how to use
Ln-L SCALE
SLIDE RULES

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PICKETT

TABLE OF CONTENTS

[2]

Introduction.

Model 1011 L and Ln on body of rule.
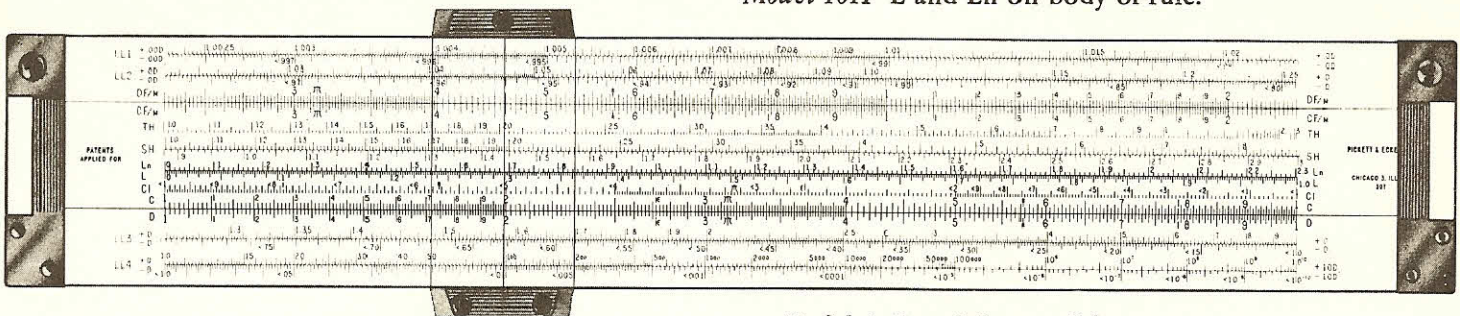
Model 4. L and Ln on slide.

The Ln scale is similar to the regular L scale. It is used for problems with base *e*. For many problems it is more convenient to use Ln than the Log Log scales on advanced models. In particular, it enables you to solve problems with *powers of e* in combined operations. Its range (from 0 to 2.3) is greater than the range of the L scale (from 0 to 1). The inclusion of the Ln scale completes the DUAL-BASE features of the Pickett Slide Rules.

By computing a "characteristic" you can use the Ln scale to find any power of *e*; thus the effective range for powers of *e* is from 0 to infinity. Since powers of *e* are read on the C (or D) scale, accuracy to 3 or 4 significant figures is obtained no matter how large or how small the numbers are. The Ln scale saves steps in many computational problems.
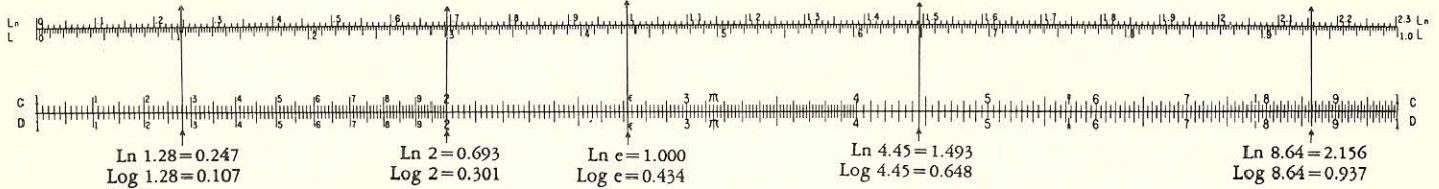
[3]

In Part 1 you will learn how to use the Ln scale to find powers of e and logarithms to base e. The Ln scale is a uniformly divided scale similar to the L scale. It is used with problems in base e just as the L scale is used with problems in base 10.

When you study the charts in this manual always set the slide and cursor on your rule like the ones pictured in the manual. Study the *slide rule* itself along with the charts. In this manual we will use the symbol Ln x for the logarithm of x to base e.

(*a*) *Finding logarithms.*

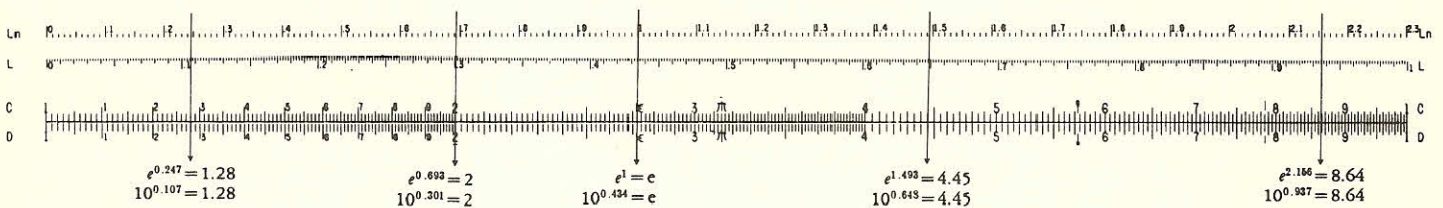| Ln 1.28 = 0.247 | Ln 2 = 0.693 | Ln e = 1.000 | Ln 4.45 = 1.493 | Ln 8.64 = 2.156 |
| Log 1.28 = 0.107 | Log 2 = 0.301 | Log e = 0.434 | Log 4.45 = 0.648 | Log 8.64 = 0.937 |

Rule: To find mantissas of logarithms: Set the number on the C (or D) scale, and read the mantissa for base 10 from L, or for base e from Ln. If L scales are on slide, set on C. If L scales are on body, set on D. Characteristic for base 10 is found by usual method. "Characteristic" for base e is explained in Section (e), page 8. Also, see below.

For $1 \leq x \leq 10$, we have $0 \leq \log x \leq 1$.
For $1 \leq x \leq 10$, we have $0 \leq \text{Ln } x \leq 2.30258$
For the same domain ($1 \leq x \leq 10$), the range of Ln is greater than the range of L.

Examples for practice. Verify that:

1. Ln 1.473 = 0.387		2. Log 1.473 = 0.168
3. Ln 2.34 = 0.850		4. Log 2.34 = 0.369
5. Ln π = 1.145		6. Ln 3.49 = 1.250
7. Ln 4 = 1.386		8. Ln 4.62 = 1.530
9. Ln 5.17 = 1.643		10. Ln 7.35 = 1.995
11. Ln 8.05 = 2.086		12. Ln 9 = 2.197
13. Ln 3.62 = 1.286		14. Log 3.62 = 0.559
15. Ln 1.91 = 0.647		16. Log 1.91 = 0.281
17. Ln 2.66 = 0.978		18. Log 2.66 = 0.425

(*b*) *Powers of* e *and of 10.* In the figure below, notice that the cursor hairlines shown are in the same positions as in Section (a) on page 4, above.

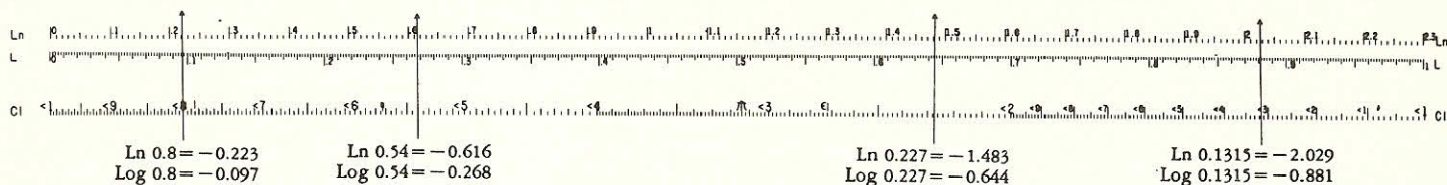| $e^{0.247} = 1.28$ | $e^{0.693} = 2$ | $e^1 = e$ | $e^{1.493} = 4.45$ | $e^{2.156} = 8.64$ |
| $10^{0.107} = 1.28$ | $10^{0.301} = 2$ | $10^{0.434} = e$ | $10^{0.648} = 4.45$ | $10^{0.937} = 8.64$ |

Rule: To find powers of e and of 10. Set the exponent of e on Ln, or of 10 on L, and read the power on C (or D). If L scales are on slide, use C; if they are on body, use D. The decimal point of the answer is found by special rules. See Section (f), page 9. Also see below.

For $0 \leq y \leq 1$, we have $1 \leq 10^y \leq 10$.
For $0 \leq y \leq 2.30258$, we have $1 \leq e^y \leq 10$.
Although the domain of y is greater for base e, the range of 10^y and e^y is the same. The exponents for this range may be set directly on the Ln or L scale.

Examples for practice. Verify that:

1. $\sqrt{e} = e^{0.5} = 1.649$	2. $e^{0.81} = 2.25$
3. $\sqrt{10} = 10^{0.5} = 3.16$	4. $e^{1.15} = 3.16$
5. $e^{1.36} = 3.90$	6. $e^{1.63} = 5.104$
7. $e^{1.875} = 6.52$	8. $e^2 = 7.39$
9. $e^{2.138} = 8.48$	10. $e^{2.252} = 9.51$
11. $10^{0.732} = 5.40$	12. $10^{0.9} = 7.95$
13. $\sqrt{e^3} = e^{1.5} = 4.48$	14. $10^{0.405} = 2.54$
15. $e^{0.1} = 1.105$	16. $e^{0.205} = 1.228$
17. $e^{0.532} = 1.702$	18. $e^{0.01} = 1.0101$
19. $e^{0.99} = 2.691$	20. $e^{1.01} = 2.746$

(c) Finding logarithms of proper fractions. The logarithm of each proper fraction is a negative number. It is written in two ways; for example, log 0.5 = −0.301 or 9.699-10; also, Ln 0.5 = −0.693 or 9.307-10. For slide rule work the form 9._ _ _ −10 is not needed.

Ln 0.8 = −0.223   Ln 0.54 = −0.616   Ln 0.227 = −1.483   Ln 0.1315 = −2.029
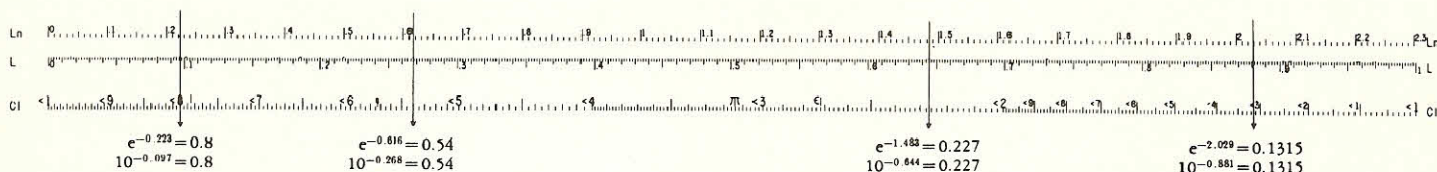Log 0.8 = −0.097   Log 0.54 = −0.268   Log 0.227 = −0.644   Log 0.1315 = −0.881

Rule: To find mantissas of logarithms of numbers between 0.1 and 1, set number on CI (or DI), read mantissa for base 10 on L, for base e on Ln. If L scales are on the slide, use CI. If L scales are on the body, use DI. For smaller numbers, see Section (f), page 9. Also, see below.

For $0.1 \leqq x \leqq 1$, we have $-1 \leqq \log x \leqq 0$.
For $0.1 \leqq x \leqq 1$, we have $-2.30258 \leqq \mathrm{Ln}\, x \leqq 0$.
For the same domain ($0.1 \leqq x \leqq 1$), the range of Ln is greater than the range of L. For *x* in this domain, the logarithm is read *directly* from the scale and written with the negative sign. For *x* not in this domain, the characteristic must be found by a special rule.

Examples for practice. Verify that:

1. Ln 0.15	= −1.897	2. Ln 0.1625	= −1.818
3. Ln 0.19	= −1.661	4. Log 0.19	= −0.721
5. Ln 0.202	= −1.599	6. Log 0.202	= −0.695
7. Ln 0.259	= −1.351	8. Log 0.259	= −0.587
9. Ln 0.3	= −1.204	10. Ln $\pi/10$	= −1.158
11. Ln 0.85	= −0.163	12. Ln 0.92	= −0.083
13. Log 0.742	= −0.130	14. Ln 0.742	= −0.298
15. Log 0.363	= −0.440	16. Ln 0.363	= −1.014
17. Log 0.178	= −0.750	18. Ln 0.178	= −1.726
19. Ln 0.120	= −2.120	20. Log 0.12	= −0.921
21. Ln 0.103	= −2.27	22. Log 0.103	= −2.274

(d) Powers for negative exponents. For negative exponents powers are all less than 1. Hence they are proper fractions. In the figure below, notice that the cursor hairlines are shown in the same positions as in Section (c) on page 6, above.

$e^{-0.223} = 0.8$ $e^{-0.616} = 0.54$ $e^{-1.483} = 0.227$ $e^{-2.029} = 0.1315$
$10^{-0.097} = 0.8$ $10^{-0.268} = 0.54$ $10^{-0.644} = 0.227$ $10^{-0.881} = 0.1315$

Rule: To find powers of *e* and of 10 for negative exponents, set the exponent of *e* on Ln, or the exponent of 10 on L, and read the power on CI (or DI). If L scales are on slide, use CI; if they are on the body, use DI. The decimal point in the answer is found by special rules. See Section (f), page 9. Also, see below.

For $-1 \leqq y \leqq 0$, we have $0.1 \leqq 10^y \leqq 1.0$.
For $-2.30258 \leqq y \leqq 0$, we have $0.1 \leqq e^y \leqq 1.0$.
Although the domain of *y* is greater for base *e*, the range of 10^y and of e^y is the same. The exponents for this range may be set directly on the Ln or the L scale.

Examples for practice. Verify that:

1.	$e^{-2} = 0.135$	2.	$10^{-0.8} = 0.1585$
3.	$e^{-1} = 0.368$	4.	$10^{-0.1} = 0.794$
5.	$e^{-0.2} = 0.819$	6.	$e^{-0.63} = 0.533$
7.	$e^{-0.27} = 0.763$	8.	$e^{-1.54} = 0.214$
9.	$e^{-1.27} = 0.281$	10.	$e^{-2.08} = 0.125$
11.	$e^{-2.29} = 0.1013$	12.	$e^{-1.65} = 0.192$
13.	$10^{-0.67} = 0.2138$	14.	$e^{-0.72} = 0.487$
15.	$10^{-0.44} = 0.363$	16.	$e^{-0.56} = 0.571$
17.	$10^{-0.25} = 0.562$	18.	$e^{-0.99} = 0.372$
19.	$e^{-1.5} = 0.223$	20.	$10^{-0.15} = 0.708$

(e) Finding the Characteristic

Base 10

The characteristic is the exponent of 10 when the number is expressed in *standard form*.

Rule. To express a number in standard form: (i) place a decimal point at the right of the first non-zero digit, (ii) start at the right of the first non-zero digit in the original number and count the digits and zeros passed over in reaching the decimal point. The result of the count is the numerical value of the characteristic, or exponent of 10. If the original decimal point is toward the right, the characteristic is positive (+). If the original decimal point is toward the left, the characteristic is negative (−). Indicate that the result of (i) is multiplied by 10 with this exponent.

Examples for practice

Number	Number in standard form	Characteristic
1. 5,790,000	5.79×10^6	6
2. 0.000283	2.83×10^{-4}	−4
3. 44	4.4×10^1	1
4. 0.623	6.23×10^{-1}	−1
5. 8.15	8.15×10^0	0
6. 461,328	4.61328×10^5	5
7. 0.0000005371	5.371×10^{-7}	−7
8. 0.0306	3.06×10^{-2}	−2

Base e

The term "characteristic" as used here will mean the number to which a reading from the Ln scale must be added to account for logarithms not in its range.

Rule: First express the number in standard form. Read the logarithm of the first factor directly from Ln, as in Section (a), page 4. Multiply 2.30258 by the exponent of 10 in the second factor. If the exponent is positive, add this result to the direct reading. If the exponent is negative, subtract the result from the direct reading.

Examples for practice. Verify that the logarithm to base *e* for the examples at the left is as follows:

1. Ln 5.79×10^6 = 1.756 + 6(2.303) = 15.574
2. Ln 2.83×10^{-4} = 1.040 − 4(2.303) = −8.170
3. Ln 4.4×10^1 = 1.482 + 2.303 = 3.785
4. Ln 6.23×10^{-1} = 1.829 − 2.303 = −0.474
5. Ln 8.15×10^0 = 2.098 − 0 = 2.098
6. Ln 4.61×10^5 = 1.528 + 5(2.303) = 13.041
7. Ln 5.371×10^{-7} = 1.681 − 7(2.303) = −14.437
8. Ln 3.06×10^{-2} = 1.118 − 2(2.303) = −3.487

For $0 < x < \infty$, we have $-\infty < \log x < +\infty$.
For $0 < x < \infty$, we have $-\infty < \text{Ln } x < +\infty$.

(f) Extending the range for 10^y and e^y.

Base 10

For y not in the interval between 0 and 1, the standard method of finding 10^y first expresses it as the product of two factors. Thus $10^{2.5} = 10^2 \times 10^{0.5}$. One factor has an integral exponent. The other factor has a fractional exponent in the interval 0 to 1. The second factor is computed by the methods of Section (b), page 5, and Section (d), page 7. The first factor then determines the position of the decimal point in the final answer.

Examples for study.

1. $10^{2.5} = 10^2 \times 10^{0.5} = 10^2 \times 3.16 = 316.$
2. $10^{4.26} = 10^4 \times 10^{0.26} = 10^4 \times 1.82 = 18,200.$
3. $10^{-5.38} = 10^{-5} \times 10^{-0.38} = 10^{-5} \times 0.417 = 0.00000417.$
4. $10^{-2.71} = 10^{-2} \times 10^{-0.71} = 10^{-2} \times 0.195 = 0.00195.$

For $-\infty < y < +\infty$, we have $0 < 10^y < +\infty$.
The effective range for powers of 10 is infinite.
Three or four significant figures of e^y can be found.

Examples for practice. Verify that:

1. $10^{3.916} = 8,240.$
2. $10^{-3.916} = 0.0001214.$
3. $10^{5.023} = 105,400.$
4. $10^{-5.023} = 0.000,009,48.$
5. $10^{14.622} = 4.19 \times 10 = 419,000,000,000,000.$
6. $10^{-23.877} = 0.000,000,000,000,000,000,000,001,327$

Base e

For y not in the interval $0 \leq y \leq 2.30258$, express e^y as the product of two factors. For example, $e^{3.5} = e^{2.3} \times e^{1.2} = 10e^{1.2}$. One method of finding these factors is to divide the exponent y by 2.30258 (or a rounded value of this divisor, such as 2.303), to determine an integral quotient, q, and a remainder, r. Then $y = 2.303q + r$, and $e^y = e^{2.303q + r} = e^{2.303q} \times e^r = (e^{2.303})^q \times e^r = 10^q \times e^r$.

The value of the second factor is computed by the methods of Section (b), page 5, and Section (d), page 7. The first factor is used to determine the decimal point.

Examples for study.

1. To find $e^{6.54}$, first divide 6.54 by 2.303, obtaining quotient 2 and remainder 1.934. Then $e^{6.54} = 10^2 \times e^{1.934}$. Set cursor to 1.934 on Ln. Read 6.92 on C (or D). Then answer is $100 \times 6.92 = 692.$
2. Find $e^{-6.54}$. As in Example 1, $e^{-6.54} = 10^{-2} \times e^{-1.934}$. Set cursor hairline to 1.934 on Ln. Read 0.114 on CI (or DI). Then $e^{-6.54} = 0.00144.$
3. Find $e^{17.4}$ Divide 17.4 by 2.303, obtaining quotient ("characteristic") 7 and remainder 1.28. Set hairline to 1.28 of Ln. Read 360 on C (or D). Multiply by 10^7.

For $-\infty < y < +\infty$, we have $0 < e^y < +\infty$. The effective range for powers of *e* is infinite. Three or four significant figures of e^y can be found.

(g) A short cut in using $Ln10 = 2.30258$.

To extend the range of Ln the number 2.30258 is needed. Suppose that, to save work, the number 2.3 is used. Some error will of course occur. For example, the remainder in division will be too large. How can we easily correct for this error? The following simple rule will serve:

Rule. **Take 1 percent of the quotient and divide it by 4. Subtract the result from the remainder to obtain the correct remainder to set on Ln.**

Example. Find $e^{17.4}$ (Compare with Example 3, page 9, under Base *e*). Divide:

$$\begin{array}{cc}
7. & 7. \\
2.3 \overline{\smash{)}17.4} & \text{or} \quad 2.30258 \overline{\smash{)}17.40000} \\
\underline{16.1} & \underline{16.11806} \\
1.3 & 1.28194
\end{array}$$

Take 1% of 7; $0.01 \times 7 = 0.07$; Divide by 4. $0.07 \div 4 = 0.02$, approx. Subtract 0.02 from 1.3, to obtain 1.28, the corrected remainder. Then $e^{17.4} = e^{1.28} \times 10^7$. The basis of this rule is explained below.

Consider $x = e^n$. Divide n by 2.30258, and denote the integral part of the quotient by q and the remainder by r. Then, $n = 2.30258q + r$, $r < 2.30258$.

We now propose to use 2.3 as divisor in place of 2.30258. We require the quotient to again be q, but get a new remainder which we denote by R, where $R > r$. Then, $n = 2.3q + R$, where $R < 2.3$.

Subtracting this from the former equation, we have $0 = 0.00258q + r - R$, or $r = R - 0.00258q$.

Thus the error, $R - r$, in the remainder is $0.00258q$. If this is rounded off to $0.0025q$, it expresses one-fourth of 1 percent of the quotient.

When the slide rule is used to divide by 2.30, proceed as follows: Set 2.30 of C over 17.4 of D. Under 1 of C read 7.56 on D. The integral part, or "characteristic", is 7. Multiply the decimal fraction 0.56 by 2.3, using the C and D scales. Obtain 1.29 as the reduced exponent of *e*.

With Model 4 rules the quotient may be obtained by merely setting the exponent on DF/M and reading the quotient on D. The relation between readings on the D and the DF/M scales may be indicated symbolically as follows:

$$(D) \times 2.30 = (DF/M) \quad \text{and} \quad (DF/M) \div 2.30 = (D).$$

For some purposes and for some exponents, this slide rule method is not sufficiently accurate.

Examples for practice.

1. Find $e^{7.61}$. Divide 7.61 by 2.3; quotient 3, remainder 0.71. Correction is $0.03/4 = .01$. Hence $e^{7.61} = e^{0.70} \times 10^3$, or 2,018.

2. Find $e^{-6.95}$. Divide 6.95 by 2.3 to get characteristic 3 and remainder 0.05. Correction is $0.03/4 = 0.0075$. Corrected remainder is 0.0425. Hence $e^{-6.95} = e^{-0.0425} \times 10^{-3}$. Set 0.0425 on Ln, read 0.958 on CI. Point off 3 places to the left, to get 0.000958.

3. Find e^9. Divide 9 by 2.3. Characteristic is 3, remainder 2.1. Correction is $0.03/4 = 0.0075$, or $.01$. $e^9 = e^{2.09} \times 10^3$. Set 2.09 on Ln, read 8.1 on C. Then $e^9 = 8100$.

4. Find e^{-9}, or $e^{-2.09} \times 10^{-3}$. Set 2.09 on Ln. Read on 0.125 on CI, point off 3 places to left to find $e^{-9} = 0.000123$.

[10]

PART 2

In Part 2 you will learn how the L and Ln scales are used in combination with other scales. The methods used when the L scales are on the body differ from those used when they are on the slide. Follow only the instructions for the type of slide rule you have.

In solving problems, first express the numbers in standard form as explained in Section (f) and Section (g), pages 9 and 10. The calculations are carried through within the ranges of L and Ln provided, and the decimal points are determined by special rules.

(h) Multiplication with powers. The scales below are set to find $16.8 \times e^{1.15}$ with L scales on the slide.

Set 1 of C over 16.8 on D   Under 1.15 on Ln,

read 530 on D

Notice that when 1 of the C scale is set over 16.8 of the D scale, the product of 16.8 and any number set on C is read on D. But by setting the cursor hairline over 1.15 of Ln the value of $e^{1.15}$ is *automatically* set on C. This number (actually 3.16) does not have to be read. The product is on D. With the log log scales, this value (3.16) must be read and transferred to C before the multiplication can be started.

Rule for $a \cdot e^v$. If L scales are on slide, set 1 of C over a on D. Move cursor hairline to y of Ln. Read figures of answer on D. Determine the decimal point by standard form method. If L scales are on body, *begin* with e^v. Set cursor hairline to y on L. Set 1 of C under cursor hairline. Move cursor to a of C. Read answer on D. With powers of 10, use L in the same way.

Set 1 of C under 1.15 on Ln   Under 16.8 on C,

L scales on body

read 530 on D

[11]
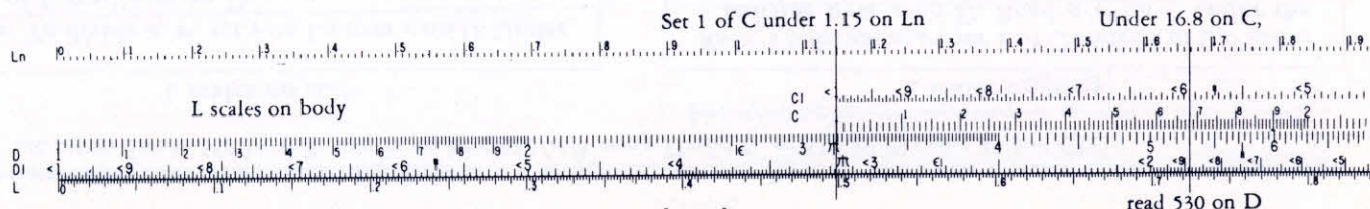
(i) Division with powers. Remember that division is the opposite of multiplication. The scales pictured in Section (h), on page 11, are set to divide 530 by $e^{1.15}$; that is, to find $530/e^{1.15}$ using D and C, or $530\,e^{-1.15}$, using D and CI.

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L scales on slide

Rule: To divide a/e^y, set y on Ln over a on D. Under 1 of C read a/e^y on D.

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L scales on body

Rule: To divide a/e^y, set 1 of C under y of Ln. Move hairline over a on D. Read a/e^y on C under the hairline.

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(j) Examples for practice.

1. Find $2.79\ e^{1.945}/3.82$. Set hairline over 2.79 on D. Move slide so 3.82 on C is under hairline. Move hairline to 1.945 on Ln. Read 5.12 on D.

2. Find $17.35\ e^{1.226}\sin 43°$. Set 1 of C over 17.35 on D. Move hairline over 1.226 on Ln. Move right index of C under hairline. Move hairline to 43 on S. Read 40.3 on D.

3. Find $0.0000452e^{7.61}$ (see Ex. 1, p. 10). Write the work in standard form: $4.52\times10^{-5}\times e^{0.70}\times10^3=4.52\times e^{0.70}\times10^{-2}$. Set index of C over 452 on D. Move hairline over 0.70 on Ln. Read 912 on D. Answer is 0.0912.

4. Find $5.27^2e^{12.7}$. First rewrite $e^{12.7}$ as $e^{1.19}\times10^5$. Set hairline over 5.27 on $\sqrt{}$. Turn rule over, and set index of C under hairline. Move hairline to 1.19 on Ln. Read 910 on D. Note 5.27^2 is about 30 or, roughly, 3×10. Also $e^{1.19}$ is about 3. Answer, then, is about $3\times3\times10\times10^5$ or 9×10^6. Correct to three significant figures, answer is 9.10×10^6.

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1′. Find $2.79\ e^{1.945}/3.82$. Set hairline over 1.945 of Ln. Set slide so 3.82 of C is under hairline. Move hairline over 2.79 on C. Read 5.12 on D.

2′. Find $17.35\ e^{1.226}\sin 43°$. Set hairline over 1.226 on Ln. Move slide so 17.35 on CI is under hairline. Move hairline to 43 on S. Read 40.3 on D.

3′. Find $0.0000452e^{7.61}$ (see Ex. 1, p. 10). Write the work in standard form: $4.52\times10^{-5}\times e^{0.70}\times10^3=4.52\times e^{0.70}\times10^{-2}$.
Set hairline over 0.70 on Ln. Move slide so index of C is under hairline. Move hairline over 452 on C. Read 912 on D. Answer is 0.0912.

4′. Find $5.27^2e^{12.7}$. First rewrite $e^{12.7}$ as $e^{1.19}\times10^5$. Set hairline over 1.19 on Ln. It is not convenient to use the A scale for 5.27^2. Move slide so 527 on CI is under hairline. Move hairline to 527 on C. Read 910 on D. Note 5.27^2 is about 30, or, roughly, 3×10. Also $e^{1.19}$ is about 3. Answer, then, is about $3\times3\times10\times10^5$ or 9×10^6. Correct to three significant figures, answer is 9.10×10^6.

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(k) Logarithms of combined operations. The scales below are set to find ln 6.78/3.24 and log 6.78/3.24 with L scales on the body.

ln 6.78/324 = 0.737 on Ln

3.24 on C

6.78 on D

Log 6.78/3.24 = 0.321 on L
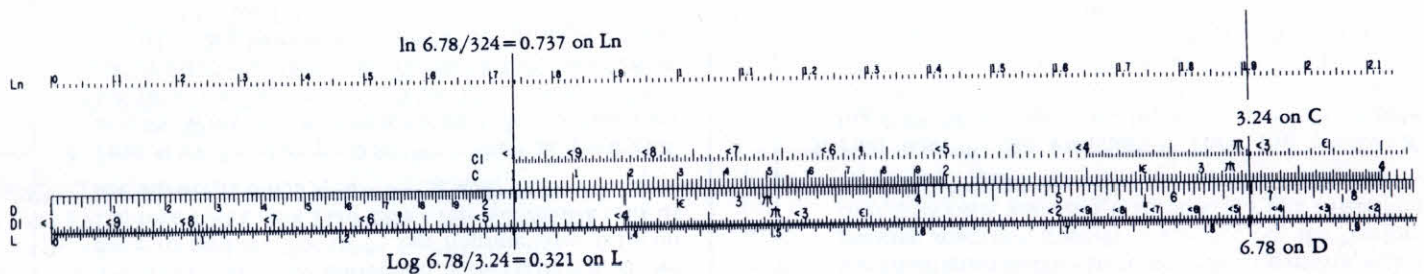
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First, divide 6.78 by 3.24 in the usual way using C and D scales. Move cursor over 1 of C. Read ln 6.78/3.24 on Ln and read log 6.78/3.24 on L. If L scales are on the slide, close rule (move slide so C and D indexes coincide) before reading from Ln or L. Or, if you prefer, first set slide so 1 of C is over 3.24 on D. Move hairline over 6.78 on D. Read logarithm from Ln or L.

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Rule: To find ln a/b or log a/b, set b on C over a on D. At index of C read ln on Ln and log on L. Characteristics must be found by a special rule.
If L scales are on slide, set 1 of C over b on D. Move indicator over a on D. Read logarithm from Ln on L. This method requires only one setting of the slide.

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ln 6.78/3.24 = 0.737 on Ln
log 6.78/3.24 = 0.321 on L

3.24 on D

6.78 on D

(*l*) **Powers of other bases.** Sometimes powers of bases other than 10 or *e* are needed. If a slide rule has Log Log scales, they may be used to find these powers within the range provided. If the slide rule does not have Log Log scales, or if the power is outside the scale range provided, one of the following methods may be used.

<table>
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Using base *e*.

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Using base 10.

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Examples

1. Find $1.5^{2.4}$. Write $e^x = 1.5$. Set 1.5 on C (or D), find $x = 0.405$ on Ln. Then $(1.5)^{2.4} = (e^{0.405})^{2.4} = e^{0.972}$ by multiplying the exponents. Using Ln again, set 0.972 on Ln, read the answer 2.65 on C (or D). This solution can be expressed in logarithmic form as follows: ln $1.5^{2.4} = 2.4$ ln 1.5.

2. Find $18.5^{-6.37}$. Set $y = 18.5^{-6.37}$. Then ln $y = -6.37 \times$ ln 18.5. Now ln $18.5 = $ ln $1.85 \times 10 = 0.615 + 2.303$ or 2.918. Now $-6.37 \times 2.918 = 18.58$ and $18.58 \div 2.3 = 8$ with remainder of 0.18. But 1% of $8 = .08$, and $.08 \div 4 = .02$, so with the correction $0.18 - 0.02 = 0.16$, we have to find $e^{-0.16} \times 10^{-8}$. Set hairline over 0.16 on Ln, read 0.852 on CI (or DI). The quotient 8 tells us the answer is $0.852 \times 10^{-8} = 8.52 \times 10^{-9}$.

3. Find $0.88^{0.25}$. Set $y = 0.88^{0.25}$. Then ln $y = 0.25 \times$ ln 0.88. Write ln $0.88 = $ ln 8.8×10^{-1}. Set hairline over 8.8 of C (or D), read 2.175 on Ln. Then ln $8.8 \times 10^{-1} = 2.175 - 2.303 = -0.128$, and $0.25 \times (-0.128) = -0.032$. Set hairline over 0.032 on Ln, read answer 0.968 on CI (or DI).

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Examples

1. Find $1.5^{2.4}$. Write $10^x = 1.5$. Set 1.5 on C (or D), find $x = 0.176$ on L. Then $(1.5)^{2.4} = (10^{0.176})^{2.4} = 10^{0.422}$, by multiplying exponents. Set 0.422 on L, read the answer 2.65 on C (or D). This solution can be expressed in logarithmic form as follows: log $1.5^{2.4} = 2.4$ log 1.5.

2. Find $18.5^{-6.37}$. Set $y = 18.5^{-6.37}$. Then log $y = -6.37 \times$ log 18.5. Now log $18.5 = 0.267 + 1 = 1.267$. Now $-6.37 \times 1.267 = -8.07$. We must now set the hairline over 0.07 on L, *reading from right to left*, or subtract 8.07 from $10.00 - 10$ to write the logarithm with a positive mantissa, namely $1.93 - 10$. Set the hairline over 0.93 on L and read 8.50 on C (or D). Then the result is 8.50×10^{-9}.

3. Find $0.88^{0.25}$. Set $y = 0.88^{0.25}$. Then log $y = 0.25 \times$ log 0.88. Write log $0.88 = $ log 8.8×10^{-1}. Set hairline over 8.8 on C (or D), read 0.944 on L. Then log $88 \times 10^{-1} = 0.944 - 1$, or $3.944 - 4$. $0.25(3.944 - 4) = 0.986 - 1$. Set hairline over 0.986 of L, read 9.68 on C (or D). Then answer is 0.968.

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(*m*) **Hyperbolic functions.** The Ln scale is very helpful in finding values of the hyperbolic functions. This is especially true for Model 1011 which does not provide Log Log scales or hyperbolic function scales. However, even with Model 4 on which these extra scales are available, the Ln scale simplifies the work in problems that fall outside the range of the scales provided.

By definition, sinh $x = (e^x - e^{-x})/2$, or sinh $x = (e^x/2) - (e^{-x}/2)$.

By definition, cosh $x = (e^x + e^{-x})/2$, or cosh $x = (e^x/2) + (e^{-x}/2)$.

By definition, tanh $x = (e^x - e^{-x})/(e^x + e^{-x}) = ($sinh $x)/($cosh $x)$.

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Rule: To find sinh x or cosh x, set hairline over x on Ln, read e^x on C (or D) and e^{-x} on CI (or DI). For sinh x, subtract e^{-x} from e^x and divide the result by 2. For cosh x, add e^x and e^{-x}, and divide by 2. To find tanh x, use Ln to find e^x and e^{-x}; divide their difference by their sum.

Rule: For x > 3, sinh x = cosh x = $e^x/2$ can be found by setting the index of the C scale over 5 on the D scale, moving the hairline to x on Ln, and reading the result on D.

Examples.

1. Find sinh 5.4 or $e^{5.4}/2$. Divide 5.4 by 2.303, obtaining quotient 2 and remainder 0.794. $(e^{5.4}/2) = (e^{0.794}/2)10^2$. Set right hand index of C over 5 of D. Move hairline to 0.794 on Ln. Read 1.107 on D. This must be multiplied by 10^2, so sinh $5.4 = 110.7$.

2. Find sinh $24 = e^{24}/2$. First write $e^{24} = e^{0.975} \times 10^{10}$. Using Ln, find $e^{0.975} = 2.65$. Then $2.65/2 = 1.32$, so sinh $24 = 1.32 \times 10^{10}$.

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Extending the ranges. Remember that if $x > 2.3$, you must first divide x by 2.3 and correct the remainder. The corrected remainder is set on Ln instead of x, and the integral quotient q is the exponent of 10 such that the factor 10^q determines the position of the decimal point.

For $x > 3$, we have $(e^{-x}/2) < 0.025$. Hence, for $x > 3$, sinh $x = $ cosh $x = e^x/2$, approximately, and tanh $x = 1$. For $x < 0.10$, we have sinh $x = x$, cosh $x = 1$, and tanh $x = x$, approximately.

On Model 4 the values of e^x and e^{-x} for $x < 23$ can be found directly on the Log Log scales by using the DF/M scale. For $x > 10$ the accuracy is poor. For $x > 23$, the Log Log scales are useless.

Examples.

3. Find cosh $4.8 = e^{4.8}/2$. Write $e^{4.8} = e^{0.195} \times 10^2$. Set left index of C over 5 on D. Move hairline over 0.195 on Ln. Read $e^{0.195}/2 = 0.608$ on D. Then cosh $4.8 = 60.8$.

4. Find tanh 1.3. Using Ln, read $e^{1.3} = 3.67$ on C (or D) and $e^{-1.3} = 0.273$ on CI (or DI). Then $3.67 - 0.273 = 3.397$ and $3.67 + 0.273 = 3.94$; hence tanh $1.3 = 0.862$.

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(n) Applied problems.

1. As an extraordinary example consider the following quotation:
   "The total N for the entire line is $N = 0.1118 \times 2000 = 223.6$ nepers, and the ratio of input to output current
   $$I_s/I_r = e^{223.6} \doteq 10^{97}"*$$
   Calculate $e^{223.6}$.
   By the method of Section (g), page 10, above, we divide 223.6 by 2.3, and correct the remainder.

   ```
        ·97
   23/223.60          0.01×97=0.97
       207.
        16.6          1/4×.97=0.24
        16.1
          .50         0.50−.24=0.26
   ```

   Set hairline over 0.26 of Ln.
   Read 1.297 on C.
   The result is 1.297×10^{97}. The result found by logarithms is 1.286×10^{97}. The error is 0.85%, or under 1%, and occurs because the correction formula uses 0.0025 instead of 0.00258. This shows that the method using Ln is sufficiently accurate for all exponents up to 100; such large exponents are exceedingly rare.

   *Ware, Lawrence A. and Reed, Henry R. *Communication Circuits.* (New York, 1942), John Wiley and Sons, Inc. p. 49.

2. A table of standard sizes for rectangular wire may be made by inserting 38 geometric means between the diameter (0.46 in.) of Gauge 0000 and the diameter (0.005 in.) of Gauge 36 of the American Wire Gauge.

   Calculate the common ratio $r = \sqrt[39]{\dfrac{0.4600}{0.005}}$, and compute the 36th term.
   First note that $r = (460/5)^{1/39}$, or $(92)^{1/39}$. Write $10^x = 92$. Set 92 on C (or D), find mantissa of x, or 0.964 on L. Then $x = 1.964$. Then $r = (10)^{1.964/39} = 10^{0.0503}$; set 0.0503 on L, read $r = 1.123$ on C (or D). The 36th term is 0.005×1.123^{35}, or $0.005 \times 10^{35} \times (0.0503) = 0.005 \times 10^{1.761}$. Set hairline over 0.761 on L, read 5.77 on C (or D). Finally, compute $0.005 \times 10 \times 5.77 = 0.289$ in., or 289 mils, approximately.

3. The formula for the current in a certain circuit is
   $$i = 1.25\,(1 - e^{-80t}),\ 0 \leq t \leq 0.01.$$
   Find i for $t = 0.006$; that is,
   $$i = 1.25\,(1 - e^{-80 \times 0.006}) = 1.25\,(1 - e^{-0.48}).$$
   Set hairline over 0.48 on Ln, read $e^{-0.48} = 0.619$ on CI (or DI). Then $i = 1.25\,(1 - 0.619) = 1.25 \times 0.381 = 0.476$.

4. In a problem similar to 3, above, the formula is $i = 4\,(1 - e^{-40t})$, $0 \leq t \leq 0.02$. Find i for $t = 0.015$; that is, $i = 4\,(1 - e^{-0.60})$. Answer: 1.804.