerically, for the Specified Pipe-Liquid combination and other numerical values of f_v

$$f_v = \frac{0.0288}{q^{\frac{1}{4}}}$$

56. In 1889 the author copyrighted a special slide rule and published results of many days' pumping of Pennsylvania Grade oil flowing steadily through long 6-inch pipe lines when using initial pressures of about 900 lbs. per sq. in. The "velocity" being approximately 5 ft. per sec. in a 6-inch pipe. The oil had an average gravity of about 42° Baumé. The 42° mark on the slide rule has been retained but later re-named as being for $f = 0.024$. The same rule has been used for all kinds of Liquids without the necessity of changing the f marks on the slide rule.

57. Assuming that Reynolds used the English **dimensions,—foot—pound—second,** it does not seem amiss to consider a Pipe Line, **one foot** in diameter, one Liquid, at one temperature, so that there is a fixed value for the density, the viscosity and the diameter, **one foot, 0.3048 meter or 12 inches.**

58. For such a **1 foot** line, the only variable in the expression, $v\ d$, is the length v, so that for such **one foot** line, one Liquid at uniform temperature and Steady Flow, f_v can be tabulated or plotted in terms of Volume measured in **cubic feet per second**, in terms of the length v or in any other units of time and Volume.

59. **Velocity** does not enter into the Pipe Line Flow Formula but, like *Density*, *Weight*, *Viscosity* and *Gravitation*, it is a very important factor in many problems that have no place in the Basic formula for the flow of Liquids through Pipes.

Numerically, Equation (*V*), for other values of q becomes

$$q^{\frac{1}{4}} = \frac{0.0288}{f_v} \qquad (VI)$$

This equation is based on the Hypothetical Liquid and the Hypothetical 1 foot Pipe for any one value of q and the corresponding value of f_v.

Tables can be constructed from Equation (*VI*) and the Basic Formulas.

* *Remark.*—The f_v in (*VI*) being a concrete number it is evident that **0.0288** must be an abstract number and not derived from

$$C = \frac{d^2 h_v}{L d} (0.7854\, d^2 v)^2$$

In this value of C there appears the constant **0.7854** and the dimensioned characters d, L, and v, therefore it is evident that the number **0.0288** must in some way be connected with h_v, the pressure measured in feet.

Comparing values of f_v with the Reynolds Number method of treating the Darcy Flow Formula it was found that, using the same data, the Reynolds Number was always **14,400** times the number f_v. This agrees with STATEMENT IV.

The number of cubic feet of water flowing per hour through a smooth pipe one foot in diameter is one-fourth the Reynolds Number for the corresponding f.

* **See Notes 33 and 34, page 18.**

60. THE REYNOLDS NUMBER is tabulated and plotted in relation to f_v and has been so treated by so many Engineers and Scientists and checked by so many tests that when a test does not agree with the curve it is a warning to pause and consider all of the factors in the flow formula. Was there a steady flow? Was q accurately measured? Was the value of h_v properly obtained? If the physical factors used are found to be correct and accurate, and there is lack of agreement, it indicates that the values of density and viscosity, if used, were not the ones that existed in the line at the time of the test. Neither density nor viscosity is included in the pipe line test but is the result of tests made outside of the pipe line. The factors v and d are the only measures required to ascertain the value of q and it is their product that must be constant in each test for each particular value of f_v.

61. Why does the Reynolds Number for any test correspond to a certain value of f_v? It is because, in any test at steady flow, there is only one liquid, one density, one temperature and one viscosity, so that density, divided by viscosity for each test, is fixed for that Liquid, that Pipe and that Temperature. In any one Steady Flow, the Volume flowing, $0.7854\, d^2 v$, is fixed, so then the value dv must also be fixed.

62. In any one specific test, the character f_v has but one numerical value. Any one f_v depends absolutely on the constants π, 0.25, the fixed concrete numbers represented by d^2, d, v and h_v which represents the only one force that is in operation during such Steady Flow.

63. In any one Steady Flow, the one value of f_v depends, absolutely, on the one value of h_v and The Pipe dimensions. The one f_v does not depend on any other thing.

For any one Pipe and any one Liquid in any one Steady Flow for any unit of length

(A) The Volume is $q = 0.7854\, d^2\, v = q_v$ (*I*)

(B) The Force is $0.7854\, d^2\, h_v$

(C) The Resistance is $\pi\, d\, L\, \Phi_v$

$$\pi\, d\, L\, \Phi_v = 0.7854\, d^2\, h_v \qquad (II)$$

When $f_v = 4\, \Phi_v$ then $d^2\, h_v = L\, d\, f_v$ (*III*)

$$q = \pi\, 0.25\, d^2\, v \left(\frac{d^2\, h_v}{L\, d\, f_v}\right)^{\frac{1}{2}} \qquad (IV)$$

Let $$C = \frac{d^2\, h_v}{L\, d}\, (0.7854\, d^2\, v)^2$$

$$q^{\frac{1}{2}} = \frac{C}{f_v} \qquad (V)$$

When $q = 1^3$ then and only then $f_1 = C$.

When the unit of length is one foot,

q is one cubic foot and
d is one lineal foot,

then $C_1 = f_1$ and $v_1 = 1.2732$

This number C_1 will be called the

Basic Pipe Line Flow Factor

The Specification numerical value of C_1 is 0.0288

For any one Pipe and any one Liquid in any one Steady Flow for any unit of length

(A) The Volume is $q = 0.7854\, d^2\, v = q_v$ *(I)*

(B) The Force is $0.7854\, d^2\, h_v$

(C) The Resistance is $\pi\, d\, L\, \Phi_v$

$$\pi\, d\, L\, \Phi_v = 0.7854\, d^2\, h_v \qquad (II)$$

When $f_v = 4\, \Phi_v$ then $d^2\, h_v = L\, d\, f_v$ *(III)*

$$q = \pi\, 0.25\, d^2\, v \left(\frac{d^2\, h_v}{L\, d\, f_v}\right)^{\frac{1}{2}} \qquad (IV)$$

Let $$C = \frac{d^2\, h_v}{L\, d}\, (0.7854\, d^2\, v)^2$$

$$q^{\frac{1}{2}} = \frac{C}{f_v} \qquad (V)$$

When $q = 1^3$ then and only then $f_1 = C$.

When the unit of length is one foot,

q is one cubic foot and
d is one lineal foot,

then $C_1 = f_1$ and $v_1 = 1.2732$

This number C_1 will be called the

Basic Pipe Line Flow Factor

The Specification numerical value of C_1 is 0.0288

64. We are interested in the relation that exists between the Any One value of $\boldsymbol{f_v}$ and the corresponding $\boldsymbol{q_v}$. (See page 17.)

65. At all times when $\boldsymbol{q} = 1^3$. Then, $\boldsymbol{f_1} = \boldsymbol{C_1}$. Such value of $\boldsymbol{C}$ is the Only Value of $\boldsymbol{C}$ that is common to both equation (***IV***) and equation (***V***). (See page 37.)

66. **See Notes 49 on page 30 to 55 on page 32.**

It makes no difference what number is selected as the value of $\boldsymbol{C}$ for the *hypothetical Liquid* flowing through the *hypothetical 1 foot Pipe* and used as a Standard or Base with which to compare other **Pipe-Liquid combinations,** and it will not change the actual value of the $\boldsymbol{C}$ **for any one Pipe-Liquid combination** that may be under consideration at any one time.

67. When there is a Steady Flow "then a change in *h*, however slight, produces a change in volume flowing and a change in *f*. **The changes in *f* are so small that they are often disregarded and *f* is considered to remain the same even when there is a considerable difference in the volume flowing.** These differences are so small that, * * * the change in *f* may be obscured by the small errors of observation." This is quoted from the *f* book of 1934.

CHAPTER V

THE PIPE LINE FLOW CONSTANT

0.0288

The numerical value of ***C*** depends on the Steady Flow of q_v cubic feet of any one Liquid through any one Pipe having a diameter of 1 foot. The formulated definition of the Number, ***C***, is

$$C = \frac{d^2 h_v}{L d} (0.7854 \, d^2 \, v)^2$$

This number, ***C***, contains only numerical values of h_v and *dimensions* of the any one Pipe System that is being tested.

In any tests that may be made on any one Pipe System, the v and the h_v are the only numbers in the number ***C*** that can change.

Such changes are always accompanied by a change in the number q and in the number f_v. The numerical relation among the characters in equations (***IV***) and (***V***) is not disturbed by these changes.

By the Specifications, one value of f_v, that is f_1, is C_1.

We are not interested in any other numerical value of the character ***C***. (See Notes 25 and 26 on page 14 and first paragraph, page 17.)

Any one Steady Flow of any one Liquid through any one Pipe is caused by a steady pressure h_v.

68. "In 1889, the author * * * noted on the special slide-rule graduated at that time, that the rule was for a velocity of 5 feet per second in a 6 inch wrought iron pipe. The factor .024 was used as being about right for Pennsylvania Grade Oil at 51° F. and 42° Baumé gravity. Tests made soon after indicated that, the head being the same, the same coefficient could also be used for WATER. By using .024 as a basic value for f, calculations can be made and * * * correction can be made according to the following formula * * * :"

TABLE I

$$\text{Factor} = (.024 \div f)^{\frac{1}{2}}$$

f	Factor	f	Factor	f	Factor
.018	1.155	.029	0.910	.040	0.775
.019	1.124	.030	.894	.05	.693
.020	1.096	.031	.880	.06	.632
.021	1.069	.032	.866	.07	.585
.022	1.044	.033	.853	.08	.548
.023	1.022	.034	.840	.09	.516
.024	1.	.035	.828	.10	.490
.025	.980	.036	.817	.20	.346
.026	.961	.037	.805	.30	.283
.027	.943	.038	.795	.40	.245
.028	.926	.039	.784	.50	.219

This table and the factor formula are applicable to all Liquids and all Pipes.

The quoted portion above is from the 1934 edition of the f book.

In any one Pipe-Liquid combination, a change in h causes a change in q, a change in v and also a very small change in f_v. (See Note 67.)

The following intimate relations among the characters ALWAYS exist:

$$d^2 h_v = L\, d\, f_v, \qquad q_v = 0.7854\, d^2\, v$$

$$q_v = 0.7854\, d^2\, v \left(\frac{d^2 h_v}{L\, d\, f_v}\right)^{0.5} \qquad (IV)$$

For any one Steady Flow of any one Liquid through any one Pipe the number C_1 does not change.

Let C_1 be the Number **0.0288**

$$q_v^{0.25} = \frac{C_1}{f_v} = \frac{0.0288}{f_v} \qquad (VI)$$

See Specifications, page 31, and Notes opposite.

Using the subscript$_1$ to represent the conditions existing under the Specifications, and

Using the subscript$_2$ to represent the conditions existing When any different diameter of pipe is used; Then

When $q_1 = 0.7854\, d_1\, (v_1\, d_1)$ ***and*** $q_2 = 0.7854\, d_2\, (v_2\, d_2)$
In order that $(d\, v)$ ***may be the same for pipes of different diameters, the v must change inversely as the diameter changes.***

A change of the unit of length does not alter the actual v, the actual q_v, the actual d or the actual L.

A change of the unit of length does not alter the actual h_v and does not change the actual f_v.

The change of the unit of length does not alter the ratio of h_v to L and it does not change the ratio of h_v to f_v.

69. The following quotation is from page 707 of Prof. Church's 1889 book. The f used is the "small f" or Φ.

"The Coefficient, f, for Friction of Water in Pipes. —See eq. (1), § 510. Experiments have been made by Weisbach, Eytelwein, Darcy, Bossut, Prony, Dubuat, Fanning, and others, to determine f in cylindrical pipes of various materials (tin, glass, zinc, lead, brass, cast and wrought iron) of diameters from $\frac{1}{2}$ inch up to 36 inches. In general, the following deductions may be made from these experiments:

"1st. f decreases when the velocity increases; e.g., in one case with the same pipe

$$f \text{ was } = .0070 \text{ for } v = 2' \text{ per sec.,}$$

while $$f \text{ was } = .0056 \text{ for } v = 20' \text{ per sec.}$$

"2dly. f decreases slightly as the diameter increases (other things being equal); * * *

"3dly. The condition of the interior surface of the pipe affects the value of f, which is larger with increased roughness of pipe."

The diameter of the pipe was not stated. The table showed that .0070 was for a "velocity" of 2 ft. per second when the diameter was 3 inches; it was .0060 when the "velocity" was 8 feet in the same size pipe, and it was .0056 when the "velocity" was 20 feet. These correspond to f values of 0.0280, 0.024 and 0.0224.

Since 1889, the author has been using $f = 0.024$ and when conditions required making a correction. (See Note 67, page 38.)

The value of $\boldsymbol{C}$ changes when the unit of length changes. BUT the Specification $\boldsymbol{C}_1$ is the $\boldsymbol{f}_1$, the constant **0.0288.**

In the formulated definition of the number $\boldsymbol{C}$ the $\mathbf{0.7854}\ \boldsymbol{d}^2\ \boldsymbol{v}$ is the Volume flowing. (Page 39.)

The actual volume does not change when the unit of length changes, but the numerical value of the Volume, of course, changes.

The ratio of $\boldsymbol{h}_v$ to $\boldsymbol{f}_v$ does not change because When the $\boldsymbol{h}$ and the $\boldsymbol{f}$ are both multiplied by the same number, their ratio does not change.

> "For first approximations, a mean value of $f = .006$ may be employed, since, in some problems, sufficient data may not be known in advance to enable us to take f from the table." (Church, Fluids, 1889, p. 708.)

The "$f = .006$" corresponds to the $f = 0.024$ as the reference is to the "small f" which is Φ, and the "table" referred to contained values of the "small f" for various velocities and Pipe diameters.

The $\boldsymbol{h}$ changes when the Density or the Viscosity changes and may change when the Pipe or the Liquid changes.

Density, Viscosity, Temperature, Cohesion and Adhesion may require a different value of $\boldsymbol{h} \div \boldsymbol{L}$ but the ratio of $\boldsymbol{h}_v$ to $\boldsymbol{f}_v$ does not change because the Force is always equal to the Resistance. These relations and a Rational Formula for the Steady Flow of any one Liquid through any one Pipe will be discussed later.

70. From the title page

f_1
The Pipe Line Flow Constant
0.0288
f The Pipe Line Flow Factor

Their relation to
Density Gravity Velocity Viscosity
and the
Reynolds Number

These relations will be discussed in the five following notes.

71. DENSITY has no direct relation to any factor in the Steady Pipe Line Flow formula. The $\boldsymbol{h}_v$ may be any number and is measured in terms of the height of a column of the Flowing Liquid. A Liquid of any Density may be used as the Specified Liquid as long as it meets all of the requirements of the Specifications.

The Relation of Density to the Steady Flow Formula.

The force which acts to produce the steady flow of a Liquid through a pipe is measured in the flow formula in terms of the head, $\boldsymbol{h}$, of a column of the Liquid.

When all resistances, except that due to gravity, are neglected, the Power required to lift one cubic unit of any substance one unit in height at the rate of one unit in height in one unit of time is numerically equal to the weight of the cubic unit. In the foot, pound-mass, second system, the Power required to lift one cubic foot

CHAPTER VI

STEADY PIPE LINE FLOW. LIQUIDS

The Pipe Line Flow Equation*

$$q_v = 0.7854\, d^2\, v\left(\frac{h_v\, d^2}{L\, d\, f_v}\right)^{0.5} \qquad (IV)$$

is for any one Steady Flow of any one Fixed Volume of any one Liquid through any one Pipe. It is, therefore, for every Steady Pipe Line Flow when considered by itself.

The equation is dimensional as it contains only fundamental relations between units of Length, Time and Mass. It is a rational equation.

Such a formula for the Steady Flow of a liquid through a Pipe must consider

(*a*) some one definite Volume of some one Liquid, and

(*b*) some one definite size of Pipe.

These can both be expressed in terms of any unit of length. The numerical lengths used will change with the unit of length; therefore, it is necessary to select some one unit of length.

*(The equation is not an empirical formula as it is independent of experience, experiment or test. It is not an exponential equation.)

of water one foot at the rate of one foot in one second is equal to 62.33* ft. lb. sec. To lift one cubic foot of salt water, having a specific gravity of 1.2, one foot in one second at a constant rate would require 74.796 ft. lb. sec. of Power. The Power required is in proportion to the relative density. In steady pipe line flow, the force is applied by the moving fluid at all points along the line and is used to overcome resistances at such points and in supplying the force to move the fluid ahead of it.

The Power required to overcome the resistance to steady flow between two points along a line is equivalent to the Power which would be required to lift the moving mass the height *h* at a steady rate in the same time.

72. GRAVITY, at any one cross-section of the Pipe, acts on every particle of the Liquid with almost exactly equal Force and has no other relation to any Steady Flow past such point. It is common practice to use the expression, " flow by gravity," when the only activating Force is that due to Gravitation.

73. VELOCITY is a very important factor in many problems but it has no rightful place in the formula for any one Steady Flow of any one Liquid through any one Pipe. " Velocity " for " incompressible fluids is equal to the Volume flowing past a given point in the line, in any given time, divided by the area of the pipe at that point." It is numerically the length of The Pipe that such Volume will fill.

* Kent's Table for Water at 68°F.

Let One Foot be The Unit of Length; then

The Cubic Foot is the Unit of Volume, i.e., Mass,

The Lineal Foot measures the Pipe, and

The Lineal Foot measures the height of a column of The Flowing Liquid.

Pressures expressed in terms of the height of a column of The Flowing Liquid can be changed to any other pressures (lbs. per sq. in., etc.) by taking into consideration the weight of a given Volume of The Flowing Liquid.

The Pipe Line Flow Formula is concerned only with dimensions of length.

(*a*) The Volume in Steady Flow;
(*b*) The diameter of the Pipe;
(*c*) The distance between two points at which the pressure is taken;
(*d*) The pressure difference between two points; all measured in terms of length.

The characters used are:

q_v The Volume in Steady Flow;
d The diameter of the Pipe;
L The distance between two points where the pressure is taken;
v The length of the Pipe* that will contain q_v;
h_v The pressure difference, and
f_v The resistance character.

The subscript $_v$ refers to one Volume.

* The length of Pipe, v, is often erroneously called "the mean velocity")

74. VISCOSITY represents the "resistance to mobility" of the Liquid. T. G. Delbridge's statement in Dr. David T. Day's "Handbook of the Petroleum Industry" (Vol. I, page 640) follows:

> "Viscosity is that property of a liquid which resists any force tending to produce flow. It should, therefore, be measured in units of force applied under definite conditions."

The definite relation of h to Viscosity will be considered in Chapter X.

Viscosity has no definite relation to density or to any factor in the Steady Pipe Line Flow Formula. The Relative Viscosity of Liquids can be obtained by using the ratios of the pressures, $\boldsymbol{h_v}$, required to produce the same volume of flow of the Liquids through the same pipe.

75. The effects of roughness can be compared by using one Liquid and one size of pipe of the same material. The effect of pipe material can be compared in a similar manner. These properties can not be represented by symbols as there is no rule or law for the effect of pipe roughness or of pipe material.

The equation which connects all Steady Flow is

$$q_v^{0.25} = \frac{C}{f_v} \qquad (V)$$

It is derived from (*IV*) by introducing C, a changing number which contains $\mathbf{0.7854\, d^2\, v}$ and also contains, among other pipe dimensions, the pressure difference h_v which always changes with q_v and f_v. (See pages 17 and 23; also Note 38, page 22.)

For any unit of length

$$\textit{When } q_1 = 1^3 \quad \textit{and} \quad d = 1$$

$$\textit{Then } f_1 = C_1 \quad \textit{and} \quad v_1 = 1.2732$$

A change of the unit of length does not change

(*a*) the actual Volume Flowing;
(*b*) the actual diameter of The Pipe;
(*c*) the actual length of The Pipe;
(*d*) the actual Pressure;
(*e*) it does not change the ratio of h_v to L or the ratio of h_v to f_v;

therefore, it makes no difference what unit of length is used.

Some one unit of length must be used to FIX the actual volume and the actual pipe.

For a 1 foot line the only variable in the expression ($v\, d$) is the length v so that for such a one foot line, one Liquid and Steady Flow, f can be tabulated or plotted in terms of Volume measured in cubic feet per second in terms of the length v or in any other units of time and Volume.

76. The Reynolds number is, numerically, 14,400 times the number of cubic feet of any one Liquid that Flows Steadily, per second, through any one Pipe one foot in diameter. It changes directly with $\boldsymbol{Q_v}$ and with $\boldsymbol{v}$. Reynolds used the Darcy experiments and not the Darcy formula to establish his law. He introduced Density and Viscosity into his formula and used them with the d and the v of Darcy. (See quotation from Reynolds paper on page 2.) Reynolds was not interested in the volume flowing but simply in the relation of the other factors to the coefficient f and to density and viscosity. Had Reynolds been interested, primarily, in the volume flowing, .7854 would have been used instead of 3.1416 in the formula for the unit diameter pipe line. That would have produced a curve of f in terms of cubic feet flowing per hour through a pipe line one foot in diameter, instead of the curve having the values of the abscissa numerically four times as large. There are 3,600 seconds in one hour.

$$4 \times 3{,}600 = 14{,}400 \quad \text{the number shown above.}$$

See Notes 60, 61, 62 and 63 on page 36; also, the first paragraph on page 5.

77. Equation (***VI***) is not based on any tests or experiments. It can not be used by substituting numerical values for the characters. The $\boldsymbol{h_v}$ is a pressure and is not a length.

When q_1 is one cubic foot and
d is one lineal foot.

Then, under the Specifications on page 31,

$$f_1 = 0.0288 \quad \textit{and} \quad v_1 = 1.2732 \textit{ feet}$$

(See Notes 49 and 50 on page 30, and the Notes on page 32.)

The numerical equation corresponding to (V) becomes

$$q_v^{0.25} = \frac{0.0288}{f_v} \qquad (VI)$$

C_1, the constant 0.0288, contains the Volume or Mass, **0.7854** $d^2\ v$ and the h_v which, with the pipe area, measures the Force. **The Mass is 1 cubic foot and ($d\ v$) is 1.2732 feet.**

Equation (VI) FIXES the numerical value of f_v when there is any one Steady Flow of any one FIXED Volume of any one Liquid through any one Pipe.

The numerical value of h_v is not Fixed.

> "A liquid having greater resistance to mobility requires greater force to keep it moving at any steady rate through a pipe line.
>
> It is evident that a force, measured by h, could be exerted sufficient to cause the same volume of a viscous liquid to pass through the same line in the same time as would be required for a less viscous liquid." ("f" 1934)

The numerical value of L is not Fixed. In the definition of C (page 15) the L appears in the denominator; therefore, the d and the v remaining unchanged, the L changes directly as the h_v changes.

78. In any one Steady Pipe Line Flow there is only one Force and one equal Resistance; they are "directed to contrary parts." See Newton's Law III, page 1.

79. For those who prefer to have $v^2 \div 2g$ in the flow formula, the *fixed number* for $2g$ may be *injected* into (IV) and it can be written either

$$q_v = 0.7854\, d^2 \left(\frac{v^2}{2g}\,\frac{d^2}{d}\,\frac{h_v}{L f_v}\right)^{0.5} \quad \text{or} \quad q = 6.302\left(\frac{h\, d^5}{l f}\right)^{0.5}$$

The latter form has been used by the author since 1889.

$$(0.25\pi \times \sqrt{2g} = 6.3\pm)$$

80. Instead of accepting and using (IV) and (VI), some writers prefer to use the more romantic $\sqrt{-1}$ in Pipe Line Flow problems. The minus sign is only ***a guide post*** to indicate a "contrary" direction. ***Always*** $(+1)^a = +1$ and $(-1)^a = -1$.

CHAPTER VII

NUMERICAL FORMULAS

If a Flow Formula is to be used in solving practical problems, the characters must each represent some actual number or dimension.

The Steady Flow of a Liquid through a cylindrical pipe is a continuing process, therefore, for any one Steady Flow there is no change in the element of time.

(See Note 2, page 6.)

The Force, which is equal to the Resistance, does not change during any one Steady Flow.

These three general formulas are used to establish the relations among the characters:

The circumference of a circle, πd,
The area of a circle, πr^2
The volume of a cylinder, $\pi r^2 l$

In what follows,

The foot is used to measure the Pipe;
The cubic foot is used to measure the Mass, i. e., Volume;
The height, in feet, of a column of the Flowing Liquid is used to measure the Pressure.
The foot, a unit of length, is used to measure The Pipe, The Liquid and The Force.

81. Numerically what is called "mean velocity" is the same as the length of the pipe that will contain q_v. In any one actual Steady Flow

The $\boldsymbol{v}$ does not change;
The $\boldsymbol{h}_v$ does not change;
The $\boldsymbol{q}_v$ does not change;
The $\boldsymbol{d}$ and the $\boldsymbol{L}$ are fixed; therefore,
The f_v can not change when the unit of length changes.

82. The difference in pressure, $\boldsymbol{h}_v$, is the only source available for furnishing the Force required to overcome the Resistance due to the Pipe surface, the Liquid, its Density, its Viscosity or any other cause.
The same $\boldsymbol{h}_v$ may apply to a pipe of any diameter.

These characters are used:

- q The Steady Flow Volume in cubic feet;
- d The Pipe diameter in feet;
- L The Pipe length in feet;
- h The Pressure measured in feet;
- v A length of The Pipe measured in feet;
- f The Resistance character, a number;
- π The number 3.1416;
- π **0.25** The number 0.7854.

The q_v, h_v and f_v indicate that some one, BUT THE SAME value of v is used throughout the equation for any one Steady Flow.

$q_v = 0.7854\ d\ (v\ d)$ for any diameter of Pipe. There is only one numerical value of $(v\ d)$ for any one value of q_v. The number v changes directly as q_v changes. The v changes inversely as the diameter changes for the same q_v. (See bottom half page 41.)

When q_v is 1 cu. ft. and d is 1 ft., then v is 1.2732 ft. and the number $(v\ d)$ is 1.2732, and, under the Specifications, this constant, 1.2732, corresponds to 0.0288 the constant in formula (*VI*) used to obtain other numerical values of f_v corresponding to q_v.

(See Table II, page 62.)

The Force that overcomes all kinds of Resistance to Steady Flow depends on h_v, the Pressure. At all times $h_v\ d^2 \div L\ d\ f_v$ is 1. To maintain this equality, it is necessary that h_v and f_v both be multiplied by the same number. The numerical value of h_v can be found by tests or by the records of many tests.

A formula, having the general form of (*IV*) was originally used almost exclusively for water. In most

83. Equation (IV) can not be used as a general equation for the reason that $Q_v \div 0.7854\, d^2\, v = 1$ and $h_v\, d^2 \div L\, d\, f_v = 1$. The value of h_v is not known.

84. By squaring only the (d^2) and placing it under the radical, the relation among the factors in (IV) is not changed but remains only for any one value of Q and the equation becomes

$$Q = 0.7854\, v \left(\frac{h_v\, d^6}{L\, d\, f_v}\right)^{0.5}$$

85. For Pipes of different diameters, for any one Steady Flow, the Q changes as the square root of the fifth power of d changes, i.e., $d^{2.5}$.

of the earlier tests, the temperature was not taken or was disregarded. The formula was known as "the Hydraulic Formula" and was later applied to all fluids which "flowed like water."

In a test on any one Pipe-Liquid combination, the only characters in (***IV***) that change are $\boldsymbol{v}$, $\boldsymbol{q_v}$, $\boldsymbol{h_v}$ and $\boldsymbol{f_v}$. These change together for *that one* Pipe-Liquid combination, but $\boldsymbol{h_v}$ changes independently for other Pipe-Liquid combinations.

From page 41, the Steady Flow formula is

$$q_v = \mathbf{0.7854}\ d^2\ v\left(\frac{d^2\ h_v'}{L\ d\ f}\right)^{0.5} \qquad (IV)$$

In a numerical formula, it is not necessary to keep the characters for d separate; therefore (***IV***) is re-written

$$q_v = \mathbf{0.7854}\ d^{2.5}\ v\left(\frac{h_v}{L\ f_v}\right)^{0.5} \qquad (IV)_n$$

Formula $(IV)_n$ does not show the relation AMONG the characters. $d^{2.5}$, "The square root of the fifth power of the diameter," has been tabulated and used for years in books on Hydraulics. The $\boldsymbol{v}$ is by definition $q_v \div \mathbf{0.7854}\ d^2$ and serves no useful purpose except in the product $(v\ d)$. The relation of $(v\ d)$ to q_v and the ratios $h_v : L$ and $h_v : f_v$ are discussed on page 41.

Under the Specifications, f_1 is the constant 0.0288 and the numerical value of f_v for the 1 ft. diameter Pipe is fixed by the formula $f_v = 0.0288 \div q_v^{0.25}$.

(See Table II, page 62.)

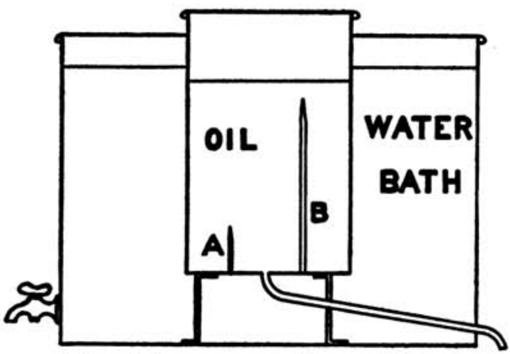
For this diameter and for Pipes of other diameters, there is only one value of the number $(v\ d)$ for any one value of q_v. (See bottom half of page 41.)

Author's Note

If Scientists and Engineers would agree on some one set of Specifications to be used as a Starting Point, all problems connected with The Steady Flow of Liquids through Pipe Lines would be simplified.

See pages 74 and 75 for Specifications and Starting Point selected for use in this Book.

Viscosity—Its Relation to the Flow Formula



A—lower hook gage *B*—upper hook gage

This simple apparatus, a thermometer and a stop-watch, were used by the author to compare certain viscous California crude petroleum, at various temperatures, with oil and other liquids having known pipe line characteristics. Based on tests made by this instrument, a pipe line 8 inches in diameter was built in 1902 and 1903. When the line was completed, it was found that the same head produced a flow of about 900 barrels per day under normal temperature conditions, but when the oil was heated and the temperature maintained at about 165° F., it was found that 18,000 barrels per day could be transported through the same line.

The numerical Pressure, $\boldsymbol{h_v}$, changes directly as the square root of the Volume changes, and, at the same time, the number, $\boldsymbol{f_v}$, changes indirectly as the fourth root of the Volume changes. The ratio $\boldsymbol{h_v} : \boldsymbol{f_v}$ changes but the ratio $\boldsymbol{q_v^{0.25}} : \boldsymbol{f_v}$ does not change. (Note 16, page 10.) Always $\boldsymbol{q_v}^{0.25} \times \boldsymbol{f_v} = C$.

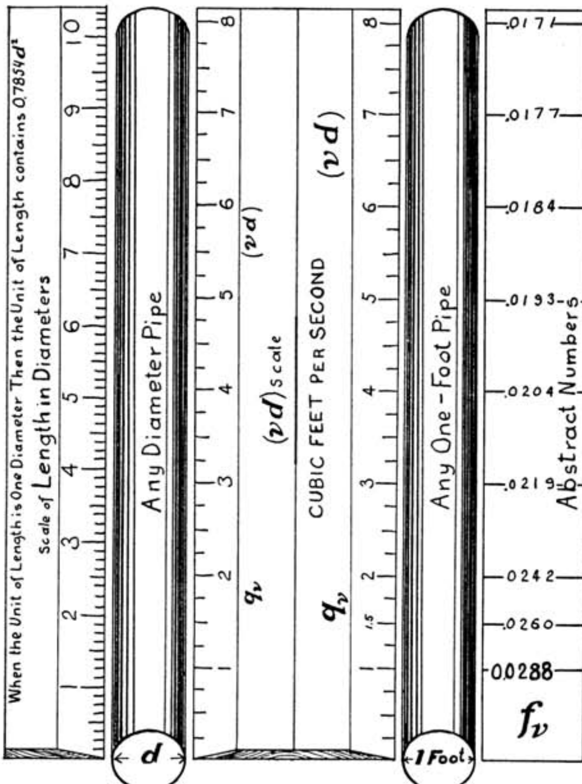
The Pipe-Liquid combination of The Specifications will be used for a series of numerical values of $\boldsymbol{h_v}$, the pressure required to produce the Steady Flow of $\boldsymbol{q_v}$ cu. ft. per second through a Pipe one foot in diameter. These values are "about right for Water" and other Liquids that "flow like Water." (The temperature of the water, the roughness of the inner surface of the Pipe and other conditions, will generally produce changes in the numerical value of $\boldsymbol{h_v}$ but not in the numerical value of $\boldsymbol{f_v}$.)

The ratio of $\boldsymbol{h_v}$ to $\boldsymbol{L}$ depends on the Liquid and the Pipe as well as on $\boldsymbol{q_v}$. The ratio of $\boldsymbol{q_v^{0.25}}$ to $\boldsymbol{f_v}$ depends on (***VI***), an entirely different numerical relation. The ratio of $\boldsymbol{h_v}$ to $\boldsymbol{f_v}$ depends on the fact that when $\boldsymbol{q_v}$ is fixed, there is a fixed value of $\boldsymbol{f_v}$ which is dependent on the product ($\boldsymbol{v\,d}$) and not entirely dependent on the diameter of the Pipe. These ratios or formulas resulting from them can not be included in any one equation.

The specified area for the Pressure is the square inch; therefore, the $\boldsymbol{d^2}$ that is in the Force is the number 144. (See Note 49, page 30.) The $\boldsymbol{d^2}$ that is in the Volume is in square feet and the Volume itself is in cubic feet.

(It is often convenient to use v in feet and d in inches for the numerical product ($v\ d$) particularly when considering the smaller sizes of Pipe. —See Note 54 on page 32. —The author has used $f_v = 0.024$ and the number 30, instead of 2.5, for a basic numerical value of such product.)

86. Formula (IV) applies to any Steady Flow of any Liquid through any Pipe BUT only to one such Steady Flow at any one time. The diagram below represents two sections of the same 12 inch Pipe. The Numbers are from Table II.



The value of $\boldsymbol{h}_v$, in feet, for the hypothetical* Pipe-Liquid combination of The Specifications, is shown in Table III, page 76. (See Notes 66 and 67, page 38.)

Contrary to the generally accepted opinion of the Scientists and Engineers, it is the $\boldsymbol{h}_v$ that is the real resistance indicator and not $\boldsymbol{f}_v$ the number.

In formula $(\boldsymbol{IV})_n$, the numerical value of all the characters can be measured or assumed and the constants and the number $\boldsymbol{f}_v$ computed or taken from a table, and the equation will not be disturbed. If the wrong number is selected for the f, a correction can be made by using the square root of the ratio of the number used for f to the true $\boldsymbol{f}_v$ as computed from the actual $\boldsymbol{q}_v$ by using equation $(\boldsymbol{VI})$. This is a separate computation and independent of the ratio $\boldsymbol{h}_v : \boldsymbol{L}$ which depends on the Viscosity of the Liquid, the roughness of the Pipe AND EVERY OTHER CAUSE which will change the pressure, $\boldsymbol{h}_v$, required to produce the steady flow of $\boldsymbol{q}_v$ through the Pipe. See Note 68 and Table I on page 40, also Note 69 on page 42 and the quotation from Church, Fluids, on page 43.

The ratio $\boldsymbol{h}_v \div \boldsymbol{L}$ is the most important relation in the Steady Pipe Line Flow formula.

> The $\boldsymbol{h}_v$ and the Pipe area fix the Force;
> The Force fixes the Resistance;
> The $\boldsymbol{h}_v \div \boldsymbol{L}$ fixes the Force per foot BUT
> The $\boldsymbol{f}_v$ is always $0.0288 \div \boldsymbol{q}_v^{0.25}$.

The $\boldsymbol{C}$ used in deriving $(\boldsymbol{VI})$ contains $\boldsymbol{h}_v$ as well as all of the other characters in $(\boldsymbol{IV})$ except $\boldsymbol{q}_v$ and $\boldsymbol{f}_v$

* It is not claimed that there is any one Liquid and any one Pipe that acts throughout unlimited conditions as the hypothetical Pipe-Liquid combination.

TABLE II

FOR THE

SPECIFIED ONE FOOT PIPE-LIQUID COMBINATION

Q_v cubic feet per second	$(v\,d)^*$, $1.2732\,Q_v$	$f_v = \frac{0.0288}{(Q_v)^{0.25}}$
.2401	.3057	.0411
.4094	.5213	.0360
.6561	.8353	.0320
.8145	1.037	.0303
1. cu. ft.	**1.2732**	**.0288**
1.5	1.9098	.0260
2.	2.5464	.0242
3.	3.8196	.0219
4.	5.0928	.0204
5.	6.3660	.0193
6.	7.6392	.0184
7.	8.9124	.0177
8.	10.1856	.0171
9.	11.4588	.0166
10. cu. ft.	12.732	.0162
11.	14.0052	.0158
12.	15.2784	.0155
14.	17.8248	.0149
16. cu. ft.	20.3712	.0144
18.	22.9176	.0140
20.	25.4640	.0136

* When $q_1 = 0.7854\,d_1\,(v_1\,d_1)$ and $q_2 = 0.7854\,d_2\,(v_2\,d_2)$. Then, in order that $(d\,v)$ may be the same for pipes of different diameters, the v must change inversely as the diameter changes.

which remain in (*VI*); therefore, (*VI*) is for any one Steady Pipe Line Flow. Neither (*IV*) nor (*VI*) shows a DEFINITE NUMERICAL RELATION between $h_v \div L$ and any of the other characters in the Steady Flow formula.

SUMMARY

The difference in Pressure, h_v, is the only source available for furnishing the Force required to overcome the Resistance due to the Pipe Surface, the Liquid, its Density or its viscosity. In makes no difference whether the resistance to the Steady Flowing stream is due to "turbulent flow" or to "parallel flow," as all of the resistance is transmitted through the Liquid to the Pipe Surface.

The Steady Flow Volume is

Always $q = 0.7854\, d^2\, v$ *i.e.* $q_v = 0.7854\, d\, (v\, d)$ (*I*)

Always there is only one value of (v d) for any one value of f_v. Read last paragraph on page 23.

Always $L\, d\, f_v = d^2\, h_v$ (*II*)

Always $\left(\dfrac{d^2\, h_v}{L\, d\, f_v}\right)^{0.5} = 1$ (*III*)

Always $q_v = 0.7854\, d^2\, v \left(\dfrac{d^2\, h_v}{L\, d\, f_v}\right)^{0.5}$ (*IV*)

Always $q_v^{0.25} = \dfrac{C}{f_v}$ (*V*)

When $q_1 = 1^3$, *then* $f_1 = 0.0288$ *which is* C_1

Always $q_v^{0.25} = \dfrac{0.0288}{f_v}$ (*VI*)

A change in the Specifications will not change the ratio h_v to L or the ratio h_v to f_v. See Notes 51 and 52 page 32.

Table based on (*VI*) appears on page 62.
Read Notes on pages 30 and 32.

87. The Foot has been adopted as the Unit of Length. Any other Unit of Length could have been adopted and if used consistently the actual ratios would not change. See Note 46, page 30.

88. The numerical value of each of the characters that appear in (IV), with the single exception of $\boldsymbol{h}_v$, are FIXED by Pipe dimensions and a Unit of Length.

89. ALL PRESSURES are expressed in terms of the height of a column of the Flowing Liquid measured in feet. See Notes on page 8. The pressure, $\boldsymbol{h}_v$, is absolutely independent of pipe dimensions. The Force depends on $\boldsymbol{h}_v$ and pipe dimensions.

90. See first paragraph on page 1.
See first paragraph on page 7.
See Notes 41, 42, 43, 44 and 45, page 28.
Read Note 46 on page 30.

Author's Note — The Number 64.4± when injected into (IV) produces the Darcy type formulas in which the number v is eliminated. See Note 36 on page 20 and the last paragraph on page 21.

The Chézy type formulas in addition inject the number 64 which is twice 2^5. There is evidence that Chézy used 2 for the diameter to avoid the use of unity as all powers and all roots of 1 are unity. See Note 50 on page 30 and the Second Specification on page 31.

CHAPTER VIII

PIPE LINE FLOW FORMULAS

RATIONAL — NUMERICAL

Rational, "5 *Math.* Expressible as the ratio of two whole numbers or entire quantities."
Funk and Wagnall's New Standard Dictionary.

Q is the number of cu. ft. per sec. which passes every point along the Pipe during *any one* Steady Flow.

The Basic numerical Formula used for such *any one* Steady Flow of *any one* Liquid through *any one* Pipe is (IV).

$$Q = \mathbf{0.7854}\ d^2 v \left(\frac{d^2\, h_v}{L\, d\, f_v}\right)^{0.5}$$

For a ONE FOOT PIPE

Always $v = 1.2732\, Q_v$ and $v^2 = 1.621\, Q_v^2$

In all that follows, the foot is used as the unit of length and the number f_v is obtained by using Formula (VI).

$$f_v = \frac{0.0288}{Q^{0.25}}$$

When the Steady Flow is 1 cu. ft. per sec. through a 1 ft. pipe and L and h have a fixed numerical value this one Steady Flow can be used as a starting point for a series of Rational-Numerical values.

91. The $\boldsymbol{f_v}$ for any cross section of a meter of the Venturi type will not change for any one $\boldsymbol{q_v}$.

"526. THE VENTURI WATER-METER.—The invention bearing this name was made by Mr. Clemens Herschel (see Trans. Am. Soc. Civ. Engineers, for November 1887), and may be described as a portion of pipe in which a gradual narrowing of section is immediately succeeded by a more gradual enlargement."—(Church, Fluids, 1889, p. 725.) See last paragraph, page 23.

For every Steady Flow

$$q = 0.7854\, d^2\, v = 0.7854\, d\, (v\, d)$$

In order that $d\,v$ may be the same for pipes of different diameters, the v must change inversely as the diameter changes. The $h_v \div f_v$ does not change with the pipe diameter but v and q_v change when the diameter changes.

A change of the unit of length does not change

(*a*) the actual Volume Flowing;
(*b*) the actual diameter of The Pipe;
(*c*) the actual length of The Pipe;
(*d*) the actual Pressure;
(*e*) it does not change the ratio of $\boldsymbol{h_v}$ to $\boldsymbol{L}$ or the ratio of $\boldsymbol{h_v}$ to $\boldsymbol{f_v}$;

By the Second Specification on page 31, the $\boldsymbol{L}$ is 10,000 ft.; the number $\boldsymbol{f}_1 = \boldsymbol{C}_1 = 0.0288$ and $\boldsymbol{h}_1$ is 7.20 ft. (See center of page 33.)

$Q = 0.7854\, \boldsymbol{d}^2 \boldsymbol{v}$ so that (IV) for *any one* value of Q for the 1 ft. Pipe may be written

$$Q = \left(\frac{Q_v^2\, h_v}{L\, f_v}\right)^{0.5} \qquad (VII)$$

with all the characters under the radical but requiring a change in the value of $\boldsymbol{h}_v$, $\boldsymbol{Q}_v^2$ and of the number $\boldsymbol{f}_v$ for every change in Q.

To increase Q requires that the Force be increased; the Resistance is increased and remains equal to the Force.

The two Ratios, h_v to L and h_v to f_v, do not change according to the same Formula.

Read Note 38 on page 22.

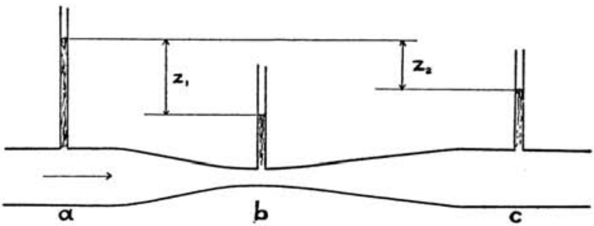
$$C = (\mathbf{0.7854}\, d^2\, v)^2 \frac{d^2\, h_v}{L\, d} \quad \text{or} \quad C = Q_v^2 \frac{d^2\, h_v}{L\, d}$$

The C contains all of (IV) except Q and $\boldsymbol{f}_v$. It contains $\boldsymbol{h}_v$ and $\boldsymbol{Q}_v^2$ which change when the Steady Flow Volume changes.

When Q increases h increases and the increase in h carries along with it an increase of f without changing the ratio of $\boldsymbol{h}_v$ to $\boldsymbol{f}_v$ BUT ***with the increase*** in Q there ***is the small decrease*** in f which is shown by (VI).

For Liquids and Pipes that require more, or less, Force than that specified, the only character in formula (IV) that changes is that, for pressure, say h'; therefore, any other combination can be compared with the *hypothetical* Pipe-Liquid by comparing the number $\boldsymbol{h}_v$ with the corresponding h'_v using the ratio $h'_v \div \boldsymbol{h}_v = \boldsymbol{x}$.

92. **The Venturi Water-Meter** is " a portion of pipe in which a gradually narrowing section is immediately succeeded by a more gradual enlargement " (see page 24). The Resistance due to such gradual reduction of the pipe area is of the same nature as the Resistance due to the pipe surface and gives the same result as a lengthening of the pipe and such a meter may be considered, for the purpose of all computations, as a longer pipe line.



The loss of pressure (Z_2) between *a* and *c* will be less than the loss of pressure (Z_1) between *a* and *b*, therefore, the "*equivalent length*" of that portion of the meter between *a* and *b* will be greater than the "*equivalent length*" of the whole meter. This is due to the fact that the piezometer only measures the standing or static pressure. Some of the static pressure is usefully employed when the velocity of the particles increases and it then becomes dynamic pressure. When the velocity of the particles decreases, some of the dynamic pressure is restored to the Steady Flowing stream as static pressure.

The $\boldsymbol{x}$ will be called the Resistance coefficient. As an example of another combination, substitute in the Specification 9.00 ft. for 7.20 ft. Comparing this new combination with the *hypothetical* Pipe-Liquid combination, the proportion is 7.20 : 9.00 : : 1 : 1.25 The Resistance coefficient for this assumed Liquid is 1.25. All values of h'_v are in this ratio, *i.e.*, 25% more than the Liquid of the Specifications. The ratio of h'_v to f'_v will not be disturbed and the other relations are the same as those in table II.

The Rational Pipe Line Steady Flow Formulas are all static equations and apply only to the one condition that can exist at any one time. They can not be used as functional equations.

Ratios are numerical relations and do not depend on any unit of length.

Q is the number of cubic feet flowing steadily through The Pipe.

C represents only one "entire quantity" at any one time.

$\boldsymbol{Q_v}$ the one Volume, is **0.7854** $\boldsymbol{d^2}$ $\boldsymbol{v}$ and is measured by three lengths, d, d and v.

$\boldsymbol{Q_v}$ is one number, **0.7854** $\boldsymbol{d}$ $(\boldsymbol{d\ v})$. The number $(\boldsymbol{d\ v})$ depends on such number. There is only one $\boldsymbol{Q_v}$ for any one $(\boldsymbol{d\ v})$.

$\pi \div 4 = 0.7854$ $\quad\quad 1 \div 0.7854 = 1.2732$

$h'_v \div \boldsymbol{h_v} = \boldsymbol{x}$ the Resistance coefficient.

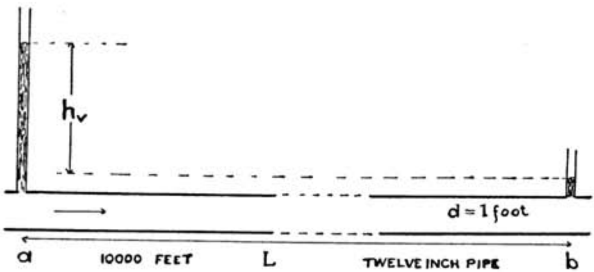
0.0288 is an abstract number. See Notes 51 and 52, page 32. For a 1 ft. Pipe Always $(\boldsymbol{d\ v}) = 1.2732\ \boldsymbol{Q_v}$.

The number of cu. ft. of Liquid flowing ***per hour*** through a 1 ft. Pipe is one-fourth the Reynolds Number for the corresponding f.

93. The only character in equation (***IV***) that does not have a fixed actual value for any one Steady Flow is $\boldsymbol{h_v}$.

94. The all important Ratio $\boldsymbol{h_v} \div \boldsymbol{L}$ does not fix the q at any one point.

95. Table III is computed from the one STARTING POINT of the SPECIFICATIONS. (*a*) A Steady Flow of one cubic foot of any Liquid through a 1 ft. Pipe each second. (*b*) The distance between the two points where the pressures are taken is 10,000 feet of level pipe, (*c*) the difference between pressures taken at those points is 7.20 feet and the number $\boldsymbol{f_v}$ is 0.0288. (*d*) The pressure difference is considered as acting on one square inch of area.



96. Any other value of $\boldsymbol{h_v}$ could have been used in The Specifications without changing any of the Basic Ratios. The Force always equals The Resistance; therefore, The Ratio $\boldsymbol{h_v} \div \boldsymbol{f_v}$ is always maintained.

The Rational Flow formulas contain no characters representing either Density, Gravity, Velocity or Viscosity. Such formulas do not contain characters representing Adhesion, Cohesion, Roughness of Pipe surface, Mass, Weight or Time. Steady Flow is a continuing operation.

EVERY TEST MADE of *any one* Steady Flow of *any one* Liquid through *any one* Pipe gives numerical values for Q_v, h_v, d and L.

(The number v is *fixed* by Q_v and d.)

Such values substituted in (IV) give values of f_v depending *exclusively* on the accuracy of such test. The h_v changes directly with Q_v^2 *but* the f_v changes inversely with $Q_v^{0.25}$.

(See Notes on page 28.)

When the Liquid or the Pipe changes and Q_v, d and L do not change, there is usually a change in the Force so that then the h_v changes *but* the f_v does not change.

Table II on page 62 is for *the one* Pipe-Liquid combination specified on page 31.

NOTES—Continued

CHAPTER IX

THE PRACTICABLE SOLUTION OF STEADY PIPE LINE FLOW PROBLEMS

" practicable-2. That can be used for an intended purpose; servicable."

The intimate relation that always exists among all the FACTORS when there is a STEADY FLOW of **any one** FIXED VOLUME of **any one** LIQUID through **any one** PIPE is established, determined and definitely FIXED in the preceding chapters.

(See first paragraph on page 1.)

The relations that always exist ***among*** Q_v, h_v, f_v, d and L are FIXED by Formula (IV) for each individual Steady Flow.

The relation between h_v and f_v is FIXED by the fact that the Force is always equal to the Resistance. Equation (II) page 11.

The numerical value of h_v IS NOT FIXED by Pipe dimensions but the numerical value of each of the other characters is FIXED by pipe dimensions.

See Table II page 62 for the numerical relation that always exists ***among*** Q_v, $(v\,d)$ and f_v.

The Ratio $h_v \div L$ changes with The Liquid and The Pipe surface.

The all important Factor in Pipe Line Flow problems is the Ratio $h_v \div L$.

See last paragraph on page 67.

These Specifications repeated from page 31.

It is practically impossible to select and specify one particular Pipe and one particular Liquid to be the two dominant Factors in a Pipe Line Flow Formula. Therefore, it is proposed to specify a hypothetical Pipe and hypothetical Liquid as a Standard or Base with which to compare other Pipe-Liquid combinations.

SPECIFICATIONS

Specifications for a Standard Pipe-Liquid Combination to be used in comparing Pipes and Liquids as to their Resistance to the Steady Flow of any one fixed Volume of any one Liquid through any one Pipe:

First— Any one Pipe one foot in diameter through which one cubic foot of any one Liquid passes in Steady Flow each second.

Second—It is further specified that when one cubic foot of such a Liquid is flowing steadily through such a one foot Pipe, the numerical value of $\boldsymbol{f}$ will be **0.0288** and $\boldsymbol{h}$ will be **7.20** feet when $\boldsymbol{L}$ is **10,000** feet of level pipe.

Third— The static pressure difference **7.20** feet will be considered as acting on one square inch area.

At This Point, it is proposed to use **the one Steady Flow of the Specifications** as a STARTING POINT and to use Formula $(VII)_s$ to determine and tabulate other values of h_v for other Steady Flows of the Specified Liquid through the Specified One Foot Pipe.

$$Q = \left(\frac{(Q_v)^2\, h_v}{L\, f_v}\right)^{0.5} \qquad (VII)_s$$

The subscript $_s$ indicates that the formula is for the Pipe-Liquid combination of The Specifications shown on the opposite page. The character d is omitted for simplicity because the numerical value of 1^α is always the number one.

In the one STEADY FLOW of the Specifications, Q is 1 cu. ft. and Q^2 is 1^2 the number used as a multiplier of 7.20 **which product is** the h_v of the Specifications. For other values of Q it **is not enough** to use Q^2 as a multiplier for the reason that when Q increases h also increases and the increase in such h carries along with it an increase of Resistance BUT does not affect the ratio of h_v to f_v as with the increase in Q there is a small decrease in f_v due to the relation shown by (VI), i.e., $C_1 \div f_v = Q_v^{0.25}$.

The value of the f_v which corresponds to the Q_v is shown in Table II page 62. When Q_v is 1 cu. ft. the value of 7.20 for h_v is for 10,000 ft. of 1 ft. Pipe, and the value of 0.0288 for f_v is for 1 ft. of such Pipe. The number 10,000 applies to h_v and also to f_v, the number Q^2 replaces the 1^2 and the number 250 replaces the Ratio $7.20 \div 0.0288$ to produce the formula

$$h_v = 250\, (Q_v)^2\, f_v \qquad (VIII)_s$$

for the Pipe and the Liquid of the Specifications.

TABLE III

FOR THE

SPECIFIED ONE FOOT PIPE-LIQUID COMBINATION

10,000 FEET LONG

Q cu. ft.	Q_v^2	$250\,Q_v^2$	f_v	h_v feet
.2401	.058	14.5	.0411	0.596
.4096	.168	42.	.0360	1.512
.6561	.430	107.5	.0320	3.440
.8145	.663	165.75	.0303	5.022
1.	1.00	250.	.0288	7.20
1.5	2.25	562.5	.0260	14.75
2.	4.	1,000.	.0242	24.20
3.	9.	2,250.	.0219	49.28
4.	16.	4,000.	.0204	81.60
5.	25.	6,250.	.0193	120.63
6.	36.	9,000.	.0184	165.60
7.	49.	12,250.	.0177	216.82
8.	64.	16,000.	.0171	273.60
9.	81.	20,250.	.0166	336.15
10.	100.	25,000.	.0162	405.00
11.	121.	30,250.	.0158	477.95
12.	144.	36,000.	.0155	558.00
14.	196.	49,000.	.0149	730.10
16.	256.	64,000.	.0144	921.60
18.	324.	81,000.	.0140	1,134.00
20.	400.	100,000.	.0136	1,360.00

Formula $(VIII)_s$ is used to compute the values of h_v in Table III. Values of Q_v are assumed, and the relative values of f_v are taken from Table II. These values of h_v apply only to the Specified Liquid and the Specified Pipe. The ratio 7.20 to 10,000; the ratio of 7.20 to f_v and the values of h_v in Table III are only for the Specified Pipe-Liquid combination.

When d is 1 foot and L is 10,000 feet, a change in h_v is necessary for other Pipe-Liquid combinations.

See example at top of page 69.

From any one Starting Point, like that of The Specifications, a series of Pipe-Liquid values can be computed and the corresponding values of h_v tabulated or charted graphically.

The character f_v does not have a FIXED numerical value. It changes for each **any one** Steady Flow of **any one** Liquid through **any one** Pipe. The function of f_v is to equate the Resistance to the Force. **Any one Specified Steady Flow** may be used without disturbing any of the Ratios.

Formula (IV) is for **any one** Steady Flow of **any one** Liquid through **any one** Pipe BUT it is only good for **ONE such Steady Flow at any one time.**

$$\textit{any one } Q = Q_v = 0.7854\ d^2\ v\left(\frac{h_v\ d^2}{L\ d\ f_v}\right)^{0.5} \quad (IV)$$

$Q_v = 0.7854\ d^2\ v$ also $(h_v d^2)^{0.5} = (L\ d\ f_v)^{0.5}$ for any one Steady Flow. By squaring $0.7854\ d^2\ v$, the Q_v^2 is placed under the radical and for any one Steady Flow (IV) becomes for any one value of Q

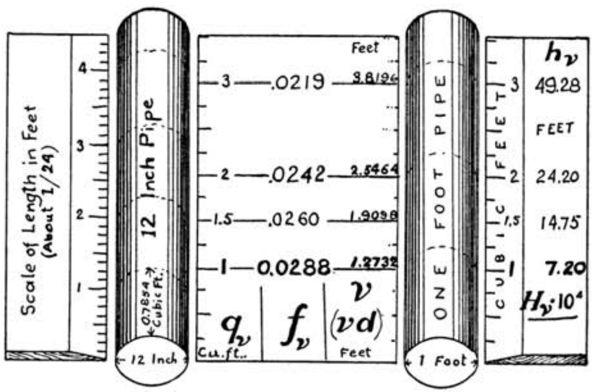
$$Q = \left(Q_v^2\ \frac{h_v\ d^2}{L\ d\ f_v}\right)^{0.5} \qquad (IV)_X$$

97. The diagram below represents two sections of a One-Foot Pipe, to scale, and shows some of the numerical values of the characters used in the equation for any one Steady Flow.

The Specifications on page 74 are used in computing the numerical values of the characters using these Basic values:

q_v' is 1 cu. ft.	v is 1.2732 ft.	d is 1 ft.
h_v is 7.20 ft.	L is 10,000 ft.	f_v is 0.0288

(The value of h_v per foot of Pipe is 0.00072 ft.)
The Derived numbers are from Table III.



It is not claimed that there is any one Liquid or any one Pipe which acts as the hypothetical Pipe-Liquid combination. The turbulence of the Steady Flowing stream may have some bearing on the result BUT the "*mean velocity*" or the velocity of any particle in the stream is not related to the Force or the Resistance by any numerical Ratio.

The Practicable Steady Flow Formulas are based on the three well-established formulas for

The circumference of a circle;
The area of a circle;
The volume of a cylinder and
The Third Law of Newton;

ALL PRESSURES being measured in terms of

The height of a column of the
Steadily Flowing Liquid.

Equation (*IV*) has been and is here used as a practicable formula for

Any one Steady Flow of
Any One Liquid through
Any One Pipe.

$$q = q_v = 0.7854\, d^2\, v \left(\frac{h_v\, d^2}{L\, d\, f_v}\right)^{0.5} \qquad (IV)$$

For any consistent system of units of length that may be used to measure The Pipe, The Volume (Mass) of the Steady Flowing Liquid and The Pressure which maintains such Steady Flow for any one duration.

98. The author's Thesis presented in June, 1935 was "a Progress Report on pipe line fluid research work undertaken at" Cornell University in October, 1934. More than 3000 tests were made later BUT NO TESTS ARE USED in this study of Pipe Line Flow problems.

99. Particular reference is made to

Flow of Water in Pipes and Pipe Fittings
John Ripley Freeman, C. E.

Published 1941 by the American Society of Mechanical Engineers. This treats of "Experiments upon the flow of water in pipe and pipe fittings made at Nashua, New Hampshire, June 28th to October 22, 1892." On page iii the Blasius formula is given as $f = \frac{0.3164}{R^{0.25}}$. In his thesis, the author used $f = .316\, R^{-.25}$

The results are practically the same as when using $f_v = 0.0288 \div Q_v^{0.25}$. See last paragraph, page 61, and Notes 50 and 51, page 32.

100. "Calling the pipe line ascertained Specific Viscosity of water at 68 degrees Fahrenheit unity; the head necessary to cause the selected flow h_1, and the head required to produce the same rate of flow of some other liquid h_2, then the following proportion can be written:

$$h_1 : h_2 :: 1 : \mu_s$$

In this, μ_s is the pipe line Specific Viscosity." This quotation is from page 23 of the 1935 Thesis. It corresponds to

$L^{-1}\, h_v' \div H_v = x$ the Resistance Coefficient.

THE PRACTICABLE **FLOW FORMULA** is

$$q_v = 0.7854\, d^2\, v \left(\frac{h_v\, d^2}{L d f_v}\right)^{0.5} \qquad (IV)$$

All of the characters except h depend for their numerical value on Pipe dimensions. The Ratios do not change with the unit of length. See Note 46, page 30. When the foot is used as the unit of length and the number 0.0288 is used as the value of f_1 as in the Specifications on page 31, then always

$$f_v = \frac{0.0288}{Q_v^{0.25}} \qquad (VI)$$

The character f does not depend on Density, Viscosity, Roughness of Pipe or any other property that may change the numerical value of h in Equation (IV).

- ***f is not a coefficient of friction.***
- ***v is a length and is not a velocity.***

For the same Volume Flowing Steadily through a Pipe of the same diameter and the same length, the h is the only character in formula (IV) that can change in actual value.

TABLE IV

DERIVED FROM THE SPECIFICATIONS

All dimensions are in feet

Only when d is 1 ft.	For any diameter of Pipe		
q_v	$\dot{v}d$	f_v	$H_v \cdot 10^4$
.2401	.3057	.0411	0.596
.4096	.5213	.0360	1.512
.6561	.8353	.0320	3.440
.8145	1.037	.0303	5.022
1	**1.2732**	**.0288**	**7.20**
1.5	1.9098	.0260	14.75
2.	2.5464	.0242	24.20
3.	3.8196	.0219	49.28
4.	5.0928	.0204	81.60
5.	6.3660	.0193	120.63
6.	7.6392	.0184	165.60
7.	8.9124	.0177	216.82
8.	10.1856	.0171	273.60
9.	11.4588	.0166	336.15
10.	12.732	.0162	405.00
11.	14.0052	.0158	477.95
12.	15.2784	.0155	558.00
14.	17.8248	.0149	730.10
16.	20.3712	.0144	921.60
18.	22.9176	.0140	1,134.00
20.	25.4640	.0136	1,360.00

The first three columns are from Table II
The first, third and fourth are from Table III

N.B. The first column only applies when the diameter is One Foot. See last paragraph on page 57.

CHAPTER X

RELATIVE VISCOSITY OF LIQUIDS

Relative Resistance to Steady Pipe Line Flow

It is impossible to Specify or Select any one actual Liquid at any one temperature which, when Flowing Steadily through any one actual Pipe, can be used as a Base or Standard with which to compare other Pipe-Liquid Combinations.

The same Pipe-Liquid Combination that is Specified on Page 31 as " a hypothetical Pipe and hypothetical Liquid " base or Standard is adopted and will be used with formula (IV).

Such a hypothetical Pipe-Liquid Combination* holds throughout unlimited conditions and depends exclusively on

(*a*) the Ratio of $\boldsymbol{h_v}$ to $\boldsymbol{L}$,
(*b*) Pipe dimensions, and
(*c*) the Axioms of Newton. See page 73

For the same Volume Flowing Steadily through a Pipe of the same diameter and the same length, the $\boldsymbol{h_v}$ is the only character in formula (IV) that can change in actual value.

*Any other such Base may be used. See Note 46, page 30.

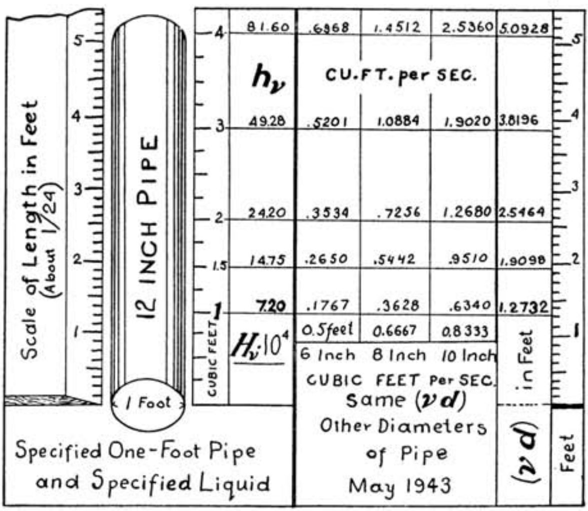
101. The diagram below shows a One-Foot Pipe and values of $\boldsymbol{h_v}$ for the Specified Pipe-Liquid combination. The Ratios $\boldsymbol{h_v}$ to $\boldsymbol{L}$ and $\boldsymbol{h_v}$ to $\boldsymbol{f_v}$ are fixed and any such Ratio may apply to a pipe of any diameter.

When $\boldsymbol{d}$ is the diameter of the 1 ft. Pipe and d is the diameter of another pipe, then for a Steady Flow through such other pipe

$$d^{2.5} : \boldsymbol{d}^{2.5} :: q_v : \boldsymbol{q_v}$$

for the same value of $(\boldsymbol{vd})$. See bottom half of page 57 and also page 75.

The table below shows cu. ft. per sec. for 6 in., 8 in. and 10 in. pipe corresponding to the 12 in. Pipe and the Specifications.



Any difference in head, $\boldsymbol{h_v}$, divided by its corresponding length, $\boldsymbol{L}$, is the head required for one foot in length of the Pipe. The $\boldsymbol{h_v}$ of the Specifications is for 10,000 feet of 1 ft. Pipe. This is a measurable quantity.

In all that follows, the $\boldsymbol{L}$ is 10,000 feet and as this does not change any of the Ratios it will usually be omitted.

Let H_v be the pressure for the Hypothetical Combination for each foot of the 10,000.

Let h'_v be the pressure for any other Combination.

For the Hypothetical Pipe-Liquid Combination there is a different Ratio, $\boldsymbol{h_v} \div \boldsymbol{L}$, for each different Volume. Each such Ratio will apply to different Volumes when flowing through pipes of different diameters. (See last half of page 41.)

¶ For each and for every other Pipe-Liquid Combination there is a complete series of different values of $h'_v \div \boldsymbol{L}$ corresponding to each of the different values of H_v for each foot of the 10,000.

When any one value of $\boldsymbol{h_v}$ of the Specified Pipe-Liquid Combination is considered as the unit of "Relative Viscosity," then for any other corresponding $\boldsymbol{h}$

$$h'_v \div \boldsymbol{h_v} = \chi$$

the Resistance Coefficient for such Pipe-Liquid Combination. (See example at the top of page 69.)

The character χ (Chi) is used to represent the Resistance Coefficient and applies to all kinds of Resistance including Viscosity and the Roughness of the Pipe.

Equation $(IV)_\chi$ contains the factor $\boldsymbol{Q}$ on one side and $(\boldsymbol{Q}_v^2)^{0.5}$ on the other side; therefore, as $\boldsymbol{h_v}\,\boldsymbol{d}^2 \div \boldsymbol{L}\,\boldsymbol{d}\,\boldsymbol{f_v} = 1$ the equation is for any one Steady Flow at any one

NOTES—Continued

Author's Notes

Various numbers, letters and other symbols have been used and arranged into formulas to represent to the mind things and the relations that exist among things that are known or believed to exist but which must be accepted as axioms for the purpose of such discussion as may follow.

These three general formulas are used to establish the relations among the characters:

The circumference of a circle, πd,
The area of a circle, πr^2
The volume of a cylinder, $\pi r^2 l$

The number f (or some other character) is used to represent the Resistance due to the Liquid and the Pipe Surface.

The number, represented by 2 g, is often injected and is a constant number in some one system of units.

Dr. Durand used $h = \frac{l}{d}\frac{v^2}{2g}$

The entire paragraph in Dr. Buckingham's statement from which the sentence on page 5 is quoted follows:

"When a liquid flows, at a constant rate, through a smooth straight pipe, the pressure gradient G may be expected to depend on the diameter D, speed S, and density ρ and viscosity μ of the liquid. So long as the pipe is full and the liquid sensibly incompressible, we do not see anything else for G to depend on; and unless we have omitted some essential circumstance, these five quantities must be time, that is $(IV)_x$ is for every Pipe Line Steady Flow and depends exclusively on Pipe Dimensions, and The Length $\boldsymbol{h_v}$ which, with Pipe Dimensions, determines the any one Force which causes the any one Steady Flow of any one $\boldsymbol{Q_v}$.

Let $\boldsymbol{Q_v}$ be the Steady Flow Volume in cu. ft. per sec. For any one value of $\boldsymbol{Q_v}$ there is but one value of $\boldsymbol{f_v}$ for the same diameter of pipe and but one value of $(\boldsymbol{v\,d})$, therefore, there is but one value of $\boldsymbol{f_v}$ for any one value of the product $(\boldsymbol{v\,d})$.

$$\boldsymbol{Q_v} = 0.7854\,\boldsymbol{d}^2\,\boldsymbol{v} = 0.7854\,\boldsymbol{d}\,(\boldsymbol{d}\,\boldsymbol{v})$$

By definition, the $\boldsymbol{v}$ is the length of The Pipe that will contain $\boldsymbol{Q_v}$. (Page 23.) In order that the product $(\boldsymbol{d}\,\boldsymbol{v})$ may be the same for pipes of different diameters, both the $\boldsymbol{v}$ and the $\boldsymbol{Q_v}$ must change inversely as the $\boldsymbol{d}$ changes.

The value of the factor $\boldsymbol{f_v}$ depends on $\boldsymbol{d}$, $\boldsymbol{d}$ and $\boldsymbol{v}$, the three pipe dimensions of the Volume $\boldsymbol{Q_v}$.

Formula (*VI*) at the bottom of page 33

$$\boldsymbol{f_v} = 0.0288 \div \boldsymbol{q_v^{0.25}}$$

is used to construct Table II, page 62.

The Ratio $\boldsymbol{h_v} \div \boldsymbol{L}$ may apply to any diameter of pipe and is the pressure utilized for each foot of the Pipe. The $\boldsymbol{f_v}$ is for one foot of the Pipe but the $\boldsymbol{Q_v}$ changes for pipes of different diameters. $\boldsymbol{Q_v}$ and $\boldsymbol{v}$ change directly with each other, so the $\boldsymbol{Q_v}$ must change inversely as the $\boldsymbol{d}$ changes for the same Ratio $\boldsymbol{h_v} \div \boldsymbol{L}$.

When $\boldsymbol{Q_v}$ does not change but flows on through another pipe of different diameter, the $\boldsymbol{f_v}$ does not change but the Ratio $\boldsymbol{h_v} \div \boldsymbol{L}$ for the different diameters will NOT be the same.

connected by some sort of relation which may be symbolized by writing $F(G, D, S, \rho, \mu) = 0$"

The subject of his paper, published in 1915, was
" Model Experiments and Emperical Equations."

Since 1915, the author has had many conferences with Dr. Durand and has been associated with Dr. Buckingham, and has, at times, worked with him on various Fluid Flow problems particularly in reference to elastic fluids. Dr. Buckingham's 1915 paper has been used by many as the basis of innumerable papers.

The Author is thoroughly convinced that the ingenious, often tricky, methods of Calculus should not be used in Pipe Line Flow Formulas or in the interpretation of any problems involving the world in which we live, when the use of Arithmetic and dimensions of Length, Area, Volume and Time can be used.

Equation (IV) is the Practicable Formula for any one Steady Flow through any one Pipe Line. It holds true for any consistent system of units. See Note 46, page 30.

$$q_v = 0.7854\ d^2\ v \left(\frac{h_v\ d^2}{L\ d\ f_v}\right)^{0.5} \qquad (IV)$$

Equation (VI) is derived from Equation (IV). See Chapter (V), page 39. It is for actual numerical values of the characters expressed in feet.

$$q_v^{0.25} = \frac{0.0288}{f_v} \qquad (VI)$$

See notes on page 30 and Specifications on page 31, also on page 74.

When any one value of H_v of The Specified Pipe-Liquid combination* is the Base for "Relative Viscosity," then any other corresponding pressure difference, h_v divided by $H_v L$ equals χ the Resistance coefficient for such any other Pipe-Liquid combination. See Note 86 on page 58 and the example at the top of page 69. Tables and charts can be prepared for the new Pipe-Liquid base. The numerical value of h_v changes only when the Pipe-Liquid combination changes. ($h_v \div L = H_v \chi$.) *See page 61.

It is not the intention of this statement of the Pipe Line Steady Flow problem to establish a table for any one Pipe-Liquid combination. Such a table for $v\ d$, f_v and $\frac{h_v\ 10^4}{L}$ can be produced by multiplying any one of the values in the $H_v \cdot 10^4$ column by the Resistance Coefficient. The more accurate and numerous the tests made to ascertain the value of the factor Chi, the more reliable the table becomes.

Author's Note

" Manuals of Engineering Practice " Bulletin No. 25 published 1942 by the Am. Soc. C. E., has recently been received. In Section I on page 1 is this statement:

> " In the literature about 1840–1850 appeared the much used pipe flow formula
>
> $$h_f = f \frac{l}{d} \frac{v^2}{2g} \qquad (3)$$
>
> the credit for which has been given to Henri Darcy, Julius Weisbach, John Thomas Fanning and Johann Albrecht Eytelwein."

This formula transposed becomes $v = \left(\frac{2gd\,h_f}{f\,l}\right)^{0.5}$

The $2g$ is a constant and is not connected in any way with any of the characters that appear in the equation. Any Steady Flow of any actual volume is a continuing operation and does not depend in any way on the unit of time.

The concluding sentence in Section I is

> " The dimensions given are readily converted to the force-length-time (F-L-T) system by the relation F = M L T^{-2}."

As The Resistance is always equal to The Force and can be measured in no other way

$$\pi\, d\, L\, \Phi_x = \pi\, d^2\, 0.25\, z_x \qquad (II)$$

$$\frac{\Phi_x}{z_x} = \frac{\pi\, d^2\, 0.25}{\pi\, L\, d} = \frac{\text{Pipe Area}}{\text{Area of The Pipe surface}}$$

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The statement in the book "*f* The Pipe Line Flow Factor," published May, 1934, can now be amended to read

> The number of cubic feet of any Liquid Flowing Steadily through any Pipe one foot in diameter in four hours is the Reynolds Number for the corresponding *f*.

Refer to second paragraph page 79.

Table II on page 62 is derived from (*VI*)

Table III on page 76 is derived from (*VIII*) on 75.

Table IV on page 82 combines three derived columns from Table II and three derived columns from Table III. The Ratios existing among the characters is not disturbed.

This is not a Theoretical discussion. It is Based on three undisputed equations:

The Circumference of a Circle $= 3.1416\ d$

The Area of a Circle $= 0.7854\ d^2$

The Volume of a Cylinder $= 0.7854\ d^2\ v$ and Newton's LAW III.

Throughout any one STEADY FLOW of any one FIXED VOLUME of any one LIQUID through any one PIPE, there is but one FIXED SERIES of RATIOS existing among the Characters in Equation IV.