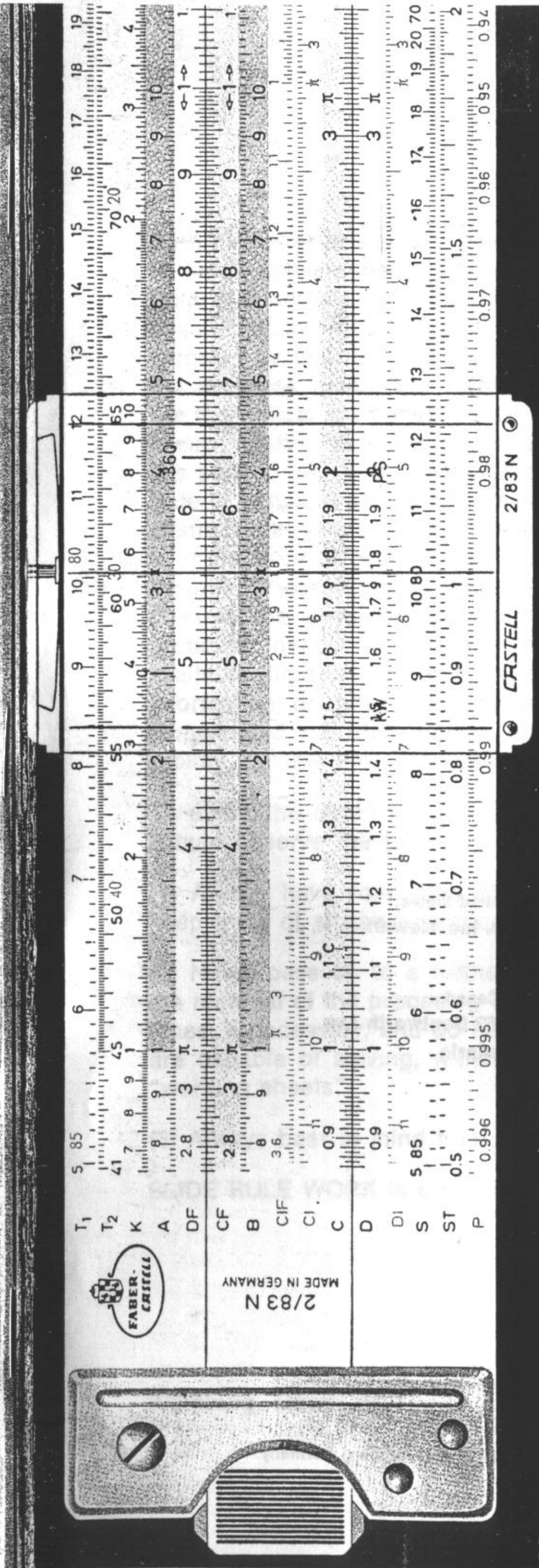
A. W. FABER-CASTELL · STEIN BEI NÜRNBERG

PETRY: USE OF THE 2/83 N AND 2/82 N SLIDE RULES



S. PETRY

Use of the 2/83 N and 2/82 N Slide Rules.

FABER-CASTELL

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FOREWORD

Both publisher and author have made it their aim to provide a modern textbook and manual which will enable the user to master the Slide Rule. Numerous illustrations and examples are provided for this purpose, in addition to a large number of problems set for practice and followed by their solutions, providing an objective means of verifying the progress made. We have deliberately omitted all discussion of the theoretical principles on which the operation of the Slide Rule is based; neither have we assumed a knowledge of these principles on the part of the reader, as it is not required for the use of the Slide Rule in actual practice. All examples and exercises are largely of the abstract mathematical kind. There are hardly any problems belonging to the sphere of practical engineering or framed in accordance with assumed concrete circumstances, for the reader will doubtless already possess the physico-mathematical knowledge relevant to his own particular needs or will be able to acquire it from a study of the available literature, all that he asks from a slide rule textbook being information on how both comparatively simple and comparatively complicated calculations can be carried out with the slide rule as rapidly, reliably and accurately as possible.

To attain the desired degree of success when using this book, you should observe the following advice:

(1) Always have your slide rule by you when studying the book and keep track of all the operations described.

(2) Never pass on to a further section until you have fully mastered the material of the preceding one. Work through the exercises several times, if necessary, and do not continue to the next section until you are capable of solving, without any errors, the problems set on the "working sheets".

(3) Always bear in mind that

SLIDE RULE WORK is a matter of PRACTICE!

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Würzburg, March 1970

Contents

1. General

1.1 Description of slide rule

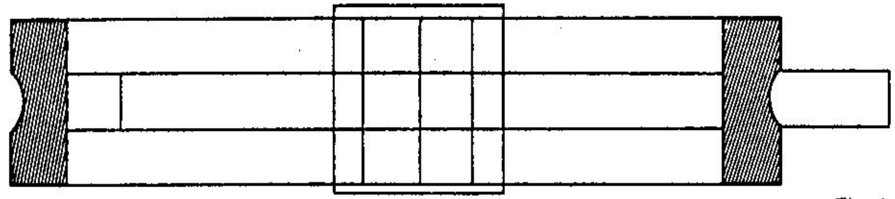


Fig. 1

The stock consists of the two end frames, interconnected by four stays. If necessary, the end frames and the ease of movement of the slide can be adjusted after releasing two screws on the stays. Rubber ridges inserted in the stays ensure that the slide and the cursor remain freely movable even when the slide rule is resting on a flat surface.

The code number for each scale is shown at the left-hand end of the slide rule (e.g. T_1, T_2, K, A, etc.). These designations will be used later in the descriptions of the calculations. On the right, next to each scale, is a mathematical formula (e.g. x^2, x^3, $\frac{10}{x}$) indicating the relationship of the scale concerned to one of the basic scales C and D, which are marked "x" at the right-hand end. The scales on the two sides of the slide rule are linked together by the long uninterrupted stroke provided in the centre of the cursor, both on the front and on the back, enabling the calculation to be continued from the front to the back of the slide rule and vice versa. The meaning of the broken and short cursor lines will be explained in due course.

An exact description of the subdivision of the main scales accompanies the diagrams at the end of this manual. Attention is also drawn to the relevant sections of the Directions for Use supplied with each slide rule.

It is particularly important to note that most of the scales (apart from exceptions which will be pointed out as and when they arise) bear simple series of figures not tied to any particular decimal place. Thus, the "4" on Scale D can be used, in calculations, for 4, 40 or 400, and also for 0.4, 0.04, 0.004 etc.

Absolute mastery of the operations of setting and reading the scales is indispensable for rapid and reliable calculation with the slide rule.

These skills can be exercised and verified by the aid of Working Sheets Nos. 1 and 2.

To set the cursor line* to a certain value, the cursor is first of all moved with o n e hand into a position close to the value required and then finally adjusted by means of the two thumbs, which will slide it along the lower narrow side of the stock of the slide rule (i.e. the narrow side closer to the user). This operation can be assisted with the two index fingers on the upper narrow side. The cursor line can thus be rapidly and accurately set to the desired value.

In the course of this operation the slide rule can be either held in the hands or placed on the table.

If necessary, you should now go through Working Sheets 1 and 2.

* In the body of the text of this manual, this will always mean, unless otherwise stated, the continuous stroke running through the middle of the cursor.

Working Sheet No. 1

It is suggested that the student first of all solve every problem, entering the answers in the respective spaces with a soft pencil. Only then should he check his answers from the key below. If necessary, they can be erased and the exercises repeated. The course of study should not be continued until the material has been thoroughly mastered.

Place the cursor line over the following value on Scale D	Move the cursor line by the following number of spaces to the right (r) or left (l)	The cursor line will now be at
242 (to be interpreted as 2-4-2)	3 r	248 (specimen answer)
775	4 r	1. ....
905	3 l	2. ....
378	3 r	3. ....
162	3 l	4. ....
455	5 r	5. ....
109	2 r	6. ....
815	4 l	7. ....
101	4 l	8. ....
111	3 r	9. ....
990	4 r	10. ....
505	2 l	11. ....
286	3 r	12. ....
665	3 l	13. ....
790	3 r	14. ....
895	2 r	15. ....
425	3 l	16. ....
107	4 l	17. ....
202	3 r	18. ....
320	3 l	19. ....
805	2 r	20. ....
199	2 r	21. ....
308	3 l	22. ....
112	4 l	23. ....
735	3 r	24. ....

Answers: 1. 795; 2. 890; 3. 384; 4. 159; 5. 480; 6. 111; 7. 795; 8. 985; 9. 114; 10. 102; 11. 495; 12. 292; 13. 650; 14. 805; 15. 905; 16. 410; 17. 103; 18. 208; 19. 314; 20. 815; 21. 202; 22. 302; 23. 108; 24. 750.

Working Sheet No. 2

For combined setting and reading exercises with the intermediate values to be estimated, we use not only Scale D but also Scale DF (on the back of the slide rule, in the case of the Castell 2/82 N).

This is subdivided on exactly the same lines as Scale D, except that the value "1" is to some extent displaced towards the middle of the scale. (As will be explained in due course, in moving upwards from D to DF we multiply by π the value to which the former has been set.)

Place the cursor line on the following value on D	The following value on DF will now be found under the cursor line
515 (to be interpreted as 5-1-5)	1619 (specimen answer)
710	1. ....
870	2. ....
915	3. ....
146	4. ....
203	5. ....
297	6. ....
365	7. ....
413	8. ....
477	9. ....
509	10. ....
641	11. ....
683	12. ....
7725	13. ....
9675	14. ....
1035	15. ....
1183	16. ....
1305	17. ....
1398	18. ....
1702	19. ....
1785	20. ....
2035	21. ....
2265	22. ....
291	23. ....
337	24. ....
320	25. ....

Answers: (Note remarks in Section 1.2!).

1. 223; 2. 2735; 3. 2875; 4. 459; 5. 638; 6. 933; 7. 1147; 8. 1297; 9. 1499; 10. 1599; 11. 2015; 12. 2145; 13. 2425; 14. 304; 15. 325; 16. 372; 17. 410; 18. 439; 19. 535; 20. 561; 21. 639; 22. 712; 23. 914; 24. 106; 25. 1005.

1.2 The accuracy of slide rule work

Owing to the logarithmic subdivision of the slide rule, the relative error possible in setting or reading a value is the same at all points on the basic scales and amounts to about 1‰, for example, in the case of a slide rule with a scale of 25 cm in length. With great care and a certain amount of practice, this can be reduced to about 0.5‰.

In a calculation requiring a total of n settings and readings, the maximum error, according to the probability theory,

is n ‰ or $\frac{n}{2}$ ‰

and the probable error $\sqrt{n}$ ‰ or $\frac{\sqrt{n}}{2}$ ‰.

In other words, for a simple multiplication or division consisting of two settings and one reading, the maximum error will be 3‰ (or 1.5‰) and the probable error $\sqrt{3}$‰ $\approx$ 1.5‰ (or 0.75‰).

These observations become important as soon as the use of the Working Sheets is commenced:

3 or 4 decimal places can be taken into account when setting or reading with the basic scales; four at the left-hand end, three at the right. The solutions to the problems for practice will thus be given with 3 or 4 significant digits.

They have been calculated with a digital computer of far higher accuracy and rounded off to the relevant number of digits in each case.

Owing to the unavoidable setting and reading error, however, it is possible and permissible for the answers found by the student using the exercises to deviate upwards or downwards from the result given, by 1 or 2 units of the last decimal place.

Example: If a result is given as 8.93, the values 8.92 and 8.94 will still be regarded as correct – as will, in the case of more extensive calculations, all values between 8.91 and 8.95.

2. Multiplication

2.1 Preliminary remarks

In all multiplications (and other calculations) the slide of the slide rule has to be adjusted in position. The following method has proved the most suitable: place the slide rule on the table or hold it between your hands. Then, using one hand only, move the slide of the slide rule into approximately the desired position and follow this up by the exact setting, using the tips of the fingers.

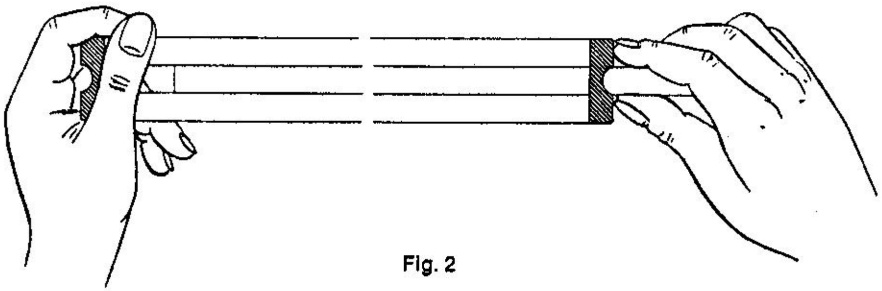


Fig. 2

Avoid compressing the end frames of the slide rule in this process, as this would render it difficult, if not impossible, to move its slide to and fro.
If it proves stiff, this can usually be remedied by the application of a little pure vaseline or silicon oil.

Multiplication can first of all be carried out with the two pairs of scales C/D and CF/DF (on the back, in the case of the Castell 2/82 N). The pair of scales last mentioned has the same graduation as the first, except that the entire scale is "displaced" towards the left by the distance π; in other words, the value π on CF or DF appears above the 1 on C or D.

2.2 Calculatory process

a · b

The multiplication of two numbers, a and b, is carried out by the following procedure:

(1) Scale D is set to the value "a" with the use of the cursor line (Example: a = 2.7).

(2) If "a" is on the left-hand half of Scale D (round: "a" to the left of 3), the 1 at the left-hand end of Scale C is placed underneath the cursor line. On the other hand, if "a" is on the right-hand end of Scale D (round: "a" to the right of 3), the 10 at the right-hand end of scale C is placed underneath the cursor line (Example: a = 6.3).
The slide thus never requires to be displaced by more than half its length, and at the same time the otherwise frequently necessary transposition (complete transfer) of slide (from 1 to 10 or vice versa) is reliably avoided. The user should thus accustom himself from the outset to this time-saving method.

(3) The cursor line is now placed over the value "b" on one of the two scales C or CF backed with green panelling, wherever this value can be reached most conveniently, i.e. with the minimum cursor movement, and where it does not project beyond the adjacent scale on the stock of the slide rule. The product, a × b, will then be found on this latter scale, underneath the cursor line.

Example: a = 2.7. The mark 1 of scale C is placed above 2.7 on D.
Abridged wording of directions: C 1 above D 2.7.
Still shorter: D 2.7 | C 1. (In this case it is D 2.7 that is mentioned first, because the cursor line is set to D 2.7 first. The vertical stroke between D 2.7 and C 1 indicates that the two values are to be placed vertically above or below each other).

1. If b = 3.2, this value can be set either on C or on CF, as desired; the setting on CF, however, only calls for a slight displacement of the cursor towards the right and is therefore of greater advantage. The result will be found vertically above it, on DF (or vertically below it, on D): 8.64.

   Abridged directions: CF 3.2 | DF: 8.64 or C 3.2 | D: 8.64.

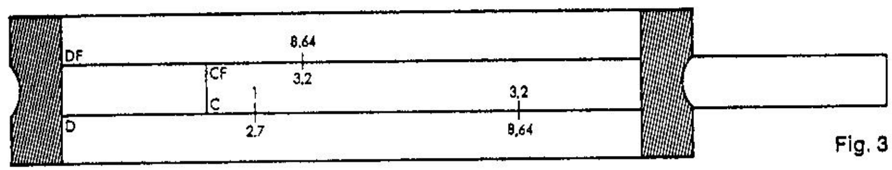


Fig. 3

2.7 × 3.2 = 8.64

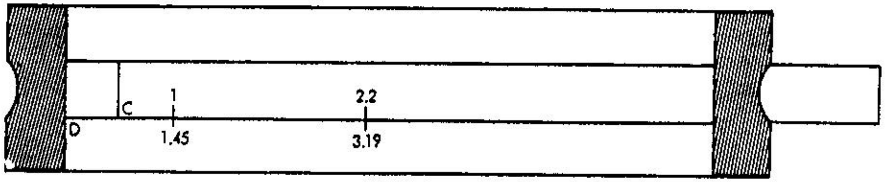
2. For b = 1.5, this value can only be set on Scale C, on CF it projects beyond DF. Result: 4.05.
   Conversely:

3. For b = 4.7, the value can only be set on CF.
   Answer: 12.7.

Summary:
The first factor is set on Scale D and C 1 or C 10 then moved into position above it. Rule: Never pull the slide of the rule out by more than one half of its length! The second factor is set on a green-backed scale (C or CF) and the product read on the adjacent (white) scale.

The following diagrams illustrate the process once again, for various sets of circumstances.

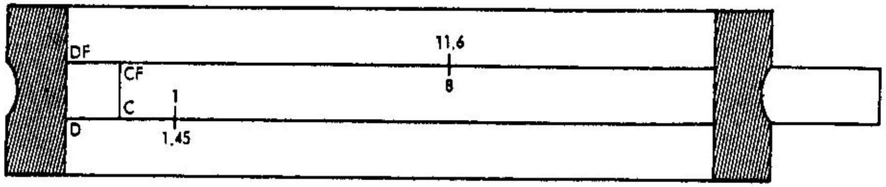
1st factor set with C 1, second on C 2:



1.45 × 2.2 = 3.19

Fig. 4

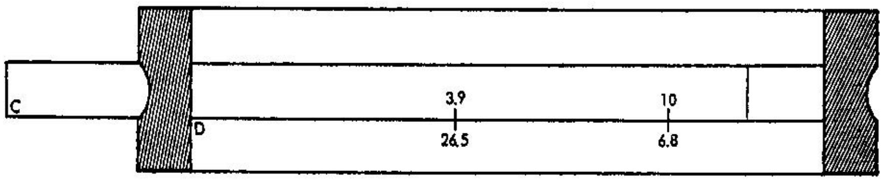
1st factor set with C 1, 2nd on CF:



1.45 × 8 = 11.6

Fig. 5

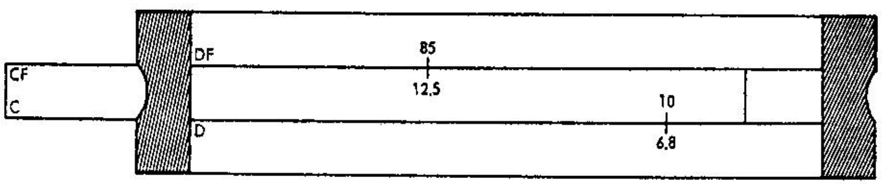
1st factor set with C 10, 2nd on C:



6.8 × 3.9 = 26.5

Fig. 6

1st factor set with C 10, 2nd on CF:



6.8 × 12.5 = 85

Fig. 7

A further tip:

As the factors may occur in any order (a × b giving the same product as b × a) an experienced slide rule user will select the number entailing the shorter movement of the slide as the factor to be set on Scale D.

Examples: 1. 2.92 × 1.28 = ? Better: 1.28 × 2.92.
2. 2.58 × 8.74 = ? Better: 8.74 × 2.58, and setting C 10.

Now go through Working Sheet No. 3.

Working Sheet No. 3

In the following multiplication exercises the order of magnitude of the product (i.e. the position of the decimal place therein) can be determined easily without any special aids.

1. 1.68 x 4.20 = ...	16. 90.4 x 1.083 = ...
2. 9.05 x 0.73 = ...	17. 11.04 x 39.9 = ...
3. 12.6 x 9.65 = ...	18. 6.91 x 303 = ...
4. 58.4 x 1.34 = ...	19. 123 030 x 0.775 = ...
5. 179 x 0.825 = ...	20. 1304 x 8.05 = ...
6. 0.0243 x 6.6 = ...	21. 912 x 4.66 = ...
7. 8460 x 0.73 = ...	22. 30.3 x 0.208 = ...
8. 0.001 04 x 4.66 = ...	23. 1.475 x 0.0864 = ...
9. 22 300 x 0.955 = ...	24. 0.693 x 106.5 = ...
10. 0.8953 x 319 000 = ...	25. 0.0277 x 7.08 = ...
11. 0.072 42 x 10.45 = ...	26. 5670 x 0.395 = ...
12. 33.86 x 9.142 = ...	27. 11.1 x 23.3 = ...
13. 644.8 x 0.1095 = ...	28. 404 x 0.152 = ...
14. 100.5 x 0.486 = ...	29. 0.764 x 0.595 = ...
15. 14.05 x 9.63 = ...	30. 138.5 x 8.86 = ...

Answers: 1. 7.06; 2. 6.61; 3. 121.6; 4. 78.3; 5. 147.7; 6. 0.1604; 7. 6180; 8. 0.004 85; 9. 21 300; 10. 286 000; 11. 0.757; 12. 309.5; 13. 70.6; 14. 48.85; 15. 135.3; 16. 97.9; 17. 440.5; 18. 2095; 19. 95 350; 20. 10 500; 21. 4250; 22. 6.30; 23. 0.1274; 24. 73.8; 25. 0.1961; 26. 2240; 27. 258.5; 28. 61.4; 29. 0.455; 30. 1227.

2.3 Determining order of magnitude of product

As already mentioned, the position of the decimal point is immaterial at the stage when the factors are being set on the scales. The reading is therefore merely a series of digits, and the position of the point within it has to be determined separately. In the samples in Working Sheet No. 3, no problems arise in this direction, the order of magnitude of the results being obvious. This, however, is not always the case, particularly where multiple products or complicated fractions are concerned. The position of the decimal point in the result then has to be determined by one of the following two methods:

1st process: The order of magnitude of the result is estimated by roughly rounding off the figures (in this case, the factors).

Examples: 1. $246 \times 0.065 \approx 200 \times \frac{6}{100} = 12$; Result: 16.0

2. $0.17 \times 0.093 \approx 0.2 \times 0.1 = 0.02$; Result: 0.01581

3. $34 \times 896 \approx 30 \times 1000 = 30\,000$; Result: 30 450

Important: The result of the 3rd example is not quite correct, as the system fails to cope with some of the places of decimals therein. The correct result is not 30 450 but 30 464, but this can no longer be read on the slide rule. In order to avoid giving an exaggerated impression of accuracy, the result should be given in the form 3.054×10^4.

The methods described below automatically preclude such defects.

2nd process: If the factors are very great or consist of decimals with a number of 0's after the decimal point, the operation of estimating the result is uncertain and awkward.

Examples: 1. $36\,400 \times 128\,000 = ?$

2. $0.034 \times 86\,120 = ?$

3. $0.0087 \times 0.000\,321 = ?$

In such cases, each factor is expressed as a product of a decimal between 1 and 10 and a power of ten.

For the above examples, this method gives the following:

$36\,400 = 3.64 \times 10^4$; $128\,000 = 1.28 \times 10^5$
$0.034 = 3.4 \times 10^{-2}$; $86\,120 = 8.612 \times 10^4$
$0.0087 = 8.7 \times 10^{-3}$; $0.000\,321 = 3.21 \times 10^{-4}$

The simplest method of finding the correct exponent is to count the number of places by which the decimal point has to be moved to the left (positive exponent) or to the right (negative exponent) to position it after the first digit of the series of digits in question.

Examples: 3 6 4 0 0 = 3.64×10^4

Decimal point 4 places to the left.

0 . 0 0 8 7 = 8.7×10^{-3}

Decimal point 3 places to the right.

After this adjustment, we commence the calculation, multiplying the decimals on the slide rule and the power of ten mentally.

Examples from above:

1. $36\,400 \times 128\,000 = 3.64 \times 10^4 \times 1.28 \times 10^5$
$= \underbrace{3.64 \times 1.28}_{\text{Calculation with slide rule}} \times \underbrace{10^4 \times 10^5}_{10^9}$ (This adjustment of the factors need not actually be written down).
$= 4.66 \times 10^9$

In the multiplication of the powers of ten, the exponents are added together (due attention being paid to their sign!).

2. $0.034 \times 86\,120 = 3.4 \times 10^{-2} \times 8.612 \times 10^4$
$= 3.4 \times 8.612 \times 10^{-2} \times 10^4$
$= 29.3 \times 10^2 = 2.93 \times 10^3$

3. $0.0087 \times 0.000\,321 = 8.7 \times 10^{-3} \times 3.21 \times 10^{-4}$
$= 8.7 \times 3.21 \times 10^{-3} \times 10^{-4}$
$= 27.9 \times 10^{-7} = 2.79 \times 10^{-6}$

This method offers the advantage, already mentioned, that it only produces places of decimals with which the system in actual fact copes. To obtain a correct result, it may be necessary for the results obtained by the 1st method to be appropriately re-written, e.g. (3rd example under 1st process) 30.5×10^3 or 3.05×10^4 instead of 30 500.

We wish you every success with Working Sheet No. 4!

Working Sheet No. 4

The student can determine the order of magnitude of the product either by estimating it or by the aid of powers of ten in each case. To obtain the correct result, however, you should express it in the form in which no "unsecured" places of decimals occur.

1. 0.0635 × 0.009 68 = ...	16. 3 760 000 × 0.005 99 = ...
2. 0.0224 × 576 = ...	17. 674 000 × 14.38 = ...
3. 16 950 × 81.3 = ...	18. 0.0706 × 0.873 = ...
4. 726 × 318 400 = ...	19. 103.5 × 492 = ...
5. 10 050 × 0.006 57 = ...	20. 0.000 0592 × 0.003 16 = ...
6. 83.2 × 0.0954 = ...	21. 1860 × 916 = ...
7. 0.801 × 0.739 = ...	22. 70 050 × 3715 = ...
8. 136.4 × 581 = ...	23. 2 310 000 × 0.008 93 = ...
9. 356 × 709 = ...	24. 62 850 × 4060 = ...
10. 1105 × 23.01 = ...	25. 0.0794 × 0.001 22 = ...
11. 80.8 × 0.000 1905 = ...	26. 0.2995 × 0.030 05 = ...
12. 0.003 79 × 0.000 207 = ...	27. 154.5 × 4049 = ...
13. 1094 × 0.00574 = ...	28. 702 × 0.000 534 = ...
14. 0.000 083 × 9056 = ...	29. 10 040 × 3905 = ...
15. 472.5 × 804 = ...	30. 0.000 574 × 0.000 606 = ...

Answers: 1. 0.000 615; 2. 12.90; 3. 1.378×10^6; 4. 2.31×10^8; 5. 66.03; 6. 7.94; 7. 0.592; 8. 7.925×10^4; 9. 2.525×10^5; 10. 2.545×10^4; 11. 0.015 39; 12. 7.84×10^{-7}; 13. 6.28; 14. 0.7515; 15. 3.8×10^5; 16. 2.25×10^4; 17. 9.7×10^6; 18. 0.0616; 19. 5.09×10^4; 20. 1.871×10^{-7}; 21. 1.740×10^6; 22. 2.602×10^8; 23. 2.063×10^4; 24. 2.552×10^8; 25. 9.69×10^{-5}; 26. 0.009 00; 27. 6.26×10^5; 28. 0.375; 29. 3.92×10^7; 30. 3.48×10^{-7}.

2.4 Tabulation of function $y = a\,x$
Products with a constant factor

$y = a \cdot x$

It is often necessary to multiply the same value, a, by various factors in succession. It is then advisable for the constant factor, a, to be set as the first factor on the Scale D (D a | C 1 or D a | C 10), because then all the products can be calculated simply by moving the cursor and without any transposition of the slide. (Advantage of "staggered" Scales CF/DF). This process is of advantage in

- conversions from one unit of measurement to another,
- currency conversions,
- percentage calculations with a constant percentage rate,
- calculations with the Horner scheme, etc.

Example: 1 inch = 2.54 cm; x ins. = 2.54 x cm.

Find number of cm in 1.8	32.7	8.9	0.15	4.8	ins.
Answers: 4.57	83.1	22.6	0.381	12.2	cm

Setting: D 2.54 | C 1. Inch and cm values corresponding to each other will then be found on Scales C and D as well as on CF and DF.

2.5 Multiplication and division by π

$\pi \cdot x$

C | CF

D | DF

Multiplication by π can be carried out with the cursor line alone, without displacement of the slide of the slide rule, because Scales CF and DF are "staggered" by the distance π in relation to Scales C and D respectively. If, therefore, the cursor line is placed above a number x on Scale C or D, the product $\pi \times x$ will appear underneath the cursor line, on CF or DF.

This property of the pair of scales CF/DF is expressed by the symbol πx on the right hand edge of the scale. The reading on the correct scale is facilitated by the green backing of the "slide scales".

$\frac{x}{\pi}$

CF | C

DF | D

Note: Setting on white – reading on white.
Setting on green – reading on green.

Conversely, the vertical transition from Scale CF or DF to Scale C or D provides the division of the selected value by π.

2.6 Multiplication and division by 3.6, 360 and 3600

$3.6 \cdot x$
$360 \cdot x$
$3600 \cdot x$

On the front of the cursor (on the back, in the case of the Castell 2/82 N) a stroke marked "360" is provided to the right of the main line and on a level with Scales CF and DF. In conjunction with the displacement of the upper scales, the distance of the stroke from the main line results in a displacement by the distance 3.6. (To check: place cursor line above C 1; the stroke "360" will then be above CF 36). The multiplication corresponding to this displacement, however, is by the factor 3.6 (or 360 or 3600).

If, therefore, the cursor mark is set to a number x on Scale C or D, a reading of the value 3.6 x or 360 x or 3600 x is provided under the mark 360 on CF or DF. (Once again, note the rule "setting on white, reading on white – setting on green, reading on green"!).

$\frac{x}{3.6}$

$\frac{x}{360}$

$\frac{x}{3600}$

Applications:

(1) Conversions from speeds in km/h to m/s and vice versa.

Rule: 1 m/s = 3.6 km/h and 1 km/h = $\frac{1}{3.6}$ m/s.

Examples: 1. 5 m/s = 18 km/h.
2. 72 km/h = 20 m/s.

Setting: 1. Central line on C 5 or D 5.
Under mark 360 on CF or DF: 18.
2. Mark 360 on CF 72 or DF 72.
Under central line on C or D: 20.

(2) For interest calculations: 1 year = 360 days. With the use of the mark 360, years can be converted into days and vice versa.

Examples: 1. $1^3/_4$ years = 1.75 years = 630 days.
2. 124 days = 0.344 years.

(3) As 1 hour = 3600 seconds, the same line is used for the conversion of hours into seconds and vice versa. The same applies to angular degrees and angular seconds (1° = 3600").

Examples: 1. 1.35 h = 4860 sec.
2. 266" = 0.0628°.

(4) Conversion of kWh into watt-seconds and vice versa (1 kWh = 3.6×10^6 watt-seconds).

2.7 Multiplication with Scale DF as basis

As the pair of scales C/D is of the same nature as the pair of scales CF/DF (apart from the displacement by π) the product a × b can also be calculated by setting Scale DF to the factor "a" and placing underneath it the mark –1– of the Scale CF.

The product a × b is then found on DF over CF b or on D under C b.

Setting of 1st factor with CF 1.

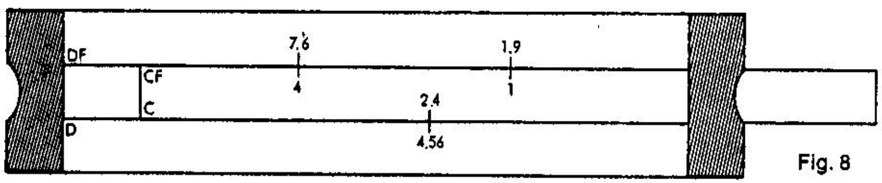


Fig. 8

1.9 × 2.4 = 4.56; 1.9 × 4 = 7.6

In principle, therefore, the process is the same as described farther back for the Scales C/D.

This method is useful:

(1) When after multiplication of a number "a" by π (by the change-over from D to DF) the answer has to be multiplied by a further factor.

Example: Table of function 4.05 π x.
Place CF 1 above D 4.05 (abbreviated instruction: D 4.05 | CF 1).
Above CF x the required value is found on DF (abbreviated instruction: CF x | DF: 4.05 π x).

x	3.6	6.45	12.4	0.225	53.9
4.05 π x	45.8	82.1	157.8	2.86	686

(2) In the calculation of a product with three factors (a × b × c) the intermediate answer a × b can only be read on Scale DF.

Example: 6.65 × 1.23 × 1.08.

Setting: D 6.65 | C 10.
C 1.23 | DF: 8.18.
Cursor on DF 8.18.
DF 8.18 | CF 1.
CF 1.08 | DF: 8.83.

We will once again explain the simple system of symbols used here:

1st line: C 10 is placed above D 6.65.
2nd line: The value 8.18 is found on DF, vertically above CF 1.23.
3rd line: The intermediate result 8.18 is held with the cursor line (but need not be read off).
4th line: CF 1 is placed vertically underneath DF 8.18.
5th line: The value 8.83 is found on DF, above CF 1.08.

The following Working Sheet No. 5 covers the material discussed in Sections 2.4, 2.5, 2.6 and 2.7. If you experience any difficulties with a problem, read through the corresponding section once again.

Working Sheet No. 5a

1. The specific gravity of iron is 7.86 g/cm^3.

What is the weight of	$3.8\ dm^3$	$0.107\ m^3$	$62.5\ cm^3$	$21.3\ mm^3$	of iron?
Answer:					

2. The density of oxygen under normal conditions is 1 429 g/dm^3.

What is the mass of	$27.1\ cm^3$	$8.15\ dm^3$	$18.6\ m^3$	$693\ mm^3$	of oxygen?
Answer:					

3. What is the circumference of a circle with

a diameter of	7.35 cm	4.96 mm	1.67 dm	0.945 m	?
Answer:					

4. What is the diameter of a cylinder with

a circumference of	3.09 m	80.5 mm	67.3 cm	9.87 cm	?
Answer:					

5. Calculate the expression $\frac{a}{\pi} \times b$ for the following values of "a" and "b":

a	8.4	17.9	37.3	106	0.202	43.5
b	1.08	7.25	0.505	0.845	123	0.85
$\frac{a}{\pi} \cdot \times b$						

6. 6.1 What distance is covered in 1 sec by a car travelling at a speed of 137 km/h? Answer: ....

6.2 A spacecraft travels at 38 600 km/h. What is its speed in metres per sec? Answer: ....

6.3 A satellite encircles the earth at a speed of 8.3 km/sec. What is its speed in km/h? Answer: ....

Working Sheet No. 5b

7. Calculate the product $\pi \times a \times b$ for the following values of "a" and "b":

a	4.15	77.5	0.148	0.148	1.48	14.8
b	6.64	0.392	2.56	68.5	4.95	0.545
$\pi \times a \times b$						

8. Calculate the product $a \times b \times c$ for the following values of "a", "b" and "c":

a	5.35	2.43	6.82	0.47	1.98	6.05
b	13.8	6.95	1.26	8.3	7.25	0.118
c	0.92	1.24	1.67	5.95	2.64	3.76
$a \times b \times c$						

9. Fill in the empty spaces in the following table:

x	1.3		73.5		4.84	
$0.765\ \pi x$		5.14		0.202		40.4

Answers: 1. 29.85 kp; 841 kp; 491 p; 0.1674 p — p (Pond), kp (Kilopond)
2. 38.75 mg; 11.65 g; 26.6 kg; 0.99 mg
3. 23.1 cm; 18.58 mm; 5.25 dm; 2.97 m
4. 0.984 m; 25.6 mm; 21.4 cm; 3.14 cm
5. 2.885; 41.3; 6.00; 28.5; 7.91; 11.77
6. 38.05 m; 10 720 $\frac{m}{s}$; 29 900 $\frac{km}{h}$
7. 86.6; 95.4; 1.190; 31.85; 23.0; 25.35
8. 67.9; 20.95; 14.35; 23.2; 37.9; 2.68

9.

x		2.14		0.84		16.8
$0.765\ \pi x$	3.12		176.6		11.6	

3. Division

3.1 Calculatory process

$\frac{a}{b}$

Division is the converse of multiplication:

1.8 × 3 = 5.4; conversely, 5.4 ÷ 3 = 1.8.

Consequently, the process of dividing "a" by "b" is the converse of that involved in multiplying them.

(1) The cursor line is placed above the dividend "a" on Scale D (cursor over D a).

(2) The divisor "b" on Scale C is placed underneath the cursor line (D a ¦ C b).

(3) The quotient, a ÷ b, is found on Scale D, on C 1 or C 10 (C 1 | D: $\frac{a}{b}$ or C 10 | D: $\frac{a}{b}$). Here again it is advisable to use the cursor line in order to obtain an accurate reading, and the "colour rule" regarding the use of the white or the green scales applies as before.

Examples:

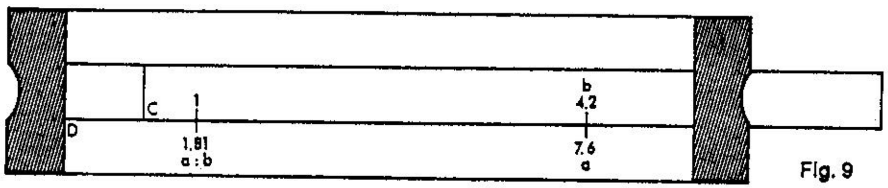


Fig. 9

7.6 ÷ 4.2 = 1.81

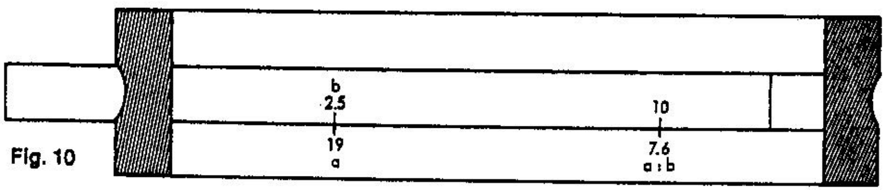


Fig. 10

19 ÷ 2.5 = 7.6

Work through the following examples for practice:

8.4 ÷ 3.36 = 2.50; 18.4 ÷ 6.45 = 2.855; 104 ÷ 5.4 = 19.26; 4.65 ÷ 2.17 = 2.14; 690 ÷ 3.24 = 213; 94.5 ÷ 77 = 1.227.

3.2 Determining the position of the decimal point in division

(1) In simpler cases the order of magnitude of the quotient can be estimated, after rounding off the dividend and the divisor.

In such cases it is often useful to remember that a number is divided by a fraction if it is multiplied by the reciprocal of the latter:

a ÷ 0.5 = 2 a; a ÷ 0.33 = 3 a; a ÷ 0.25 = 4 a; a ÷ 0.2 = 5 a;
a ÷ 0.167 = 6 a; a ÷ 0.1 = 10 a; a ÷ 0.05 = 20 a; etc.

(2) It is frequently useful, before the rough calculation, to shorten or extend the fraction by a suitable power of ten (i.e. to move the decimal point of divisor and dividend to the left or right by an equal number of places), so that the divisor becomes a number between 1 and 10.

Examples: 428 ÷ 0.059 = 42 800 ÷ 5.9 ≈ 7000
0.813 ÷ 240 = 0.00813 ÷ 2.4 ≈ 0.004
0.0308 ÷ 0.76 = 0.308 ÷ 7 6 ≈ 0.04

(3) If the dividend is smaller than the divisor, it is best to start by determining the position of the decimal point as in written division operations.

Example: 0.146 ÷ 26.4 = 0 · 0 0 533

0 by 26 gives (→ 0)
When the decimal point is passed, the decimal point is placed in the quotient (→ ·)
1 under this; 1 by 26 gives 0
4 under this; 14 by 26 gives 0
6 under this; 146 by 26 gives: the sequence of digits which now follows is calculated with the slide rule.

(4) In cases where it is difficult to obtain a clear view and in complicated calculations (compound multiplication and division operations) it is again useful to divide up each of the numbers concerned into a factor, between 1 and 10, and a power of ten.

Examples:

1. $\frac{0.0036}{418} = \frac{3.6 \times 10^{-3}}{4.18 \times 10^{2}} = 0.861 \times 10^{-5}$

2. $\frac{56\,200}{0.079} = \frac{5.62 \times 10^{4}}{7.9 \times 10^{-2}} = 0.711 \times 10^{6}$

3. $\frac{329}{60\,400} = \frac{3.29 \times 10^{2}}{6.04 \times 10^{4}} = 0.545 \times 10^{-2}$

In this process the powers of ten are divided by subtracting the exponent of the denominator (taking its sign into account!) from that of the numerator.

In working through the next Sheet (No. 6) you are again at liberty to choose the method by which you determine the order of magnitude of the quotient.

Working Sheet No. 6a

1. 630 ÷ 13 = ...	16. 1086 ÷ 874 = ...
2. 2.94 ÷ 6.49 = ...	17. 80.4 ÷ 0.003 47 = ...
3. 34.9 ÷ 2.59 = ...	18. 0.198 ÷ 0.267 = ...
4. 96.8 ÷ 431 = ...	19. 1 ÷ 56.3 = ...
5. 0.867 ÷ 4.31 = ...	20. 1 ÷ 257 = ...
6. 0.624 ÷ 0.034 = ...	21. π ÷ 5.34 = ...
7. 2430 ÷ 1.73 = ...	22. π ÷ 28.3 = ...
8. 9.86 ÷ 97.52 = ...	23. 100 ÷ 766 = ...
9. 8.51 ÷ 1.015 = ...	24. 31.6 ÷ 0.1795 = ...
10. 18.4 ÷ 72 400 = ...	25. 620 ÷ 89 = ...
11. 32 000 ÷ 2.34 = ...	26. 0.001 ÷ 7.05 ....
12. 1 ÷ 13.4 = ...	27. 0.0585 ÷ 0.0647 ....
13. 4.18 ÷ 6.81 = ...	28. 0.004 94 ÷ 23.2 = ...
14. 0.0296 ÷ 0.283 = ...	29. 76.2 ÷ 0.038 = ...
15. 0.0069 ÷ 0.000 782 = ...	30. 10 π ÷ 6.35 = ...

Answers: 1. 48.5; 2. 0.453; 3. 13.47; 4. 0.2245; 5. 0.201; 6. 18.35; 7. 1405; 8. 0.1011; 9. 8.38; 10. 2.54×10^{-4}; 11. 1.367×10^{4}; 12. 0.0746; 13. 0.614; 14. 0.1046; 15. 8.825; 16. 1.243; 17. 2.315×10^{4}; 18. 0.742; 19. 0.017 76; 20. 3.89×10^{-3}; 21. 0.588; 22. 0.111; 23. 0.1305; 24. 176.0; 25. 6.97; 26. 1.418×10^{-4}; 27. 0.904; 28. 2.13×10^{-4}; 29. 2005; 30. 4.95.

Working Sheet No. 6b

1. 14.65 ÷ 297.5 = ...	16. 0.005 98 ÷ 0.000 472 = ...
2. 156 ÷ 25 = ...	17. 0.582 ÷ 4780 = ...
3. 0.4729 ÷ 0.467 = ...	18. 3920 ÷ 0.0881 = ...
4. 38.4 ÷ 1.38 = ...	19. 10.05 ÷ 0.006 74 = ...
5. 0.0463 ÷ 0.002 79 = ...	20. 0.0714 ÷ 0.006 45 = ...
6. 64.8 ÷ 0.0035 = ...	21. π ÷ 0.000 784 = ...
7. 79.44 ÷ 0.067 = ...	22. 83 500 ÷ 2 420 000 = ...
8. 386.5 ÷ 14.28 = ...	23. 0.0603 ÷ 496 = ...
9. 0.0473 ÷ 26.42 = ...	24. 0.002 08 ÷ 0.000 718 = ...
10. 29.37 ÷ 0.0564 = ...	25. 100 π ÷ 86 400 = ...
11. 29.43 ÷ 37.64 = ...	26. 11 400 ÷ 0.053 = ...
12. 0.0796 ÷ 0.0452 = ...	27. 0.009 02 ÷ 0.0394 = ...
13. 68 400 ÷ 3.92 = ...	28. 1 ÷ (16.4×10^{-6}) = ...
14. 23.8 ÷ 14.7 = ...	29. 1 ÷ (5.05×10^{3}) = ...
15. 596 ÷ 0.1125 = ...	30. 10^{4} ÷ (7.8×10^{6}) = ...

Answers: 1. 0.0492; 2. 6.24; 3. 1.013; 4. 27.85; 5. 16.59; 6. 1.851×10^{4}; 7. 1186; 8. 27.05; 9. 1.79×10^{-3}; 10. 521; 11. 0.782; 12. 1.761; 13. 1.745×10^{4}; 14. 1.619; 15. 5.3×10^{3}; 16. 12.67; 17. 1.218×10^{-4}; 18. 4.45×10^{4}; 19. 1491; 20. 11.07; 21. 4.01×10^{3}; 22. 0.0345; 23. 1.216×10^{-4}; 24. 2.895; 25. 3.635×10^{-3}; 26. 2.15×10^{5}; 27. 0.229; 28. 6.1×10^{4}; 29. 1.98×10^{-4}; 30. 1.282×10^{-3}.

3.3 Use of pair of Scales CF/DF in division

(1) The result of a division commenced with scales C and D as described can also be read above CF 1 on scale DF, provided the mark CF 1 does not extend beyond the scale DF.

Example: $3.7 \div 6.8 = 0.544$. This result can be read both under C 10 and above CF 1.

(2) The entire division can also be carried out with the pair of scales CF/DF:
Cursor above dividend, a, on Scale DF.
Divisor, b, to be placed under the cursor, on Scale CF.
Quotient, $a \div b$, can be read above CF 1 on DF (or under C 1 or C 10 on D).

This method often enables lengthy displacements of the slide of the slide rule to be avoided and offers the additional advantage that dividend and divisor are to be found one above the other in the same way as in the case of a fraction (dividend above, divisor below); the sliding joint between the scales CF and DF then at the same time does duty for the stroke through the fraction.

This fact is particularly useful in the proportions (see 3.5).

The problems set on Working Sheets 6a and 6b can be used to practise this method.

3.4 A second division process: division with constant factor

$$y = \frac{x}{b}$$

If different numbers are to be divided by one and the same divisor, b, (e.g. in forming a table for the function $y = x/b$) it is preferable to use a different division method, which is also useful for combined calculations in conjunction with other scales (e.g. the sine scale). This method too is based on the reversal of the multiplication process and on the fact that

$$b \times \frac{a}{b} = a.$$

Setting: C 1 or C 10 above D b.

The quotient, $\frac{a}{b}$, is then shown on C, above D a.

Here again, needless to say, we can change over to the pair of scales CF/DF.

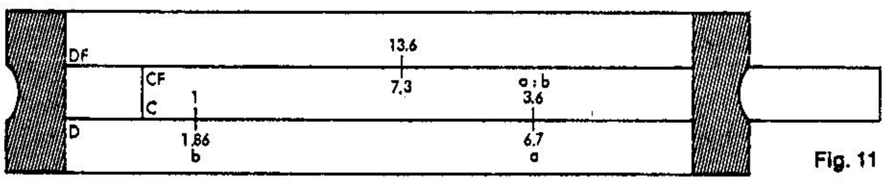


Fig. 11

$6.7 \div 1.86 = 3.6$; $13.6 \div 1.86 = 7.3$

This method likewise can be practised with the problems set on Working Sheets 6a and 6b.

3.5 Proportions – formation of tables

In the calculation of a quotient,

$$\text{e.g. } 9 \div 12 = \frac{9}{12} = 0.75$$

with the pair of scales CF/DF, it can be noticed that with the setting DF 9 | CF 12 the quotient of all numbers opposite each other on CF and DF have the same value, i.e. 0.75, e.g. $3 \div 4$, $4.5 \div 6$, $15 \div 20$ etc.

The same applies to numbers opposite each other on Scales C and D, except that they are now arranged in the converse manner; the dividend is below (on D) and the divisor above (on C).

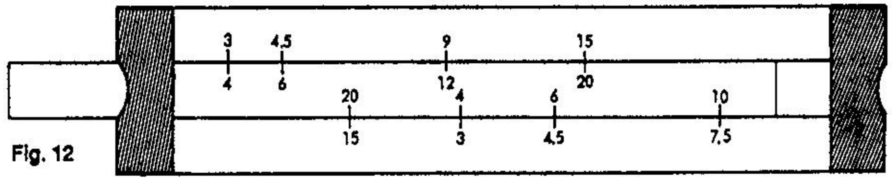


Fig. 12

$$\frac{9}{12} = \frac{3}{4} = \frac{4.5}{6} = \frac{15}{20} = \frac{7.5}{10}$$

If, therefore, $a_1, a_2, a_3, \ldots$ are some numbers on DF or D, and $b_1, b_2, b_3, \ldots$ are the numbers to be found below them, or above them, on CF or C respectively, then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \ldots$$

This fact provides a simple means of solving proportional equations (proportions), i.e. equations of the type

$$(1)\ \frac{x}{a} = \frac{b}{c} \text{ and } (2)\ \frac{a}{x} = \frac{b}{c}.$$

Setting: DF b | CF c

The numbers b and c will then be found one above the other, as on the right-hand side of the equation, and the sliding joint corresponds to the stroke through the fraction.

Equation (1): CF a | DF: x (Fig. 13).
Equation (2): DF a | CF: x (Fig. 14).

In both cases, x and a will be found opposite one another, as in the corresponding vulgar fraction.

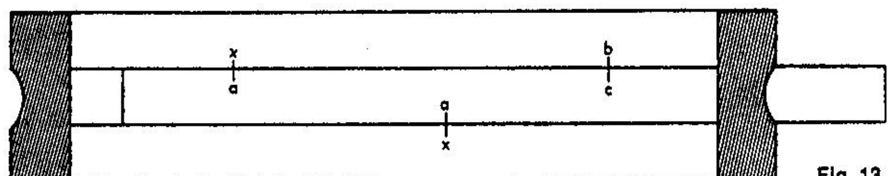


Fig. 13

$$\frac{x}{a} = \frac{b}{c}$$

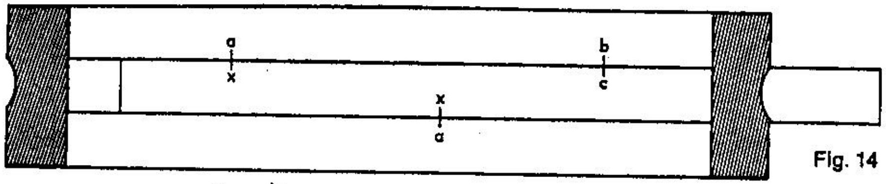


Fig. 14

$$\frac{a}{x} = \frac{b}{c}$$

The range within which readings can be taken will in some cases be too small.

Example: $\frac{x}{7.5} = \frac{4.2}{2.8}$

The position of the mark C 10 (or in the case of other value: C 1) is then marked with the cursor line, after which C 1 in place of C 10 (or vice versa) is placed underneath the latter. (Slide moved back, or pushed through to the end).
Result: x = 11.25.

A further possible method: Set the ratio 4.2 : 2.8 on the scales C/D from the outset (C 4.2 | D 2.8). In the transition to the pair of scales CF/DF, however, it must then be borne in mind that the top and bottom are interchanged. The "colour rule" – "(white) is to (green) as (green) is to (white)" – is nevertheless reliable in this case likewise.

Example: $\frac{24}{3.8} = \frac{x}{5.2} = \frac{6.95}{y} = \frac{z}{8.7}$

Either the setting DF 24 | CF 3.8
CF 5.2 | DF: x = 32.8
D 6.95 | C: y = 1.1
and after pushing the slide through
CF 8.7 | DF: z = 54.9
or
C 8.7 | D: z = 54.9

or the setting D 24 | C 3.8
C 5.2 | D: x = 32.8
DF 6.95 | CF: y = 1.1
C 8.7 | D: z = 54.9

By the aid of proportions, calculations with the rule of three with a direct ratio, and also calculations of percentages, can be carried out in a simple manner, the three types of percentage calculation (calculation of rate of percentage, of value represented by percentage, and basic number) being treated by the same scheme.

Examples:

(1) A car requires 39.4 litres of petrol for 386 Km. What is its consumption for 100 Km?

Problem set: $\frac{39.4\ l}{386\ Km} = \frac{x}{100\ Km}$

Setting of slide rule: DF 39.4 | CF 386
CF 100 | DF: 10.2

Any desired number of further pairs of values can be obtained with the same setting, e.g. the consumption for 260 Km (26·55 litres) or the distance that can be travelled with 5 litres (49 Km). It must simply be noted that in this case the upper scale (DF) represents litres and the lower scale (CF) kilometres.

(2) An article costs \$ 283 (including turnover tax of 11%).
What is the nett cost and what is the amount of the tax?

Problem set: $\frac{111\ \%}{\$\ 283} = \frac{100\ \%}{x} = \frac{11\ \%}{y}$

In the above, x is the price of the article without tax, while y is the tax.

Setting on slide rule: DF 111 | CF 283 — DF: Percentage scale.
CF: \$ scale.

Answer: x = \$ 254.90, y = \$ 28.10.

(3) A distance of 421 cm is measured with a maximum error (△ 1) of 0.3 cm.
What is the relative uncertainty, $\frac{\Delta l}{l}$, expressed as a percentage?

Problem set: $\frac{100\ \%}{421\ cm} = \frac{x}{0.3\ cm}$ — DF: Percentage scale.
CF: centimetre scale.

Result: x = 0.0713%.

For exercises in this connection, see Working Sheet No. 7.

Working Sheet No. 7

1. A spring, when loaded with 43.5 p, undergoes an elongation of 8.73 cm.
   Complete the following table:

Loading	26.7 p		7.92 p		136 p
Elongation		21.4 cm		4.7 mm	

2. Complete the following:

Basic number (100%)	64.7 m	24.8 kg		$ 7.35	
Percentage rate	86.4%		8%		0.3%
Amount represented by percentage		37.3 kg	19.4 l	$ 42.50	6.9 kg

3. Calculate the unknown quantities:

$$\frac{13.7}{96.5} = \frac{x}{42.8} = \frac{225}{y} = \frac{0.68}{z} = \frac{7.15}{u}$$

x = ... y = ... z = ... u = ...

$$\frac{44.5}{27.3} = \frac{a}{16.4} = \frac{8.92}{b} = \frac{1320}{c} = \frac{d}{0.074}$$

a = ... b = ... c = ... d = ...

Answers:

1.

	106.6 p		2.34 p	
5.36 cm		1.59 cm		27.3 cm

2.

		242.5 l		2300 kg
	150.4%			
55.9 m			578%	

3. x = 6.075 y = 1585 z = 4.79 u = 50.35

4. a = 26.75 b = 5.47 c = 810 d = 0.1206

4. The reciprocal scales CI, DI and CIF

4.1 General

The letter I (standing for "inverse") in the code letters for these scales indicates that they are reciprocal scales, while the letters C, D or CF indicate the main scales of which they are reciprocals. If, therefore, one of the three scales C, D or CF is set by means of the cursor line to a series of digits x, the series of digits representing their reciprocal, i.e. 1/x, will be found on the corresponding scale CI, DI or CIF, as the case may be – and vice versa.

The reciprocal of a number can thus be determined in 6 different ways:

	1.	2.	3.	4.	5.	6.
Setting on:	C	CI	D	DI	CF	CIF
Reading on:	CI	C	DI	D	CIF	CF

The red colouring of the reciprocal scales is intended to draw constant attention to the fact that the scales run in the reverse direction, which also applies to all other scales marked in red. The order of magnitude of the reciprocal value must once again be estimated – or found by the aid of powers of ten.

The pair of scales C/CI, moreover, is provided a second time, on the back of the slide of the slide rule (on the front, in the case of the Castell 2/82 N).

Examples and exercises. (Use different methods of setting for the different problems):

x	1.69	3.83	7.14	23.5	0.67	458	0.008	5800	0.094
1/x	0.592				1.492				

Answers: 0.261; 0.14; 0.0426; 0.00218; 125; 1.724×10^{-3}; 10.64.

4.2 Division by reciprocal scales

Instead of dividing by a number, we can multiply by its reciprocal:

$$a \div b = a \times \frac{1}{b}.$$

This method is of particular use when the same number, a, has to be divided by different numbers in succession, i.e. when a table is to be formed for a function $y = \frac{a}{x}$.

$$\boxed{y = \frac{a}{x}}$$

This method is also advantageous in the case of multiple divisions (see 5th section).

Example: 16.4 ÷ 2.8; 16.4 ÷ 2.22.

Setting: Cursor line above dividend 16.4 on Scale D.
Mark 1 (or 10) of Scale CI under cursor line. Quotient can be read on D or DF opposite 2.8 or 2.22 on Scale CI or CIF respectively.

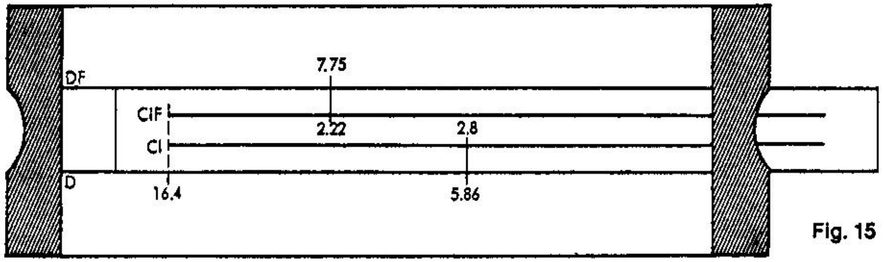


Fig. 15

16.4 ÷ 2.8 = 5.86; 16.4 ÷ 2.22 = 7.39

It is advisable, for purposes of practice, to solve at least some of the problems on Working Sheets 6a and 6b once again, using the above method.

4.3 Multiplication by reciprocal scales

Instead of multiplying by a certain number one can divide by its reciprocal,

e.g. $a \times b = a \div \frac{1}{b}$.

Example: 2.5 x 3.2

Setting: Cursor line above 2.5 on D.
3.2 on CI, under the cursor line.
The product, 8, will be found on D, opposite C 10.

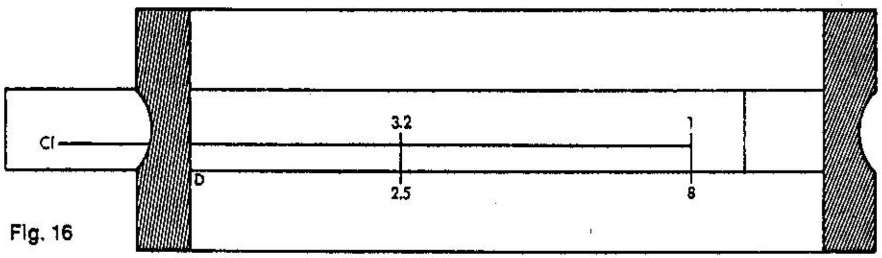


Fig. 16

2.5 x 3.2 = 8

This method is particularly useful with products having more than two factors (see Sect. 5).

For exercise, at least some of the problems on Working Sheets 3 and 4 should be solved by the above method.

4.4 Problems based on rule of three, with inverse (indirect) proportions

Example: A certain quantity of gas has a volume V_1 at pressure p_1.
What is the pressure of this gas at volume V_2, with unchanged temperature?

The Boyle-Mariott Law applies:

$$p_1 V_1 = p_2 V_2 \text{ or } \frac{V_1}{1/p_1} = \frac{V_2}{1/p_2}$$

Pressure and volume are thus in inverse proportion to one another.

Indirect proportions are quite easy to solve with the Reciprocal Scales.

Example: A quantity of gas has a volume of 26 litres at 3.4 at.
What is the pressure with a volume of 17.8 litres?

Formula: $\frac{26\,l}{1/3.4\text{ at}} = \frac{17.8\,l}{1/p}$ DF: Volume scale. CIF: Pressure scale.

Setting: DF 26 | CIF 3.4
DF 17.8 | CIF: 4.965

Answer: p = 4.965 at

Needless to say, any desired number of further pairs of values can be obtained.

Such calculations, of course, can also be carried out with Scales D and DI as well as with DI and C; furthermore, as may be required in the following example, we can change over from the pair of scales DF/CIF to the pair of scales D/CI, and vice versa, in the course of the calculation.

Example: With the use of three Trucks of the same type, a certain quantity of gas can be transported in 7.5 hrs.
How long would 2 or 5 Trucks require?

Problem set: 3 x 7.5 = 2 x x = 5 x y or

$$\frac{3\text{ Trucks}}{1/7.5\text{ h}} = \frac{2\text{ Trucks}}{1/x} = \frac{5\text{ Trucks}}{1/y}$$

Answer: x = 11.25 h; y = 4.5 h

Example: With resistances connected in parallel, the currents are in inverse proportion to the resistances:

$$\frac{R_1}{1/I_1} = \frac{R_2}{1/I_2}$$

Numerical example: A current of 0.86 Amps. flows through a resistance of 36 Ohms. What is the current in a parallel resistance of 23.4 Ohme?

Problem set: $\frac{36\text{ Ohm}}{1/0.86\text{ Amps.}} = \frac{23.4\text{ Ohm}}{1/x}$

Answer: x = 1.323 Amps. (read on Scale CI above D 23.4).

Further important applications for Scale DI will emerge from trigonometrical problems (see Sect. 9.5).

4.5 Calculation of $\frac{1}{\pi x}$ and $\frac{\pi}{x}$

$\frac{1}{\pi \cdot x}$ | C | CIF | CIF | C

1. If the cursor line is placed above a series of digits, x, on Scale C, the series for $\frac{1}{\pi x}$ will be found on Scale CIF, underneath the cursor line. (Attention is also drawn to this fact by the formula $\frac{10}{\pi x}$ at the right-hand end of the Scale CIF.)
   The same applies to the change-over in the reverse direction, i.e. from CIF to C, which is particularly important when further operations have to be carried out with the value $\frac{1}{\pi x}$.

Example: $\frac{1}{\pi 7.15} \times 194$ Setting: D 194 | C 1
CIF 7.15 | D: 8.64

$\frac{\pi}{x}$ | DF | DI

2. As we already know, the change-over from DF to D (or from CF to C) results in a division of the selected value, x, by π, i.e. $\frac{x}{\pi}$.
   The subsequent change-over to the Reciprocal Scale DI or CI then results in $\frac{\pi}{x}$.

Example: $\frac{\pi}{1.83}$ Setting: DF 1.83 | DI: 1.717
Multiplication by this value can continue by the aid of Scale CI.

Example: $\frac{\pi}{1.83} \times 3.43$
Setting: DF 1.83 | DI: 1.717
DI 1.717 | C 10 (= CI 1)
CI 3.43 | DI: 5.89

5. Multiple and compound multiplication and division operations

5.1 Products with more than two factors

a · b · c

The calculation of products with three or more factors by the simple repetition of the multiplication process requires the slide of the rule to be displaced after each multiplication, in order to bring the mark 1 or 10 into position above the intermediate product, thus obtaining the starting position for the next multiplication. If the reciprocal scale is used, however, only one movement of the slide is required.
Example: 1.35 × 5.3 × 6.85
The first multiplication (1.35 × 5.3) is replaced by the division by the reciprocal (1.35 ÷ 1/5.3), after which the multiplication by 6.85 can be performed immediately.
Setting: D 1.35 | CI 5.3
(C 10 | D: 7.15 no reading need be taken of the intermediate result)
C 6.85 | D: 49

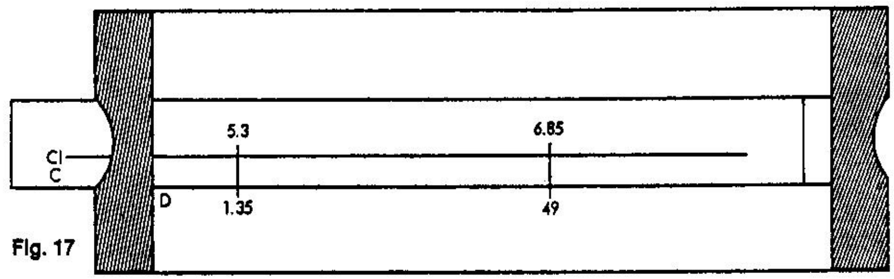


Fig. 17

Here again, of course, the change-over to Scales CF/DF/CIF can be effected.

5.2 Fractions of the form $\frac{a \times b}{c}$, $\frac{a \times b \times c}{d \times e}$ etc.

$\frac{a \cdot b}{c}$ | $\frac{a \cdot b \cdot c}{d \cdot e}$

The calculation of such expression is carried out in "zigzag fashion":

$$\frac{a \cdot b \cdot c}{d \cdot e},$$

We thus start with a division, followed by a multiplication, followed by a further division, and so on. This enables two calculations to be carried out with one single movement of the slide of the slide rule, since each division operation automatically moves it into the starting position for the subsequent multiplication.

Example: $\frac{7.35 \times 1.38 \times 6.3}{4.75 \times 8.1}$
Setting: D 7.35 | C 4.75
C 1.38 | D: 2.135 is held with the cursor line.
D 2.135 | C 8.1
C 6.3 | D: 1.661

Here again, recourse to the pair of scales CF/DF is of advantage.

Example: $\frac{3.82 \times 1.57}{7.6}$

Setting: D 3.82 | C 7.6
CF 1.57 | DF: 0.789

5.3 Fractions taking the form $\frac{a}{b \times c}$, $\frac{a \times b}{c \times d \times e}$, etc.

$$\frac{a}{b \cdot c} \qquad \frac{a \cdot b}{c \cdot d \cdot e}$$

In this case likewise, the zigzag process described in 5.2 is adopted but leaves a division operation at the end. This is replaced by the process of multiplying by the reciprocal.

Example: $\frac{24.5}{4.95 \times 2.23}$

Setting: D 2.45 | C 4.95
CI 2.23 | D: 2.22

5.4 Fractions of the form $\frac{1}{a \times b}$

$$\frac{1}{a \cdot b}$$

1st method: Multiply with the pair of scales C/D as usual, but read the result on DI instead of on D.

Example: $\frac{1}{6.9 \times 0.312}$ Setting: D 6.9 | C 10
C 3.12 | DI: 0.465

The operation of transposing the slide of the rule from the right to left index digit, as is sometimes necessary in this method, can be reliably avoided by the following method:

2nd method: Using the scales DI and CI, multiply the reciprocals of factors a and b:

$$\frac{1}{a \cdot b} = \frac{1}{a} \cdot \frac{1}{b}$$

Example: $\frac{1}{8.35 \times 1.64}$ Setting: DI 8.35 | C 1
CI 1.64 | D: 0.073

In this process the operation of transposing the slide is saved by the change-over to the pair of scales CIF/DF.

Example: $\frac{1}{3.88 \times 2.06}$ Setting: DI 3.88 | C 1
CIF 2.06 | DF: 0.1251

Practice these important calculations thoroughly by going through Working Sheet No. 8.

Working Sheet No. 8a

Simplify the following multiplication operations by using the reciprocal scales as far as possible:

1. $7.2 \times 3.2 \times 6.4$ = ...
2. $4.96 \times 3.28 \times 320\,000$ = ...
3. $0.028 \times 7.85 \times 1.19$ = ...
4. $0.246 \times 930 \times 1480 \times 5.8$ = ...
5. $0.0059 \times 0.0627 \times 8.92 \times 15$ = ...
6. $87 \times 4390 \times 3.45$ = ...
7. $0.718 \times 19.6 \times 25.6 \times 0.064$ = ...
8. $27\,000 \times 3.14 \times 5.75 \times 4.2$ = ...
9. $0.825 \times 13.6 \times 14.2 \times 48$ = ...
10. $0.000\,63 \times 54.5 \times 96 \times 318$ = ...
11. $2.9 \times 47.4 \times 10.73$ = ...
12. $0.803 \times 0.598 \times 29.06$ = ...
13. $0.35 \times 0.46 \times 0.85$ = ...
14. $9.05 \times 378 \times 0.004\,49 \times 13.4$ = ...
15. $875 \times 486 \times 3.14 \times 7.37 \times 0.003\,87$ = ...
16. $940 \times 380 \times 683 \times 0.0048 \times 0.573$ = ...
17. $0.3205 \times 680\,000 \times 5.41 \times 8.24 \times 0.003$ = ...
18. $312 \times 0.65 \times 1.8 \times 74.6 \times 0.1565$ = ...
19. $1.30 \times 2.70 \times 5.40 \times 11$ = ...
20. $8.09 \times 0.0732 \times 0.289 \times 0.006\,34 \times 0.4$ = ...

Answers: 1. 147.5; 2. 5.21×10^6; 3. 0.2615; 4. 1.964×10^6; 5. 495; 6. 1.318×10^6; 7. 23.05; 8. 2.045×10^6; 9. 7.65×10^3; 10. 1048; 11. 1475; 12. 13.95; 13. 0.1368; 14. 206; 15. 3.81×10^4; 16. 6.71×10^4; 17. 2.915×10^4; 18. 4.26×10^3; 19. 208.5; 20. 4.34×10^{-4}.

Working Sheet No. 8b

In the following problems it is useful to determine the order of magnitude of the result by separating powers of ten.

1. $\frac{1.26 \times 8.65}{0.0654} =$ ....

2. $\frac{4.82 \times 394}{16.7} =$ ....

3. $\frac{0.0047 \times 4.05 \times 14.6}{64.8 \times 18.4} =$ ....

4. $\frac{4.31 \times 6980 \times 98 \times 0.003\,71}{0.002\,71 \times 543 \times 62.4} =$ ....

5. $\frac{3840 \times 96.5 \times 5.05}{8060 \times 0.94} =$ ....

6. $\frac{0.004\,08 \times 0.89 \times 37.6}{319 \times 67.5 \times 9.04} =$ ....

7. $\frac{74.2}{18.7 \times 459} =$ ....

8. $\frac{0.000\,108}{0.0795 \times 0.34} =$ ....

9. $\frac{67\,400}{286 \times 843} =$ ....

10. $\frac{7.41 \times 56.3}{0.95 \times 438 \times 86} =$ ....

11. $\frac{102.5 \times 46.8}{54.3 \times 87.9 \times 370} =$ ....

12. $\frac{0.0648 \times 0.403 \times 0.088}{78.6 \times 10.02 \times 0.0302 \times 57} =$ ....

13. $\frac{1}{792 \times 45.9} =$ ....

14. $\frac{1}{0.76 \times 0.0204} =$ ....

15. $\frac{1}{24.7 \times 94.3 \times 0.604} =$ ....

Answers: 1. 166.7; 2. 113.7; 3. 2.33×10^{-4}; 4. 119.2; 5. 247; 6. 7.015×10^{-6}; 7. 8.64×10^{-3}; 8. 3.995×10^{-3}; 9. 0.2795; 10. 0.011 66; 11. 2.715×10^{-3}; 12. 1.695×10^{-6}; 13. 2.75×10^{-5}; 14. 64.5; 15. 7.11×10^{-4}.

6. The square scales A and B

6.1 Graduation of scales A and B

The scales A and B bear the graduation of scales C and D on a smaller scale of 1:2 and in two parallel rows.

The marking runs from 1 to 100, so that on the right-hand half a 0 must be mentally added in each case after the figures 2 to 9.

6.2 Multiplication and division

With the pair of scales A/B multiplication and division can be carried out in exactly the same way as with the pair of scales C/D or CF/DF, but as the length of the scale is halved the accuracy is reduced accordingly.

Product a x b: Factor "a" is set on the scale A by means of the cursor line. This can be done in the left-hand or in the right-hand half of the scale.

B 1, B 10 or B 100 is then placed underneath the cursor line, whichever is the most convenient. The product a x b is then read on A, vertically above B b (no matter whether on the left-hand or on the right-hand half of the scale).

Examples:
1. $4.2 \times 5 = 21$; Setting: A 4.2 | B 1; B 5 | A: 21
2. $76 \times 0.45 = 34.2$; Setting: A 76 | B 100; B 0 45 | A: 34 2
3. $8.6 \times 39 = 336$; Setting: A 8.6 | B 10; B 39 | A: 336

The above can be practised by using Working Sheet 4 once again.

Quotient a ÷ b: A a | B b
B 1 (or B 10 or B 100) | A: (a ÷ b).

Examples:
1. $\frac{76}{138} = 0.55$. Setting: A 76 | B 138 (both values near middle of scale)
B 10 | A: 0.55

2. $\frac{112}{69} = 1.62$. Setting: A 112 | B 69 (both values near middle of scale)
B 10 | A: 1.62

Working Sheets 6a and 6b can be used again in order to practise the above.

Multiplication and division on scales A/B is only of advantage, however, if it is desired to multiply or divide by the square of a given number (see below) and if the smaller degree of accuracy is accepted.

By means of the root scales on the 2/83 N (Section 10) these and the following calculations can be performed with greater accuracy.

6.3 Calculation of squares

a^2
D | A
C | B

The change-over from D to A or from C to B provides the square of the sequence of figures to which D, or C as the case may be, has been set:

D x | A: x^2 and C x | B: x^2

If "a" is a number between 1 and 10, the above method also indicates the correct order of magnitude for a^2.

Examples: $2.3^2 = 5.29$; $7.28^2 = 53$.

If "a" is not between 1 and 10, the order of magnitude of the square must be estimated or else found by means of powers of 10.

Examples: 1. $23\,600^2 = (2.36 \times 10^4)^2 = 2.36^2 \times 10^8 = 5.57 \times 10^8$

2. $0.008\,63^2 = (8.63 \times 10^{-3})^2 = 8.63^2 \times 10^{-6} = 74.5 \times 10^{-6}$

With the squares thus found, calculations can continue immediately on scales A and B.

6.3.1 Products and quotients with squares: $a^2 b$, $\frac{a^2}{b}$, $\frac{a}{b^2}$, $\frac{a \times b^2}{c}$, $\frac{a \times b}{c^2}$ etc.

In this case the products and quotients are calculated on the pair of scales A/B, and the squares required for this purpose are obtained simply by the change-over C | B or D | A.

Examples:

1. $2.36^2 \times 4.85 = 27.0$; Setting: D 2.36 | B 1
   B 4.85 | A: 27
2. $72 \times 5.05^2 = 1840$; Setting: A 72 | B 100
   C 5.05 | A: 1840
3. $\frac{16.7^2}{19.4} = 14.4$; Setting: D 16.7 | B 19.4
   B 100 | A: 14.4
4. $\frac{8.7}{9.35^2} = 0.0995$; Setting: A 8.7 | C 9.35
   B 100 | A: 0.0995
5. $\frac{34 \times 3.02^2}{79} = 3.93$; Setting: A 34 | B 79
   C 3.02 | A: 3.93 (Multiplication by 3.02^2)
6. $\frac{265 \times 68}{53.5^2} = 63$; Setting: A 265 | C 53.5
   (Division by 53.5^2)
   B 68 | A: 63

Examples for practice will be found on Working Sheet No. 9.

6.3.2 Table of function $a\,x^2$

$a\,x^2$

Example: Table $6.4\,x^2$

Setting: A 6.4 | B 10

C x | A: $6.4\,x^2$

x	2	3	5	1.41	21.2	8.45	93
$6.4\,x^2$	25.6	57.6	160				

Answers: 12.72 2875 457 5.535×10^4

6.3.3 Table of function $a^2 x$

$a^2 x$

Example: Table $0.81^2 x$

Setting: D 0.81 | C 10 (= B 100)

B x | A: $0.81^2 x$

x	2	3	7.5	11.2	0.35	0.86
$0.81^2 x$	1.31	1.968				

Answers: 4.92 7.35 0.2295 0.564

6.3.4 Table of function $\frac{a}{x^2}$

$\frac{a}{x^2}$

Example: Table $\frac{54}{x^2}$

Setting: A 54 | B 100

CI x | A: $\frac{54}{x^2}$

x	2	3	12.5	16.1	7.2	9.4
$\frac{54}{x^2}$	13.5	6				

Answers: 0.3455 0.2085 1.042 0.611

6.3.5 Table of function $a^2 x^2$

$a^2 x^2$

Setting: D a | C 1 (10)

C x | A: $a^2 x^2$

6.3.6 Table of function $\frac{x^2}{a^2}$

$\frac{x^2}{a^2}$

Setting: DI a | C 1 (10)

C x | A: $\frac{x^2}{a^2}$

6.3.7 Table of function $\frac{a^2}{x^2}$

$\frac{a^2}{x^2}$

Setting: D a | CI 1 (10)

CI x | A: $\frac{a^2}{x^2}$

Practise with Working Sheet No. 9.

Working Sheet No. 9

1.	14.2^2 = ...	6.	4180^2 = ...
2.	73.5^2 = ...	7.	0.078^2 = ...
3.	0.43^2 = ...	8.	$62\,400^2$ = ...
4.	359^2 = ...	9.	$0.000\,97^2$ = ...
5.	0.0027^2 = ...	10.	$694\,000^2$ = ...
11.	$(4.86 \times 2.03)^2$ = ...	16.	$(0.742 \div 3.14)^2$ = ...
12.	$(0.605 \times 0.038)^2$ = ...	17.	$(84.5 \div 0.67)^2$ = ...
13.	$(34.2 \times 105)^2$ = ...	18.	$(0.695 \div 0.048)^2$ = ...
14.	$(0.072 \times 0.0053)^2$ = ...	19.	$(319 \div 7600)^2$ = ...
15.	$(812 \times 94)^2$ = ...	20.	$(0.0014 \div 0.057)^2$ = ...
21.	27.5×3.60 = ...	26.	$\frac{42.6^2}{85}$ = ...
22.	$1.93^2 \times 86$ = ...		
23.	$5.12^2 \times 97$ = ...	27.	$\frac{0.29}{0.93^2}$ = ...
24.	3.24×4.8^2 = ...	28.	$\frac{32.4^2}{17.4}$ = ...
25.	86×34.2^2 = ...		
29.	$\frac{1080}{76^2}$ = ...	32.	$\frac{5.63^2 \times 7.6}{2.4^2}$ = ...
30.	$\frac{92.1 \times 6.73^2}{424}$ = ...	33.	$\frac{13.1^2 \times 0.45}{9.2^2}$ = ...
31.	$\frac{2.06 \times 73}{7.8^2}$ = ...	34.	$\frac{4.7^2 \times 3.18^2}{8.12^2}$ = ...

Answers: 1. 201.5; 2. 5.4×10^3; 3. 0.1849; 4. 1.289×10^5; 5. 7.29×10^{-6}; 6. 1.747×10^7; 7. 6.08×10^{-3}; 8. 3.89×10^9; 9. 9.41×10^{-7}; 10. 4.815×10^{11}; 11. 97.3; 12. 5.29×10^{-4}; 13. 1.29×10^7; 14. 1.456×10^{-7}; 15. 5.825×10^9; 16. 5.43; 17. 3205; 18. 1.113×10^{-3}; 19. 5.88×10^{12}; 20. 6.37×10^{-10}; 21. 2.72×10^3; 22. 320.5; 23. 2545; 24. 53.9; 25. 1.006×10^5; 26. 21.35; 27. 0.335; 28. 60.3; 29. 0.187; 30. 9.84; 31. 2.47; 32. 41.8; 33. 0.912; 34. 3.39.

Further examples for practice will be found on Working Sheet No. 16 (p. 82).

6.4 Extraction of square roots

$\sqrt{a}$

A | D

B | C

The change-over from Scale A or B to D or C respectively provides the square root of any number between 1 and 100. It should be borne in mind that numbers between 1 and 10 must be set on the left-hand half and those between 10 and 100 on the right-hand half of Scale A or B.

Positive numbers below 1 and numbers above 100 are shown as the product of a number 1 and 100 and a power of ten with an **even** exponent. (The decimal point should be moved by 2 places at a time, until a number between 1 and 100 is obtained).

Examples:

1. $\sqrt{14\,800} = \sqrt{1.48 \times 10^4} = \sqrt{1.48} \times 10^2 = 1.217 \times 10^2$
2. $\sqrt{148\,000} = \sqrt{14.8 \times 10^4} = \sqrt{14.8} \times 10^2 = 3.85 \times 10^2$
3. $\sqrt{0.00459} = \sqrt{45.9 \times 10^{-4}} = \sqrt{45.9} \times 10^{-2} = 6.77 \times 10^{-2}$
4. $\sqrt{0.0459} = \sqrt{4.59 \times 10^{-2}} = \sqrt{4.59} \times 10^{-1} = 2.14 \times 10^{-1}$

By means of the root scales of the 2/83 N, square roots can be calculated more accurately (see Sect. 10.6).

Now solve problems 1-14 on Working Sheet 10.

6.4.1 Table of function $a\sqrt{x}$

$a\sqrt{x}$

Example: $13.4 \times \sqrt{x}$ Setting: D 13.4 | C 1

B x | D: $13.4 \times \sqrt{x}$

x	2	3	12	6.65	17.4	110
$13.4\sqrt{x}$	18.95	23.2	46.4			

Answers: 34.55 55.9 140.5

6.4.2 $\frac{\sqrt{a}}{b}$

$\frac{\sqrt{a}}{b}$

Example: $\frac{\sqrt{19.2}}{4.6}$ Setting: A 19.2 | C 4.6

C 10 | D: 0.953

6.4.3 $\frac{a}{\sqrt{b}}$

$\frac{a}{\sqrt{b}}$

Example: $\frac{5.45}{\sqrt{47}}$ Setting: D 5.45 | B 47

C 10 | D: 0.795

6.4.4 $\frac{1}{\sqrt{a}}$

$\frac{1}{\sqrt{a}}$ | B | CI | A | DI

Example: $\frac{1}{\sqrt{13.4}}$ Setting: B 13.4 | CI: 0.273
or: A 13.4 | DI: 0.273

6.4.5 Table of function $\frac{a}{\sqrt{x}}$

$\frac{a}{\sqrt{x}}$

This problem is solved by **multiplication on the two reciprocal scales CI and DI,** with which, of course, multiplication (and division) can be carried out just as accurately as with Scales C and D.

Example: 1.26 x 2.88 Setting: DI 1.26 | CI 1
CI 2.88 | DI: 3.63

We will now turn our attention to the actual problem:

Example: Table $\frac{12.4}{\sqrt{x}}$

Setting: DI 12.4 | CI 1
B x | DI: $12.4/\sqrt{x}$

On the 2/83 N D 12.4 | C 1
BI x | C: $12.4/\sqrt{x}$

x	10	20	45	8.1	170
$\frac{12.4}{\sqrt{x}}$	3.92	2.77			

Answers: 1.848 4.36 0.951

Now work the examples 15-23 on Working Sheet 10.

Working Sheet No. 10

1. $\sqrt{7.4}$ = ...	8. $\sqrt{0.0039}$ = ...
2. $\sqrt{36.3}$ = ...	9. $\sqrt{6590}$ = ...
3. $\sqrt{204}$ = ...	10. $\sqrt{47\,500}$ = ...
4. $\sqrt{730}$ = ...	11. $\sqrt{0.000\,56}$ = ...
5. $\sqrt{0.87}$ = ...	12. $\sqrt{220\,000}$ = ...
6. $\sqrt{0.43}$ = ...	13. $\sqrt{0.000\,092}$ = ...
7. $\sqrt{0.0077}$ = ...	14. $\sqrt{9500}$ = ...
15. $\sqrt{83.5} \times 7.9$ = ...	21. $\frac{1}{\sqrt{5600}}$ = ...
16. $3.89 \times \sqrt{0.56}$ = ...	22. $96 \div \sqrt{115}$ = ...
17. $\sqrt{420} \div 13.4$ = ...	23. $7.8 \div \sqrt{0.83}$ = ...
18. $\sqrt{0.072} \div 0.37$ = ...	24. $\sqrt{216} \times \sqrt{0.083}$ = ...
19. $\frac{1}{\sqrt{305}}$ = ...	25. $\sqrt{57.5} \div \sqrt{9.6}$ = ...
20. $\frac{1}{\sqrt{0.042}}$ = ...	

Answers: 1. 2.72; 2. 6.025; 3. 14.28; 4. 27.0; 5. 0.933; 6. 0.656; 7. 0.087 75; 8. 0.0625; 9. 81.2; 10. 218; 11. 0.023 65; 12. 469; 13. 0.009 59; 14. 97.5; 15. 72.2; 16. 2.91; 17. 1.53; 18. 0.0726; 19. 0.057 25; 20. 4.88; 21. 0.013 36; 22. 8.95; 23. 8.56; 24. 4.235; 25. 2.45.

6.5 Square roots from products and quotients

$$\sqrt{a \cdot b} \qquad \sqrt{\frac{a}{b}} \qquad \sqrt{\frac{a \cdot b}{c}}$$

With expressions taking the form $\sqrt{a\,b}$, $\sqrt{\frac{a}{b}}$, $\sqrt{\frac{a\,b}{c}}$ and the like the radicand is first of all calculated with the pair of scales A/B, the root then being extracted by changing over to Scale D.

Example: $\sqrt{3.4 \times 7.8} = 5.15$. Setting: A 3.4 | B 1
B 7.8 | D: 5.15

In the calculation of the product, owing to the subsequent extraction of the square root, the two factors must in each case be found in the correct "decade" (half of scale), i.e. factors between 1 and 10 on the left and those between 10 and 100 on the right. Furthermore, the operation of setting the slide to the first factor must only be carried out with the mark B 1 or B 100.

Examples: $\sqrt{17.2 \times 71} = 34.94$; Setting: A 17.2 | B 100
B 71 | D: 34.95

but: $\sqrt{1.72 \times 71} = 11.05$; Setting: A 1.72 | B 1
B 71 | D: 11.05

If necessary, powers of ten must be separated from the factors beforehand:

$$\sqrt{326 \times 4800} = \sqrt{3.26 \times 10^2 \times 48 \times 10^2} = \sqrt{3.26 \times 48 \times 10^4} = 12.51 \times 10^2 = 1251$$

Setting: A 3.26 | B 100
B 48 | D: 12.51

Similar considerations apply when the radicand is a quotient:

$\sqrt{\frac{53}{8.7}} = 2.47$; Setting: A 53 | B 87
B 1 | D: 2.47

but:

$\sqrt{\frac{53}{87}} = 0.781$; Setting: A 53 | B 87
B 100 | D: 0.781

$$\sqrt{\frac{21\,500}{8\,300}} = \sqrt{\frac{2.15 \times 10^4}{83 \times 10^2}} = \sqrt{\frac{2.15}{83}} \times 10 = 0.161 \times 10 = 1.61$$

Setting: A 2.15 | B 83
B 100 | D: 0.161

$$\sqrt{\frac{345 \times 69}{5.6}} = \sqrt{\frac{3.45 \times 10^2 \times 69}{5.6}} = \sqrt{\frac{3.45 \times 69}{5.6}} \times 10 = 6.52 \times 10 = 65.2$$

Setting: A 3.45 | B 5.6
B 69 | D: 6.52

These calculations likewise can be carried out with greater accuracy on the root scales of the 2/83 N.

Now solve the problems set in Working Sheet No. 11.

Working Sheet No. 11

1. $\sqrt{18 \times 1.28} =$ ....
2. $\sqrt{1630 \times 0.290} =$ ....
3. $\sqrt{479 \times 0.00894} =$ ....
4. $\sqrt{76.8 \times 4500 \times 0.147} =$ ....
5. $\sqrt{8.4 \times 689 \times 0.062} =$ ....
6. $\sqrt{\frac{0.987}{0.012}} =$ ....
7. $\sqrt{\frac{6560}{0.159}} =$ ....
8. $\sqrt{\frac{86\,000}{45.7}} =$ ....
9. $\sqrt{\frac{492}{16.3}} =$ ....
10. $\sqrt{\frac{63.4}{102}} =$ ....
11. $\sqrt{\frac{586 \times 1.39}{10.2}} =$ ....
12. $\sqrt{\frac{1.35 \times 31.6}{0.062}} =$ ....
13. $\sqrt{\frac{4.98}{563 \times 0.0714}} =$ ....
14. $\sqrt{\frac{3.26 \times 235}{422 \times 0.953}} =$ ....
15. $\sqrt{\frac{5.47 \times 644}{0.0025 \times 12}} =$ ....
16. $\sqrt{\frac{873 \times 3.42 \times 4.2}{7.31 \times 0.455}} =$ ....
17. $\sqrt{\frac{1054 \times 78.5 \times 643}{304 \times 0.767}} =$ ....
18. $\sqrt{\frac{0.0365 \times 47.3}{0.146}} =$ ....
19. $\sqrt{\frac{6050}{20.5 \times 123}} =$ ....
20. $\sqrt{\frac{74}{0.98 \times 0.056}} =$ ....

Further exercises will be found on Working Sheet 16 (p. 82).

Answers: 1. 4.80 2. 21.75; 3. 2.07; 4. 225; 5. 18.95; 6. 9.07; 7. 203; 8. 43.4; 9. 5.49; 10. 0.788; 11. 8.94; 12. 26.23; 13. 0.352; 14. 1.38; 15. 343; 16. 61.4; 17. 478; 18. 3.44; 19. 1.549 (calculate the reciprocal on pair of scales A/B and read result on DI); 20. 36.7.

6.6 Area of circle and volume of cylinder

6.6.1 Diameter and area of a circle

For the area q (cross section) of a circle of diameter d:

$$q = \frac{\pi}{4} d^2$$

The diameter must thus be squared and then multiplied by $\frac{\pi}{4}$. The two calculations are carried out simultaneously if the cursor line is placed above the value d on Scale C or D and the result is read under the stroke "q" of the cursor, on Scale A or B respectively.

Reasons: The change-over from C to B or from D to A gives d^2, and the subsequent change-over from the central stroke to the stroke "displaced" by the distance $\frac{\pi}{4}$ i.e. the stroke "q", provides the multiplication by $\frac{\pi}{4}$.

Examples: 1. d = 2.92 cm; q = 6.7 cm²
2. d = 0.78 m; q = 0.478 m²

The calculation of the diameter of a circle from a given cross section is carried out in the converse manner, care merely having to be taken to set the correct half of the scale A or B to q (see Sect. 6.4).

Examples: 1. q = 4.45 mm²; d = 2.38 mm
2. q = 17 cm²; d = 4.65 cm

Exercises:

d	4.65 m		80.5 cm			
q		0.00194 m²		0.0295 km²	720 cm²	0.68 m²

Results:

	0.0497 m		0.194 km	30,3 cm	0.931 m
16.98 m²		5090 cm²			

6.6.2 Calculation of circular cylinders

The volume of a circular cylinder of diameter "d" and length "l" is

$$V = \frac{\pi}{4} d^2 l$$

1. Find V for a given d and l.

   Cross section calculated with the stroke q; multiplication by l carried out with the pair of scales A/B.

   Example: d = 0.237 m. l = 1.46 m.
   Setting: A 1.46 | B 1 (preparation for multiplication by 1.46).
   Cursor on C 0.237.
   Reading under "q" on A: 0.0645, V = 0.0645 m³.

2. Find d for a given l and V.
   In this case V is first of all divided by l, by means of scales A/B.
   Example: V = 0.0204 m³, l = 0.89 m.
   Setting: A 20.4 ($\times 10^{-2}$) | B 189. (In this case the position of the decimal point is immaterial, so the most convenient setting, i.e. 18.9, should be adopted).
   Mark "q" on B 10.
   Reading under cursor mark on C: 0.357, d = 0.357 m.

3. Find l for a given V and d.
   Example: V = 8.65 dm³, d = 1.24 dm.
   Setting: Cursor on D 1.24. (The reading 1.208 will then be provided under "q", on A, so that q = 1.208 dm²).
   Place B 1 under "q".
   A 8.65 | B: 7.16, i.e. l = 7.16 dm.

Exercises:

d	7.8 cm	2 mm			2.7 dm	4.6 cm
l	93.5 cm	1300 m	5.40 m	1.85 m		
V			12.6 dm³	0.037 m³	64 dm³	0.89 dm³

Results:

		5.45 cm	0.1595 m		
				1.12 m	53.7 cm
4.47 dm³	4.085 dm³				

6.7 Quadratic proportions

Example: With cylinders of equal length and of the same material, the weights G are in proportion to the squares of the radii r (or of their diameters).

$$\frac{G_1}{r_1^2} = \frac{G_2}{r_2^2}$$

Setting: A G_1 | C r_1 or A G_1 | CF r_1

The G values will then be found opposite their respective r values on Scales A and C (or CF) respectively (A: weight scale, C or CF: radius scale).

Numerical example: A cylinder with a radius of 6.3 cm weighs 86 kp.
Setting: A 86 | C 6.3

Weight in kp	86	145	50.5	220		
Radius in cm	6.3	8.81			2.72	18.5

Results:

		16.02	7.41
4.83	10.8		

Exercise: The electrical conductivity of wires of the same length and of the same material is proportional to the square of their diameter. Complete the following table:

Conductivity in mS*	320	500	800	90			
Diameter in mm	0.8				0.6	0.12	1.2

Results:

			180	7.2	720
1.0	1.265	0.424			

6.6 Indirect quadratic proportions

Example: The illumination E, for a given light incidence, is inversely proportional to the square of the distance r from the light source.

$$\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2} \quad \text{or} \quad \frac{E_1}{1/r_1^2} = \frac{E_2}{1/r_2^2}$$

Setting: A E_1 | CI r_1 or A E_1 | CIF r_1
Values of E will then be found opposite their respective r values on Scales A and CI (or CIF) respectively. (A: Illumination scale, CI or CIF: distance scale).

In the Castell 2/82 N the setting DI E_1 | D r_1 is likewise possible.
The slide rule then need not be turned over, but the operation of transposing the slide will sometimes be necessary.

Numerical example: At a distance of 1.3 m from a source of light the illumination is 320 Lux.
Setting: A 320 | CI 1.3 or BI 320 | D 1.3

Luminous intensity in Lux:	120	790	590	960		
Distance in m:	2.12	0.827			1	3.2

Results:

		540	52.7
0.957	0.750		

* mS = millisiemens.

7. The Cube Scale K

7.1 The graduation of the cube scale

This covers the three powers of ten, from 1 to 1000, so that each figure from 2 to 9 must be thought of as being followed by one 0 in the range for the second power of ten and by two 0's in that for the third power.

7.2 Finding the cube of a number

a^3 | D | K

The change-over from D to K provides the third power of the sequence of figures to which D has been set:

D x | K: x^3

If "a" is a number between 1 and 10, this method also indicates the correct order of magnitude of a^3.

Examples: $1.6^3 = 4.1$; $3.45^3 = 41.1$; $7.44^3 = 412$.

If "a" is not between 1 and 10, the order of magnitude of the third power must again be estimated, or found by means of powers of 10.

Examples: 1. $68^3 = (6.8 \times 10)^3 = 6.8^3 \times 10^3 = 314 \times 10^3$
2. $0.036^3 = (3.6 \times 10^{-2})^3 = 3.6^3 \times 10^{-6} = 46.7 \times 10^{-6}$

Exercises: Working Sheet 12a, Ex. 1-7.

7.3 Extraction of cube roots

$\sqrt[3]{a}$ | K | D

The change-over from K to D provides the third root for any number between 1 and 1000:

K x | D: $\sqrt[3]{x}$

In this process, the first third of the Scale K is to be set to radicands between 1 and 10, the second third to radicands between 10 and 100 and the last third to radicands between 100 and 1000.

Radicands not between 1 and 1000 must first of all be subdivided into a factor between 1 and 1000 and a power of ten of which the exponent is a number divisible by 3.

Examples: 1. $\sqrt[3]{43\,000} = \sqrt[3]{43 \times 10^3} = \sqrt[3]{43} \times 10 = 3.5 \times 10 = 35$
2. $\sqrt[3]{580\,000} = \sqrt[3]{580 \times 10^3} = \sqrt[3]{580} \times 10 = 8.34 \times 10 = 83.4$
3. $\sqrt[3]{0.000\,0032} = \sqrt[3]{3.2 \times 10^{-6}} = \sqrt[3]{3.2} \times 10^{-2} = 1.473 \times 10^{-2} = 0.014\,74$

Exercises: Working Sheet 12a, Ex. 8-20.

7.4 Further calculations with the cube scale

$\frac{1}{a^3}$ | DI | K

7.4.1 $\frac{1}{a^3}$

Example: $\frac{1}{4.15^3}$ Setting: DI 4.15 | K: 0.014.

7.4.2 $\frac{1}{\sqrt[3]{a}}$

Example: $\frac{1}{\sqrt[3]{182}}$ Setting: K 182 | DI: 0.1746

$\frac{1}{\sqrt[3]{a}}$
K | DI

7.4.3 $a^{2/3} = \sqrt[3]{a^2} = (\sqrt[3]{a})^2$

Example: $56^{2/3}$ Setting: K 56 | A: 14.63

$a^{2/3}$
K | A

7.4.4 $a^{3/2} = \sqrt{a^3} = (\sqrt{a})^3$

Example: $46.5^{3/2}$ Setting: A 46.5 | K: 317

$a^{3/2}$
A | K

Exercises: Working Sheet 12a, Ex. 21.

7.4.5 Cubic proportions

Example: A cube with an edge length of 3.2 cm weighs 127 p.
What is the weight of cubes of the same material and with edge lengths of 1.65 cm, 4.8 cm and 0.41 cm respectively?

Problem:

$$\frac{127\ p}{(3.2\ cm)^3} = \frac{x}{(1.65\ cm)^3} = \frac{y}{(4.8\ cm)^3} = \frac{z}{(0.41\ cm)^3}$$

Setting: K 127 | C 3.2
C 1.65 | K: 17.4 $x = 17.4$ p.

Only one movement of the cursor in each case is thus required in order to find:
$y = 429$ p; $z = 0.267$ p.

Exercise: What is the edge length of a cube of 7.6 p, 84 p, 1600 p?

Results: 1.25 cm; 2.785 cm; 8.05 cm.

7.4.6 Expressions of the form $a\,b^3$ and $\frac{a}{b^3}$

$a \cdot b^3$

$\frac{a}{b^3}$

Examples:

1. $27.8 \times 3.06^3 = 797$; Setting: K 27.8 | C 1
   C 3.06 | K: 797
2. $590 \times 7.4^3 = 2.39 \times 10^5$; Setting: K 590 | C 10
   C 7.4 | K: 2.39×10^5
3. $168 \div 2.74^3 = 8.17$; Setting: K 168 | C 2.74
   C 1 | K: 8.17
4. $630 \div 0.87^3 = 957$; Setting: K 630 | C 0.87
   C 10 | K: 957

Exercises:

1. $5.3 \times 3.8^3 =$ ....
2. $420 \times 0.46^3 =$ ....
3. $0.073 \times 13.6^3 =$ ....
4. $4650 \div 5.75^3 =$ ....
5. $9.4 \div 0.082^3 =$ ....
6. $730 \div 34^3 =$ ....

Results: 1. 291; 2. 40.9; 3. 183.5; 4. 24.5; 5. 1.705×10^4; 6. 0.0186.

8. The Pythagorean Scale P

8.1 General

$\sqrt{1-a^2}$

If the cursor line is placed above a number x (which must now be between 1 and 10, with the correct "place-value") on Scale D, Scale P underneath shows the value $y = \sqrt{1-(0.1x)^2}$. Scale P is graduated in reverse and therefore coloured red.

Example: $\sqrt{1-0.364^2}$ Setting: D 3.64 | P: 0.9315.

From $y = \sqrt{1-(0.1x)^2}$, $x = 10\sqrt{1-y^2}$ follows.

The calculation can therefore also be performed in the reverse order.

Example: $\sqrt{1-0.59^2}$ Setting: Either D 5.9 | P: 0.808
or P 0.59 | D: 8.08

For the maximum accuracy in the calculation of $\sqrt{1-a^2}$ it is advisable,
where "a" is less than 0.7, to select the first method (setting on D, reading on P) and,
where "a" is at least equal to 0.7, to adopt the second method (setting on P, reading on D),
as both the setting and the reading are then more accurate.

Examples:

1. $\sqrt{1-0.1525^2} = 0.9883$ D | P
2. $\sqrt{1-0.9822^2} = 0.188$ P | D

Compare the accuracy obtained with the converse setting!

Exercises: Working Sheet 12b, Ex. 22.

8.2 Further applications of the Pythagorean Scale

8.2.1 Conversion of sin α to cos α and vice versa

In view of $\sin\alpha = \sqrt{1-\cos^2\alpha}$ and $\cos\alpha = \sqrt{1-\sin^2\alpha}$, the change-over from D to P or vice versa enables the cosine to be calculated from the sine of an angle and vice versa. This is why the Pythagorean Scale is particularly important in trigonometrical calculations (see Section 11).

8.2.2 Triangular calculations with the Theorem of Pythagoras

$\sqrt{c^2-a^2}$

Example: Of a right-angled triangle, the hypotenuse c is known to be 24.5 cm and one side, a, to be 16.8 cm.
To find the other side, b:

We have $b = \sqrt{c^2-a^2} = c\sqrt{1-(a/c)^2} = 24.5\ cm\ \sqrt{1-(16.8/24.5)^2}$

Setting: D 16.8 | C 24.5
C 10 | D: 0.686 | P: 0.7285
D 0.7285 | C 10
C 24.5 | D: 17.85

Exercises: Working Sheet 12b, Ex. 23.

8.2.3 The function $1 - x^2$

This is obtained by changing over from P to A, as $1 - x^2$ is $(\sqrt{1-x^2})^2$.

Example: $1 - 0.71^2$. Setting: P 0.71 | A: 0.496.

$1 - x^2$

8.2.4 The function $(1 - x^2)^{3/2}$

This is calculated by changing over from P to K.

Example: $(1 - 0.86^2)^{3/2}$. Setting: P 0.86 | K: 0.133

$(1 - x^2)^{3/2}$

8.2.5 The function $\frac{1}{\sqrt{1-x^2}}$

This is obtained by changing over from P to DI.

Example: $\frac{1}{\sqrt{1-0.54^2}}$ Setting: P 0.54 | DI: 1.188.

$\frac{1}{\sqrt{1-x^2}}$

Exercises: Complete the following table:

x	0.685	0.974	0.818	0.42	0.9921	0.2
$1 - x^2$						
$(1 - x^2)^{3/2}$						
$\frac{1}{\sqrt{1-x^2}}$						

Results:

x	0.685	0.974	0.818	0.42	0.9921	0.2
$1 - x^2$	0.531	0.0513	0.331	0.824	0.015 75	0.96
$(1 - x^2)^{3/2}$	0.387	0.0116	0.190	0.747	0.001 98	0.941
$\frac{1}{\sqrt{1-x^2}}$	1.372	4.415	1.738	1.101	7,97	1.02

Working Sheet No. 12a

1. 0.26^3 = ...	11. $\sqrt[3]{910}$ = ...
2. 7.53^3 = ...	12. $\sqrt[3]{0.346}$ = ...
3. 10.7^3 = ...	13. $\sqrt[3]{0.097}$ = ...
4. 0.0864^3 = ...	14. $\sqrt[3]{0.001\,31}$ = ...
5. 31.6^3 = ...	15. $\sqrt[3]{4450}$ = ...
6. 149^3 = ...	16. $\sqrt[3]{73\,000}$ = ...
7. $0.004\,04^3$ = ...	17. $\sqrt[3]{560\,000}$ = ...
8. $\sqrt[3]{49.5}$ = ...	18. $\sqrt[3]{1\,720\,000}$ = ...
9. $\sqrt[3]{186}$ = ...	19. $\sqrt[3]{0.000\,59}$ = ...
10. $\sqrt[3]{2.75}$ = ...	20. $\sqrt[3]{0.000\,087}$ = ...

21. Complete the following table:

a	64	0.57	112	0.086	604	0.0026
$\frac{1}{a^3}$						
$\frac{1}{\sqrt[3]{a}}$						
$a^{2/3}$						
$a^{3/2}$						

Working Sheet No. 12b

22. Complete the following:

x	0.366	0.218	0.605	0.9662	0.1345	0.843
$\sqrt{1-x^2}$						

23. Let "a" and "b" be the sides of a right-angled triangle and "c" its hypotenuse. Complete the following table:

a	3.06		67	420
b		112		
c	5.25	189	98	570

Answers:

21.

$\frac{1}{a^3}$	0.382×10^{-5}	5.40	0.712×10^{-6}	15.72	0.452×10^{-8}	0.569×10^{8}
$\frac{1}{\sqrt[3]{a}}$	0.25	1.206	0.2075	2.265	0.1183	7.275
$a^{2/3}$	16.0	0.6875	23.25	0.195	71.4	0.0189
$a^{3/2}$	512	0.43	1185	0.252	1.485×10^{4}	1.33×10^{-4}

Answers:

1. 0.0176; 2. 427; 3. 1225; 4. 0.645×10^{-3}; 5. 3.16×10^{4}; 6. 3.31×10^{6};
7. 6.59×10^{-8}; 8. 3.67; 9. 5.71; 10. 1.401; 11. 9.69; 12. 0.702; 13. 0.459;
14. 0.1094; 15. 16.45; 16. 41.8; 17. 82.4; 18. 119.8; 19. 0.0839; 20. 0.0443;
22. 0.9308; 0.976; 0.796; 0.258; 0.9909; 0.5385;
23.

	152.3		
4.265		71.5	385

8.2.6 The functions $a\sqrt{1-x^2}$ and $\frac{a}{\sqrt{1-x^2}}$

These frequently occur, for instance, in the formulae of the Theory of Relativity and can be tabulated as follows:

Examples: 1st table: $1.49\sqrt{1-x^2}$

Setting: D 1 | C 1.49 (the change-over from D to C then provides the multiplication by 1.49).

P x | C: $1.49\sqrt{1-x^2}$

x	0.85	0.97	0.98	0.994	0.7	0.6	0.5
$1.49\sqrt{1-x^2}$	0.785	0.362					

Results: 0.2965; 0.163; 1.064; 1.192; 1.291.

2nd table: $\frac{2.28}{\sqrt{1-x^2}}$

Setting: DI 1 | CI 2.28 (the change-over from DI to CI then provides a multiplication by 2.28)

P x | CI: $2.28/\sqrt{1-x^2}$

x	0.7	0.975	0.2	0.3	0.91	0.98	0.99	0.995
$\frac{2.28}{\sqrt{1-x^2}}$	3.19	10.27						

Results: 2.335; 2.39; 5.5; 11.45; 16.17; 22.8.

9. The trigonometric scales and marks

9.1 General

The trigonometric scales S, ST, T_1 and T_2 are marked in degrees (old degrees, i.e. 90° to the right angle); the degrees are subdivided decimally. The conversion of angular minutes to tenths of a degree is carried out on the basis of the simple equation: $1^0 = 60'$ (D 6 | C 10).

Examples: $42.5' = 0.708^0$; $8.4' = 0.14^0$; $0.225^0 = 13.5'$.

9.2 The Scale S (sine-cosine scale)

The scale S is marked in black from 5° to 90° for the sine and red for the cosine (in reverse, from 85° to 0°).
The change-over from the Scale S **black** to scale D provides the sine and the change-over from the Scale S **red** to scale D the cosine of the angle selected, a 0 having to be mentally added in front of the figure on Scale D in each case. Conversely, the arc sine and the arc cosine of the value are obtained respectively.

Examples:

α	24°	29.2°	34.6°	52.5°	79°
sin α	0.407				
cos α	0.9135				

x	0.86	0.695	0.151	0.203	0.535
arcsin x	59.3°				
arccos x	30.7°				

Results:

sin α	0.488	0.568	0.793	0.982
cos α	0.873	0.823	0.609	0.191

arcsin x	44°	8.69°	11.7°	32.3°
arccos x	46°	81.31°	78.3°	57.7°

As $\cos\alpha = \sqrt{1-\sin^2\alpha}$ and $\sin\alpha = \sqrt{1-\cos^2\alpha}$, the sine and the cosine of an angle can be obtained by the aid of the Pythagorean scale P and with one single cursor setting:

If S (**black**) is set to an angle α, the value sin α is shown vertically above it, on Scale D, while the cos α appears vertically below it, on Scale P.

If, on the other hand, α is set on S (**red**) the value cos α will be found vertically above it, on D, and the value sin α below it, on P.
The following "colour rule" applies:

The change-over between two scales of the same colour (black to black, or red to red) gives the sine or the arc-sine, and the change-over between two scales of different colour (black to red, or red to black) the cosine or arc-cosine.

Each functional value sin α, cos α, arcsin x and arccos x can thus be determined in two different ways. The following should be noted:

(1) **With angles under 45° the black marking of the Scale S is the more accurate, while with angles over 45° the red marking is the more accurate.**

(2) **With values below 0.7 the greater accuracy is obtained with Scale D, and with values over 0.7 the greater accuracy is obtained with Scale P.**

Examples: Most satisfactory method:

		Setting:	Reading:
To find:	sin 27.2°	S black	D 0.457
	sin 64.5°	S red	P 0.9026
	cos 72.3°	S red	D 0.304
	cos 12.6°	S black	P 0.9759
			S black 22.7°
			S red 74.7°
			S red 78.64°
			S black 10.63°

The calculation of sin α from cos α and vice versa is effected by the direct change-over from Scale D to Scale P and vice versa (see Section 8.2.1).

9.3 Scales T_1 and T_2 (tangent and cotangent scales)

These scales likewise are doubly marked:

Scale T_1 in black from 5° to 49° and in red from 85° to 41°, and scale T_2 in black from 41° to 85° and in red from 49° to 5°. Together with Scale D they form a tangent table (black text) and a cotangent (red). In using Scale T_1 a "0" must be mentally noted before the figures on Scale D in each case.

Examples: $\tan 17.6^0 = 0.317$; $\tan 82.2^0 = 7.30$; $\cot 62.4^0 = 0.523$; $\cot 35^0 = 1.428$
$\arctan 0.392 = 21.4^0$; $\arctan 2.92 = 71.1^0$
$\text{arccot}\ 0.69 = 55.4^0$; $\text{arccot}\ 4.1 = 13.7^0$

As $\cot\alpha = 1/\tan\alpha$, the tangent and the cotangent can be determined by the aid of Scale DI and with one single cursor setting.

Examples: 1. $\tan 19.5^0 = 0.354$, $\cot 19.5^0 = 2.824$
Setting: T_1 19.5° | D: 0.354 | DI: 2.824
2. $\tan 83.5^0 = 8.78$, $\cot 83.5^0 = 0.1139$
Setting: T_2 83.5° | D: 8.78 | DI: 0.1139

9.4 Measuring and drawing angles with the use of the tangent scales.

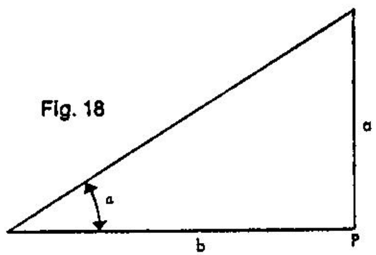
In conjunction with a set square (or compasses) and a ruler the tangent scales provide a high-precision goniometric system.

9.4.1 Measuring a given angle.

At a point P on one side, as far as possible from the apex, let it intersect the other side, and measure the two distances "a" and "b" (see Fig. 18).

Using the ruler, calculate the quotient a/b = tan α and take a reading of the associated angle, above the result, on T_1 (for a tan α value between 0.087 and 1) or T_2 (for a tan α value between 1 and 11.4).

Fig. 18



9.4.2 Drawing an angle of a given magnitude.

Converse of process described under 9.4.1: first calculate tan α from α and then draw the lines "a" and "b" in such a way that a/b = tan α, e. g. b = 1 dm, a = tan α dm.

Example: For α = 62.5°, tan α = 1.92.
Then draw b = 1 dm, a = 1.92 dm, or b = 5 cm, a = 9.6 cm.

Working Sheet No. 13

1. Complete the following table as accurately as possible:

α	16.4°	32.6°	58.8°	79.1°				
sin α					0.828		0.9911	
cos α						0.256		0.146

2. Complete the following table:

β	8.4°	42.8°	79.4°				
tan β				0.278		8.45	
cot β					1.04		0.098

3. Complete the following table with only one movement of the slide:

x	2	4	6	8	10
x × sin 27.2°					

4. Do the same with the following table:

x	2	4	6	8	10
$\frac{\cos 52.3^\circ}{x}$					

5. Calculate the expression $\frac{3.74 \times 1.43}{\tan 83.1^\circ}$ with only one movement of the slide.

6. Do the same with the expression $\frac{6.25}{1.68 \times \cot 64.1^\circ}$

7. Do the same with the expression $1.87 \times 3.52 \times \sin 33.8^\circ$.

8. Complete the following table with only one movement of the slide:

β	a	a × sin β	a × cos β	a × tan β	a × cot β
17.4°	12.4				

9. Complete the following table:

β	$\sin^2 \beta$	$\sin^3 \beta$	$\cos^2 \beta$	$\cos^3 \beta$	$\tan^2 \beta$	$\cot^2 \beta$
22.3°						

Answers:

1.

α					55.9°	75.16°	85.35°	81.6°
sin α	0.2825	0.539	0.855	0.9820		0.9667		0.9893
cos α	0.9593	0.8425	0.518	0.1891	0.561		0.1331	

2.

β				15.54°	43.9°	83.25°	84.4°
tan β	0.1477	0.926	5.34		0.961		10.2
cot β	6.77	1.08	0.1871	3.60		0.1183	

3. 0.914; 1.828; 2.74; 3.655; 4.57

4. In this case the division is effected as a multiplication by the reciprocal (Scale CI and CIF).
0.306; 0.153; 0.1019; 0.0764; 0.0612.

5. In the division by tan 83.1° = 8.26, the functions of the two scales C and D are interchanged.
Setting: T_2 83.1° | C 3.74 (The quotient 3.74 ÷ 8.26 = 0.453 is now to be found above D 10).
DF 1.43 | CF: 0.647 (The multiplication by 1.43 must be carried out by the aid of the "displaced" scales).
Answer: 0.647

6. Division 6.25 ÷ cot 64.1° as under No. 5. The subsequent division by 1.68 is carried out with the scale DI.
Setting: T_1 red 64.1° | C 6.25
DI 1.68 | C: 7.66 Answer: 7.66

7. Setting: S 33.8° | CI 1.87
C 3.52 | D: 3.665 Answer: 3.665

8. Multiplication with interchanged scales.
Setting: D 1 | C 12.4
S 17.4° | C: 3.71 = a sin β
S 17.4° | P: 0.9542 = cos β
DF 0.954 | CF: 11.83 = a cos β
T_1 17.4° | C: 3.89 = a tan β
T_2 red 17.4° | C: 39.6 = a cot β

9. 0.144; 0.0547; 0.856; 0.792; 0.1681; 5.94.

9.5 Trigonometric calculations

9.5.1 Calculation of right-angled triangle

With the angular functions the right-angled triangle can be calculated in the known manner from a sufficient number of known factors. We shall now show how this can be done with only one movement of the slide (and possibly moving the slide through).

1st case: Given: 2 sides, "a" and "b".

Formulae used: $a \times \frac{1}{b} = \tan \alpha$ $\beta = 90° - \alpha$

$a \times \frac{1}{c} = \sin \alpha$

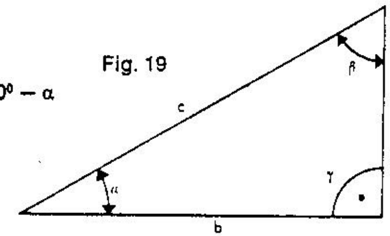


Fig. 19

Example: a = 8.4 cm, b = 6.3 cm
Setting: D 8.4 | CI 1
CI 6.3 | T_2: 53.1° = α (for tan α < 1, take reading on T_1!)
S 53.1° | CI: 10.5 c = 10.5 cm

The calculation of β = 90° − α if the angle set on S by the cursor is read from the red marking: β = 36.9°.

Exercises: Complete the following table:

a	7.05 cm	4.45 cm	4.95 cm	1.3 dm
b	4.8 cm	5.9 cm	2.08 cm	3.04 dm
α				
β				
c				

Results:

α	55.75°	37°	67.2°	23.17°
β	34.25°	53°	22.8°	66.83°
c	8.52 cm	7.39 cm	5.36 cm	3.31 dm

2nd case: Given the hypotenuse, c, and an angle (e.g. α).

Formulae used: a = c sin α; b = c cos α; β = 90° − α

Examples: c = 13.4 cm, α = 37.2°

Setting: D 1 | C 13.4 (in this case the functions of scales C and D being interchanged in the multiplication; the first factor is set on C).
S 37.2° | C: 8.1 $a = 8.1$ cm
S 37.2° **red** | C: 10.68 $b = 10.68$ cm
$\beta = 52.8°$

Exercises: Complete the following table:

c	21.2 cm	27.4 cm	9.25 cm	10.6 cm
α	25.8°	41.5°		9.6°
β			34.3°	
a				
b				

Answers:

α			55.7°	
β	64.2°	48.5°		80.4°
a	9.23 cm	18.17 cm	7.64 cm	1.77 cm
b	19.1 cm	20.55 cm	5.21 cm	10.45 cm

3rd case: Given the hypotenuse, c, and one side (e. g. a).

The most convenient method is the use of the sine formula:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma\ (=1)} = \frac{b}{\sin\beta}$$

Example: $c = 8.65$ cm, $a = 5.85$ cm

Setting: D 10 | C 8.65 Proportion!
C 5.85 | S: 42.5° $\beta = 47.5°$
S 47.5° | D: 6.38 $b = 6.38$ cm

Exercises: Complete the following table:

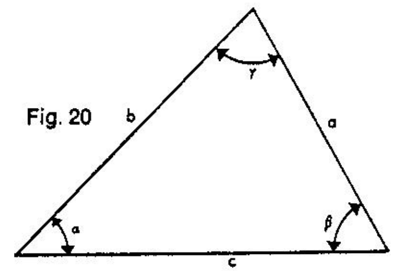
c	7.9 cm	14.8 cm	11.4 cm	19.1 cm
a	3.15 cm	8.6 cm		5.4 cm
b			7.8 cm	
α				
β				

Answers:

a			8.32 cm	
b	7.25 cm	12.07 cm		18.3 cm
α	23.45°	35.5°	46.85°	16.42°
β	66.55°	54.5°	43.15°	73.58°

9.5.2 Calculation of scalene triangle

Fig. 20



1st case: Given one side and two angles (a–s–a).

Solution obtained with sine equation: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

Example: $a = 6.75$ cm, $\beta = 58.4°$, $\gamma = 72.5°$
The preliminary result of the above is $\alpha = 180° - (\beta + \gamma) = 49.1°$.

Setting: S 49.1° | C 6.75
S 58.4° | C: 7.6 $b = 7.6$ cm
S 72.5° | C: 8.52 $c = 8.52$ cm

2nd case: Given two sides and the angle opposite one of the two sides (s–s–a).

Here again the sine equation is employed. It should be noted, however, that the solution is ambiguous if the angle given is opposite the smaller of the two sides given.

Example: $a = 6.3$ cm, $b = 8.2$ cm, $\alpha = 37°$

Setting: S 37° | C 6.3
C 8.2 | S: 51.6° thus $\beta_1 = 51.6°$, $\beta_2 = 180° - \beta_1 = 128.4°$
and $\gamma_1 = 91.4°$, $\gamma_2 = 14.6°$

We thus also find: $\sin 91.4° = \sin 88.6°$
S 88.6° | C: 10.45 i.e. $c_1 = 10.45$ cm
and S 14.6° | C 2.64 i.e. $c_2 = 2.64$ cm

Exercises: Complete the following table:

a	8.4 cm			4.15 cm		50 cm
b	4.85 cm	9.1 cm	9.8 cm		12 cm	
c		7.7 cm	6.25 cm			
α	71°			48.4°	71°	10.2°
β		51.2°		60°		46.5°
γ			38.3°		54°	

Answers:

a		11.65 cm	9.15 cm 6.24 cm		13.85 cm	
b				4.81 cm		205 cm
c	8.62 cm			5.26 cm	11.85 cm	236 cm
α	71°	87.5°	65.2° 38.2°			
β	33.1°		76.5° 103.5°		55°	
γ	75.9°	41.3°		71.6°		123.3°

The following two cases require the use of the cosine equation or other complicated equations for an accurate solution. Approximation processes, providing the answer more rapidly, will be found more convenient.

3rd case: Given two sides and the angle enclosed by them (s–a–s).

Example: $a = 6.4$ cm, $b = 5.8$ cm, $\gamma = 59^0$.

Once again, start with $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$, and endeavour to find an α and β value such that $\alpha + \beta + \gamma = 180^0$.

For this purpose, we take α as equal to 60⁰, for example, as a 1st approximation, determine the corresponding β value, calculate the sum of the three angles and then correct the value for α, and so on:

	1.	2.	3.
α	60°	65°	65.5°
β	51.7°	55.2°	55.5°
γ	59°	59°	59°
Sum	170.7°	179.2°	180.0°

The 1st and 2nd approximation give too small an angular sum, so that the assumed value for α will always be greater. With the correct value, we calculate $c = 6.03$ cm.

Exercises: see below.

4th case: Given, three sides (s–s–s).

Here again we use the proportion resulting from the sine equation and determine the three angles on an approximate basis, so that their sum amounts to 180⁰.

Examples: $a = 46$ cm, $b = 34$ cm, $c = 42$ cm.

Experimentally we take α as equal to 70⁰, for example.

	1.	2.	3.
α	70°	75°	73.7°
β	44°	45.6°	45.2°
γ	59.1°	61.9°	61.1°
Sum	173.1°	182.5°	180°

Exercises: Complete the following table:

a		17 cm	4.56 cm	51 cm
b	87.2 cm	12 cm	4.25 cm	65 cm
c	63.3 cm		3.28 cm	45 cm
α	80°			
β				
γ		59.3°		

Answers:

a	98.5 cm			
b				
c		14.99 cm		
α		77.2°	75°	51.4°
β	60.7°	43.5°	61°	85°
γ	39.3°		44°	43.6°

9.6 Scale ST and Mark ϱ

9.6.1 Conversions between radians and degrees.

The Scale ST interacts with Scale D and enables radians to be converted to degrees and vice versa, for angles between 0.5° and 6.5°. In this case a "0.0" has to be mentally noted in front of the figures on Scale D.

Examples: 0.85° = 0.1484 rad, 0.069 rad = 3.955°.

The scale can also be used, however, for the conversion of greater or smaller angles by imagining the values associated with each other to be multiplied or divided by one and the same power of ten.

Examples: As 0.056 rad = 3.21°
It follows that: 0.56 rad = 32.1°
5.6 rad = 321°
0.0056 rad = 0.321° etc.

The following equation is often useful in enabling the position of the decimal point to be determined:

$$2\pi \text{ rad} = 6.28 \text{ rad} = 360^\circ$$
$$\text{and } 1 \text{ rad} = 57.3^\circ$$
$$\text{and } 1^\circ = 0.01745 \text{ rad.}$$

The conversion of degrees to radians and vice versa can also be effected by means of the mark ϱ (1745) on Scales C, D, CF, DF, W_1 and W_1'. This is only of importance, however, when the greater accuracy of the Scales W_1 and W_1' is required (see Section 10).

9.6.2 Sine and tangent of small angles.

As sufficiently small angles α can be coped with by the equations

$$\sin\alpha \approx \tan\alpha \approx \text{arc}\,\alpha \approx \cos(90^\circ - \alpha) \approx \cot(90^\circ - \alpha)$$

the scale ST can also be used for calculating the sine and tangent of angles between 0.5° and 6°, as well as the cosine and cotangent of angles between 84° and 89.5°.

Example: $\sin 2.5^\circ \approx \tan 2.5^\circ = \cot 87.5^\circ \approx \cos 87.5^\circ \approx 0.0436$

The slight deviation of tan α from arc α, for α ≧ 4°, is taken into account by correction-marks on the right of the marks for 4°, 5° and 6°. (For α > 5°, however, the scale T should be used for preference).

Example: tan 5° = 0.0875.

If the angle is between the full degrees having correction-marks, the correction interval must be judged visually and transferred accordingly.

Example: tan 4.6° = 0.0805.

If the functional value is known and the angle sought, the cursor line must be thought of as having been moved to the left by the width of the "correction interval", before the reading is taken.

Example: arctan 0.0787 = 4.5°.

With small angles, the mark ϱ can also be used for determining the sine and tangent. C ϱ is placed above D 1, and the scale D (and DF) then provides the "angle scale", C and CF being the sine-tangent-arc scale.

Example: $\sin 2^\circ \approx \tan 2^\circ \approx \text{arc}\, 2^\circ = 0.0349$
$\sin 0.2^\circ \approx \tan 0.2^\circ \approx \text{arc}\, 0.2^\circ = 0.00349.$

9.6.3 Tangent of angles in the vicinity of 90° (85° < α < 90°) and cotangent of small angles (α < 5°).

The functional values extending beyond the range of Scale T_2 are found, on the basis of the following considerations, by the aid of scales ST and DI:

We have

$$\tan\alpha = \frac{1}{\cot\alpha} = \frac{1}{\tan(90^\circ - \alpha)} \approx \frac{1}{\text{arc}\,(90^\circ - \alpha)} \quad \text{(for } \alpha \text{ in the vicinity of } 90^\circ)$$

and

$$\cot\alpha = \frac{1}{\tan\alpha} \approx \frac{1}{\text{arc}\,\alpha} \quad (\text{for } \alpha < 5^\circ)$$

Examples:

1. $\tan 89^\circ = \frac{1}{\cot 89^\circ} = \frac{1}{\tan 1^\circ} \approx \frac{1}{\text{arc}\, 1^\circ} = 57.3$

   Setting: ST 1° | DI: 57.3

2. arctan 26.4 = 87.83°

   Setting: DI 26.4 | ST: 2.17°; 90° − 2.17° = 87.83°

3. $\cot 3.4^\circ = \frac{1}{\tan 3.4^\circ} \approx \frac{1}{\text{arc}\, 3.4^\circ} = 16.83$

   Setting: ST 3.4° | DI: 16.83

4. arccot 32.8 = 1.746°

Setting: DI 32.8 | ST: 1.746°

5. cot 0.35° = 163.7

Setting: ST 3.5° | DI: 163.7
(3.5° is here shown with the different position of the decimal point for 0.35°).

Exercises: Complete the following table:

α	85.8°		88.65°		89.7°	
tan α		18.3		72.5		320

α	1.85°		0.22°		0.1°	
cot α		43.5		181		415

Answers:

α		86.87°		89.21°		89.821°
tan α	13.62		42.45		191	

α		1.317°		0.3165°		0.138°
cot α	31		261		573	

10. The root scales W_1, W_1', W_2, W_2'

10.1 General

On the back of the 2/83 N a pair of scales is provided, with a graduated length of 50 cm, subdivided in the middle ($\sqrt{10} = 3{\cdot}162$), the right-hand half being situated on the upper part of the slide rule.

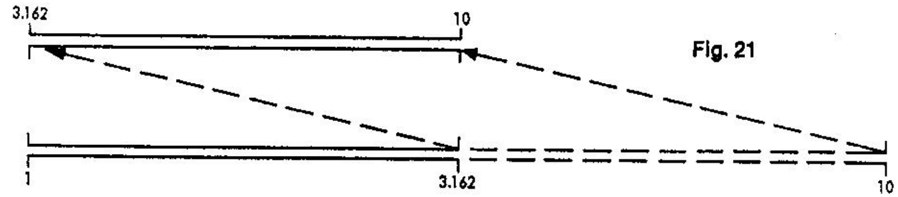


Fig. 21

This subdivided pair of scales provides the accuracy of a 50-cm slide rule in multiplication, division, squaring and the extraction of square roots, as well as in determining common logarithms, without sacrificing the ease of handling a slide rule of normal length. After brief practice the user will be fully familiar with this pair of scales and with the required new rule for calculation.

The graduation of the 50-cm scale is explained on the diagram in the appendix.

10.2 Multiplication

To calculate the product "a × b", first of all find the first factor (a) on the (fixed) scale of the stock, in the usual manner. With the subdivided scale, however, (a) is either on the lower scale W_1 only or on the upper scale W_2 only (if one disregards the additional graduations).

If "a" is on W_1, then either the mark "1" or the red stroke at $\sqrt{10} = 3{\cdot}162$ of the adjacent scale W_1' is placed above it. If "a" is on W_2, then either the mark "10" or the red stroke at $\sqrt{10}$ of the adjacent scale W_2' is placed below it. The factor "b" is then sought on the scale W_1' or W_2' as the case may be.

The product (a × b) is now to be found vertically below or above the factor "b" – on the stock scale, **resting against** the factor "b", if the first factor was set with the mark "1" or "10", or on the stock scale **situated opposite** the factor "b", if the first factor was set with one of the two red strokes.

Examples:

(1a) $1.3 \times 2.1 = 2.73.$ (1b) $1.3 \times 5.8 = 6.24.$

Setting with mark "1". Reading taken on adjacent scale of stock of slide rule.

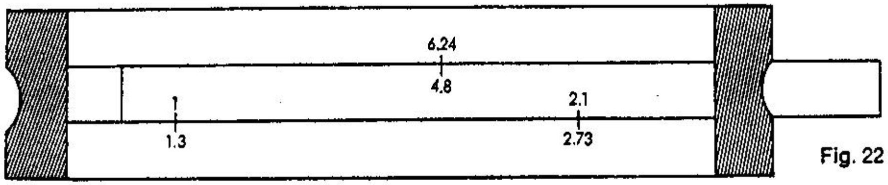


Fig. 22

(2a) $2.4 \times 2.1 = 5.04.$ (2b) $2.4 \times 6 = 14.4.$

Setting with mark "▽". Reading taken on stock scale situated opposite.

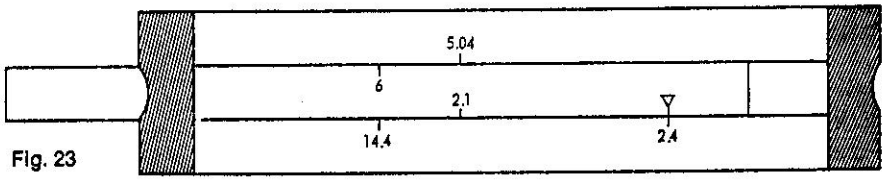


Fig. 23

(3a) $8.3 \times 5.2 = 43.15.$ (3b) $8.3 \times 1.9 = 15.77.$

Setting with mark "10". Reading taken on adjacent stock scale.

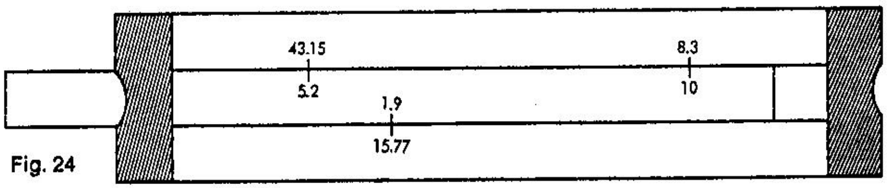


Fig. 24

(4a) $3.8 \times 5 = 19.$ (4b) $3.8 \times 2 = 7.6.$

Setting with mark "△". Reading taken on stock scale situated opposite.

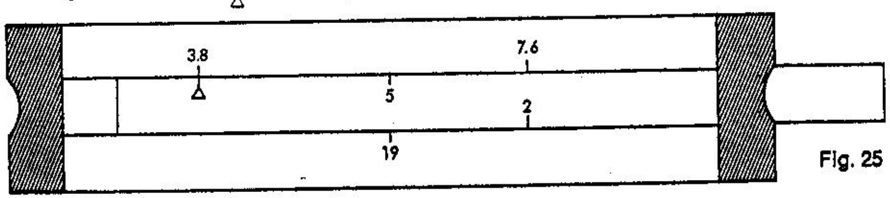


Fig. 25

Working Sheet No. 14

1. $15.45 \times 1.925 =$	11. $504 \times 0.1865 =$ ....
2. $139 \times 2.74 =$	12. $72.8 \times 41.6 =$ ....
3. $24.2 \times 19.75 =$	13. $137.5 \times 8.34 =$ ....
4. $29.7 \times 8.46 =$	14. $64.5 \times 15.35 =$ ....
5. $239 \times 5.88 =$	15. $27.6 \times 8.22 \times 0.197 =$ ....
6. $106.5 \times 9.72 =$	16. $117.5 \times 0.306 \times 4.84 =$ ....
7. $87.6 \times 5.08 =$	17. $92.8 \times 0.785 \times 1.98 =$ ....
8. $792 \times 0.658 =$	18. $40.3 \times 6.88 \times 15.3 =$ ....
9. $944 \times 2.36 =$	19. $124.5 \times 0.0634 \times 2.08 =$ ....
10. $46.3 \times 5.12 =$	20. $26.7 \times 48.9 \times 0.804 =$ ....

Answers: 1. 29.74; 2. 380.9; 3. 478; 4. 251.3; 5. 1405; 6. 1035; 7. 445; 8. 521; 9. 2228; 10. 237; 11. 94; 12. 3028; 13. 1147; 14. 990; 15. 44.7; 16. 174; 17. 144.4; 18. 4242; 19. 16.42; 20. 1050.

For further practice you can also use Working Sheets 3 and 4 once again.

10.3 Division

To calculate the quotient a/b, the cursor line is placed above the dividend "a", which is found either on W_1 or on W_2. The divisor, b, is then – on W_1' or W_2' – placed underneath the cursor line. We shall then find "a" and "b" either on adjacent or on mutually opposite scales.

If "a" and "b" are on adjacent scales, the quotient will be found on W_1 or W_2 under the mark "1" of W_1' or above the mark "10" of W_2'.

If, on the other hand, "a" and "b" are situated on mutually opposite scales, the quotient will be found on W_1 or W_2 under or above one of the two red strokes on W_1' or W_2' respectively.

Examples:

(1a) 2.4 ÷ 1.8 = 1.333. (1b) 8 ÷ 6 = 1.333.

Setting on adjacent scales, reading taken under mark "1".

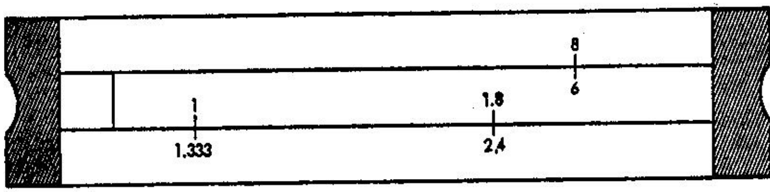


Fig. 26

(2a) 21 ÷ 24 = 0.875. (2b) 3.5 ÷ 4 = 0.875.

Setting on adjacent scales, reading taken above mark "10".

Fig. 27

(3a) 24 ÷ 8 = 3. (3b) 57 ÷ 19 = 3.

Setting on mutually opposite scales, reading taken under mark " ▽ ".

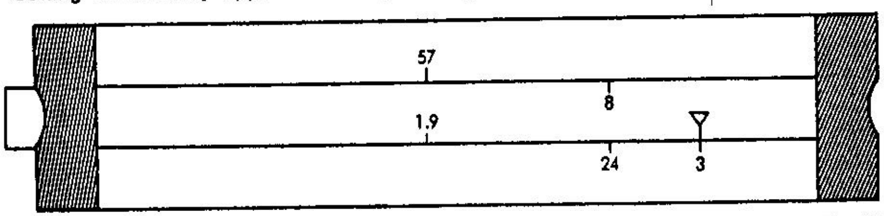


Fig. 28

(4a) 27 ÷ 6 = 4.5. (4b) 9.9 ÷ 2.2 = 4.5.

Setting on mutually opposite scales, reading taken above mark " △ ".

Fig. 29

If it becomes necessary to move the slide through to the end, in the course of compound multiplication and division operations, it should be borne in mind that this changes the method of setting, so that the reading likewise has to be taken differently.

Example:

$$\frac{2.9 \times 7.5}{1.4}$$

Setting: W_1 2.9 | W_1' 1.4 Setting on adjacent scales.
W_1' 1 | W_1: 2.071. (Reading need not be taken).

In the multiplication by 7.5 which now follows, the slide must be moved through to the end. In this operation the setting with the mark "1" is replaced by the setting with the mark " ▽ "; the reading must therefore be taken on mutually opposite scales: W_2 7.5 | W_1: 15.54.

Working Sheet No. 15

1.	2.435 ÷ 1.975 = ...	13.	0.688 ÷ 0.019 15 = ...
2.	304.5 ÷ 0.267 = ...	14.	591 ÷ 2465 = ...
3.	48.7 ÷ 362 = ...		
4.	0.607 ÷ 39.8 = ...	15.	$\frac{2.72 \times 4.18}{15.45}$ = ...
5.	1.805 ÷ 29.1 = ...	16.	$\frac{14.05 \times 8.92}{53.1}$ = ...
6.	13.62 ÷ 0.2505 = ...		
7.	40.35 ÷ 59.3 = ...	17.	$\frac{494 \times 0.1875}{26.7}$ = ...
8.	6410 ÷ 751 = ...		
9.	164 ÷ 53.6 = ...	18.	$\frac{76.5 \times 19.4}{37.3}$ = ...
10.	287 ÷ 412.5 = ...	19.	$\frac{0.1885 \times 93.8}{4.22}$ = ...
11.	106.5 ÷ 8.06 = ...		
12.	3815 ÷ 212.5 = ...	20.	$\frac{90.2 \times 3.87 \times 15.7}{23.2 \times 8.93}$ = ...

Answers: 1. 1.233; 2. 1140; 3. 0.1345; 4. 0.015 25; 5. 0.062; 6. 54.4; 7. 0.6805; 8. 8.535; 9. 3.06; 10. 0.696; 11. 13.21; 12. 17.95; 13. 35.93; 14. 0.2398; 15. 0.736; 16. 2.36; 17. 3.47; 18. 39.8; 19. 4.19; 20. 26.45.

For further practice, you can work through Sheets 6a, 6b and 8b once again.

10.4 Formation of tables; proportions.

For the formation of tables and the calculation of proportions, we once again place an interrelated pair of values, on adjacent or opposite scales, vertically one above the other. The reading of values corresponding to one another is effected on either adjacent or opposite scales, according to the setting method adopted, and on the following principle:

Setting on adjacent scales – reading from adjacent scales.

Setting on mutually opposite scales – reading from mutually opposite scales.

If the slide has to be moved through, during the calculation, the method of setting and thus the method of taking the reading are thereby changed!

Examples:

(1) 1 nautical mile = 1.852 km.

Setting: W_1 1 | W_1' 1852 (adjacent scales) — W_1/W_2: nautical-mile scale. W_1'/W_2': km scale.

We then find, for example, 1.512 n.m. = 2.80 km; 8.4 km = 4.535 n.m.
Also, after moving the slide through (= setting on mutually opposite scales!):
2.42 n.m. = 4.482 km
5.3 km = 2.86 n.m.

(2) 26 inches = 66 cm.

Setting: W_1 26 | W_2' 66. (mutually opposite scales). — W_1/W_2: inch scale. W_1'/W_2': cm scale.

The following readings, for example, are then taken from opposite scales:
19 inches = 48.22 cm
89 inches = 225.9 cm

Also, after moving the slide through (= setting on adjacent scales!):
11.7 ins. = 29.7 cm
91 cm = 35.85 ins.

Exercises:

(1) Given: a proportion $\frac{x}{y} = \frac{62.4}{4.47}$

Complete the following table:

x	7.82		18.25		36.9	
y		33.4		1.905		0.297

(2) Given: a proportion $\frac{a}{b} = \frac{7.72}{15.1}$

Complete the following table:

a	56.4		34.1		116	
b		1.87		29.2		3.92

Results:

x		466.3		26.6		4.146
y	0.560		1.307		2.643	

a		0.956		14.93		2.004
b	110.3		66.7		226.9	

10.5 Finding the square of a number.

A number is squared by changing over W_1 or W_2 to D or from W_1' or W_2' to C. It is immaterial whether scales C and D on the back or on the front of the slide rule are used for this purpose.

Examples: $15.65^2 = 245$; $0.264^2 = 0.0697$; $4.17^2 = 17.4$; $79.2^2 = 6.27 \times 10^3$

10.6 Finding the square root.

This is done by changing over from Scale C to Scale W_1' or W_2' or from Scale D to Scale W_1 or W_2. In this process, the roots of numbers between 1 and 10 will be found on scale W_1' or W_1, while those of numbers between 10 and 100 will be found on scale W_2' or W_2.

Radicands less than 1 or greater than 100 are once again to be subdivided into a factor between 1 and 100 and a power of ten with an even index (see Sect. 6·4).

Examples: $\sqrt{4.1} = 2.025$; $\sqrt{41} = 6.405$; $\sqrt{410} = 20.25$; $\sqrt{4100} = 64.05$;
$\sqrt{0.41} = 0.6405$; $\sqrt{0.041} = 0.2025$

Solve the problems in Working Sheet 10 for exercise.

10.7 Calculations with squares and square roots.

After a square or a square root has been found by means of the square scales, further calculation is always possible, the accuracy being that of the normal scales (25-cm graduations) after squaring a number and as high as that of the 50-cm scales after the extraction of a square root. For example, the following expressions can be simplified without difficulty:

$a^2 \cdot b$

1st example: $18.2^2 \times 4.7 = 1557$
Setting: W_1 18.2 | C 10
C 4.7 | D: 1557

2nd example: $6.84^2 \times 10.8 = 505$
Setting: W_2 6.84 | C 1
C 10.8 | D: 505

$a \cdot b^2$

1st example: $23.7 \times 1.935^2 = 88.7$
Setting: D 23.7 | W_1' 1
W_1' 1.935 | D: 88.7

2nd example: $8.45 \times 7.62^2 = 491$
Setting: D 8.45 | W_2' 10
W_2' 7.62 | D: 491

$\frac{a^2}{b}$

1st example: $2.61^2 \div 0.382 = 17.83$
Setting: W_1 2.61 | C 0.382
C 1 | D: 17.83

2nd example: $4.13^2 \div 6.05 = 2.82$
Setting: W_2 4.13 | C 6.05
C 10 | D: 2.82

$\frac{a}{b^2}$

1st example: $58.5 \div 14.6^2 = 0.2745$
Setting: D 58.5 | W_1' 14.6
C 1 | D: 0.2745

2nd example: $338 \div 9.22 = 3.98$
Setting: D 338 | W_2' 9.22
C 10 | D: 3.98

$\frac{a^2 \cdot b}{c}$

1st example: $\frac{2.64^2 \times 1.3}{4.4} = 2.06$
Setting: W_1 2.64 | C 4.4
C 1.3 | D: 2.06

2nd example: $\frac{4.82^2 \times 3.5}{6.15} = 13.22$
Setting: W_2 4.82 | C 6.15
C 3.3 | D: 13.22

$\frac{a^2 \cdot b}{c^2}$

1st example: $\frac{7.42^2 \times 6}{19.4^2} = 0.878$
Setting: W_2 7.42 | W_1' 19.4
C 6 | D: 0.878

2nd example: $\frac{16.9^2 \times 2.4}{9.38^2} = 7.79$
Setting: W_1 16.9 | W_2' 9.38
CF 2.4 | DF: 7.79

1st example: $\frac{6.9 \times 3.2}{2.85^2} = 2.72$

$\frac{a \cdot b}{c^2}$

Setting: D 6.9 | W_1' 2.85
C 3.2 | D: 2.72

2nd example: $\frac{38.8 \times 9.25}{7.22^2} = 6.88$

Setting: D 38.8 | W_2' 7.22
C 9.25 | D: 6.88

Exercises in connection with the above will be found on Working Sheets Nos. 9 and 16.

In the case of expressions taking the form

$$\sqrt{a \times b},\ \sqrt{\frac{a}{b}},\ \sqrt{\frac{a \times b}{c}},\ \sqrt{a \times b \times c} \text{ etc.}$$

the radicand is first of all calculated in the known manner, by means of scales C and D (and possibly CI). The extraction of the root is then effected by the change-over to W_1 or W_2. Whether the result is to be found on W_1 or on W_2 is then decided after estimating the order of magnitude of the radicand, taking the rule in 10.6 into account.

Example: $\sqrt{8.15 \times 3.08} = 5.01$

$\sqrt{a \cdot b}$

Setting: D 8.15 | C 10
C 3.08 | W_2: 5.01

The radicand can be estimated at about 25, i.e. between 10 and 100, so that the root is found on W_2.

Example: $\sqrt{\frac{64}{9.4}} = 2.61$

$\sqrt{\frac{a}{b}}$

Setting: D 64 | C 9.4
C 10 | W_1: 2.61

The radicand is approximately 7, so that the reading is taken on W_1.

Example: $\sqrt{\frac{2.92 \times 31.6}{0.57}} = 12.72$

$\sqrt{\frac{a \cdot b}{c}}$

Setting: D 2.92 | C 0.57
C 31.6 | W_1: 12.72

For this and similar problems it is advisable to decide on the following basis whether the reading of the result is to be taken from W_1 or W_2: the value of the radicand is about 160 (intermediate result underneath the cursor line, on D: 162). The root of the latter must be approximately 13. We now have to decide between the sequence of figures 1-2-7-2, to be found on W_1, underneath the cursor line, and the sequence 4-2-5, to be found on W_2 undrneath the cursor line. Result: 12.72.

Example: $\sqrt{76.5 \times 0.38 \times 1.56} = 6.74$

$\sqrt{a \cdot b \cdot c}$

Setting: D 76.5 | CI 0.38
C 1.56 | W_2: 6.74

Estimated: radicand $\approx$ 50, result $\approx$ 7.

Expressions taking the form

$$\sqrt{a \times b} \times c \text{ and } \sqrt{\frac{a}{b}} \times c$$

can likewise be calculated with one single movement of the slide:

$\sqrt{a \cdot b} \cdot c$

1st example: $\sqrt{21.8 \times 5.65} \times 18.2 = 202$

Setting: D 21.8 | CI 5.65
W_1' 18.2 | W_1: 202

Division by the reciprocal; the subsequent change-over to the W scale provides the root, which is then multiplied by 18.2.

2nd example: $\sqrt{16.4 \times 1.89} \times 6.68 = 37.2$

Setting: D 16.4 | CI 1.89
W_2' 6.68 | W_2: 37.2

1st example: $\sqrt{\frac{136}{590}} \times 894 = 429$

$\sqrt{\frac{a}{b}} \cdot c$

Setting: D 136 | C 590
W_2' 894 | W_2: 429

2nd example: $\sqrt{\frac{369}{16.4}} \times 5.73 = 27.18$

Setting: D 369 | C 16.4
W_2' 5.73 | W_1: 27.18

Exercises on this section will be found on Working Sheets 9, 11 and 16.

Working Sheet No. 16

1. $\frac{7.08^2 \times 9.85}{36.2} =$ ....

2. $\frac{18.45^2 \times 5.38^2}{69.5^2} =$ ....

3. $\frac{122 \times 3.41}{6.02^2} =$ ....

4. $\frac{8.36^2 \times 37.2}{25.3^2} =$ ....

5. $4.79^2 \times 3.18 \times 0.164^2 =$ ....

6. $10.85^2 \times 0.87^2 \times 5.15 =$ ....

7. $\frac{7.62^2 \times 4.55^2 \times 0.374}{2.14^2 \times 7.75} =$ ....

8. $\sqrt{19.1 \times 28.7} =$ ....

9. $\sqrt{0.785 \times 0.434} =$ ....

10. $\sqrt{\frac{571}{0.658}} =$ ....

11. $\sqrt{\frac{0.935}{13.8}} =$ ....

12. $\sqrt{\frac{0.628 \times 112}{0.843}} =$ ....

13. $\sqrt{\frac{193 \times 27.6}{418}} =$ ....

14. $\sqrt{129 \times 6.45 \times 0.303} =$ ....

15. $\sqrt{88.5 \times 73.2 \times 2.7} =$ ....

16. $\sqrt{16.2 \times 5.17 \times 2.285} =$ ....

17. $\sqrt{46.7 \times 16.3} \times 1.405 =$ ....

18. $\sqrt{\frac{344}{57.6}} \times 10.7 =$ ....

19. $\sqrt{\frac{198}{7.15}} \times 9.04 =$ ....

20. $\sqrt{4.35 \times 13.4} \div 32.6 =$ ....

21. $\frac{246}{\sqrt{36.4 \times 1.88}} =$ ....

22. $\sqrt{\frac{745}{16.9}} \div 29.3 =$ ....

Results: 1. 13.64; 2. 2.04; 3. 11.48; 4. 4.06; 5. 1.962; 6. 459; 7. 12.67; 8. 23.41; 9. 0.584; 10. 29.46; 11. 0.2603; 12. 9.135; 13. 3.57; 14. 15.88; 15. 132.25; 16. 20.91; 17. 37.76; 18. 26.15; 19. 47.57; 20. 0.2342; 21. 29.74; 22. 0.2266.

Further exercises of this type will be found on Working Sheets 9 and 11.

10.8 Calculations of circles and cylinders with root scales.

10.8.1 Diameter and area of a circle.

The strokes "d" and "q" on the back of the cursor enable areas and diameters of circles to be calculated without moving the slide through and with the increased accuracy of the root scales.

If the diameter of the circle is set with the mark "d" on the Scale W_1 or W_2 (or W_1' or W_2'), the area of the circle will be shown under the mark "q" on Scale D (or C, respectively).

Examples: 1. d = 0.718 mm; q = 0.405 mm²
2. d = 2.27 mm; q = 4.05 mm²

In the converse calculation, the area of the circle, q, is found either on W_1 (or W_1') or on W_2 (or W_2'), according to the order of magnitude of the diameter, d. The rule given in Section 10.6 then once again applies.

Examples: 1. q = 1.84 cm²; d = 1.53 cm
2. q = 18.4 cm²; d = 4.84 cm

Exercises:

d	1.57 mm	3.08 cm	64.3 m			
q				6.85 cm²	28.6 mm²	490 cm²

Results:

d				2.953 cm	6.035 mm	24.98 cm
q	1.935 mm²	7.45 cm²	3245 m²			

10.8.2 Calculation of circular cylinders.

The use of root scales is of particular advantage in calculating volumes and weights (or masses) of cylinders.

1st example: To find volume, V, of a cylinder of diameter d = 7.2 cm and length of l = 87 cm.

Formula: $V = q \times l$

Setting: Mark "d" on W_2 7.2.
C 10 under mark "q".
C 87 | D: 3540.

Answer: V = 3540 cm³.

2nd example: To find weight, G, of a brass bar (specific gravity $\gamma = 8.4$ p/cm³) of diameter, d, of 2.4 cm, and length, l, of 1.60 m.

Formula: $G = q \times l \times \gamma$

The multiplication of "q" by "l" is carried out by means of the scale CI, as a division by the reciprocal, so that the multiplication by γ can then be effected without again moving the slide through.

Setting: Mark "d" on W_1 2.4.
Place CI 1.6 under the mark "q".
C 8.4 | D: 6080.

Answer: G = 6080 p = 6.08 kp.

3rd example: What must be length of a cylinder of lead ($\gamma = 11.35$ p/cm³) of diameter 5.8 cm if it is to weigh 26 kp?

Method: First divide the weight by γ to obtain the volume. The length is obtained from this latter and from the diameter, which is already known.

Setting: D 26 | C 11.35
Mark "d" on W_2 5.8.
The answer will then be found under the mark "q" on CI:
l = 86.7 cm.

Exercises:

d	5.20 m	1.6 mm	0.94 m	8.75 cm		24.6 cm
l	7.6 m	2600 m	38 cm		35 cm	1.55 m
γ in p/cm³	1	8.93	2.3	13.6	5.6	
G in kp				5.4	135	171

Answers:

d					29.6 cm	
l				6.6 cm		
γ in p/cm³						2.32
G in kp	161.4×10^3	0.0467	607			

10.9 Further uses for the root scales.

In conjuction with the scales on the front of the 2/83 N the root scales enable a number of further calculation to be carried out, e. g. finding x^4 by changing over from $W_{1.2}$, to A, or calculating $\sqrt[4]{x}$ by changing over from A to $W_{1.2}$.

Exercises:

Using the abbreviated notation which has been adopted here, give hereunder the change-over by which the following calculations can be carried out:

x^6	$\sqrt[4]{1-x^2}$	$\sqrt{\sin x}$	$\sqrt[6]{x}$	$\sqrt{\cot x}$	$\sqrt{\frac{1}{x}}$	$\sqrt{\frac{x}{\pi}}$	$\sqrt{\frac{1}{\pi x}}$	arcsin x^2

πx^2	$\frac{1}{x^2}$	$\frac{1}{\pi x^2}$	$\sqrt{1-x^4}$

Answers: $W_{1.2}$ | K   P | $W_{1.2}$ S | $W_{1.2}$ K | $W_{1.2}$ $T_{1.2}$ red | $W_{1.2}$ DI | $W_{1.2}$
CIF | $W'_{1.2}$ $W_{1.2}$ | S $W_{1.2}$ | DF $W_{1.2}$ | DI $W'_{1.2}$ | CIF $W_{1.2}$ | P

11. The mantissa scale L

This interacts with the root scale, as may be seen from its designation at the right-hand end: $\frac{1}{2}\log x$ is equivalent to $\log \sqrt{x}$. The most convenient method is to use the mantissa scale in conjunction with the scales W_1' and W_2', as it is then not necessary to place the slide in the exact zero position.
In the change-over from W_1' to L the markings on the left of the individual graduation marks apply, while in the change-over from W_2' those to the right apply, these being always 0.5 greater than those to the left.

Reason: It is $\log \sqrt{10x} = \frac{1}{2}\log 10x = \frac{1}{2}(\log 10 + \log x)$
$= \frac{1}{2}(1 + \log x) = 0.5 + \frac{1}{2}\log x$

Examples: log 2.5 = 0.398; log 7.91 = 0.898

To form the logarithm, the characteristic, which is obtained from the following table, has to be added to the mantissa provided by Scale L:

Antilogarithm	Characteristic	
1–10:	0	Rule: Characteristic = number of places before decimal point, minus 1.
10–100:	1	
100–1000:	2	
etc.	etc.	
1–0.1:	0. . . . –1	Rule: Characteristic = 0. . . . less number of noughts preceding the first figure which is other than a nought (including the nought in front of the decimal point).
0.1–0.01:	0. . . . –2	
0.01–0.001:	0. . . . –3	
etc.	etc.	

Examples:

Antilogarithm	Mantissa	Characteristic	Logarithm
1.875	.273	0	0.273
18.75	.273	1	1.273
187.5	.273	2	2.273
18 750 000	.273	7	7.273
0.1875	.273	0. . . . –1	0.273–1
0.018 75	.273	0. . . . –2	0.273–2
0.000 000 187 5	.273	0. . . . –7	0.273–7

If the logarithm is known and the antilogarithm sought, the sequence of figures representing the number (antilogarithm) is found from the mantissa and the position of the decimal point from the characteristic – by reversing the above operation.

Examples: log x = 2.197; x = 157.4
log y = 5.848; y = 705 000
log z = 0.911–4; z = 0.000 815

Exercises:

a	3490		83 700		0.005 06		
log a		4.723		0.001		0.091–3	–2.348

b	1.08		962		0.073		46.2
log b		– 0586		1.03		0.08–2	

Answers:

a		52 850		1.0025		0.001 233	0.004 487
log a	3.5428		4.9227		0.7042–3		

b		0.2594		10.7		0.0631	
log b	0.0334		2.9832		0.8633–2		1.6646

12. The exponential scales LL

e^n

12.1 Description. Powers of e.

The 2/83 N is provided on the back with two groups of four scales each, for the powers of "e" (2.71828), i.e.

- the group LL_0, LL_1, LL_2 and LL_3, for positive exponents between 0.001 and 11;
- the group LL_{00}, LL_{01}, LL_{02} and LL_{03}, for negative exponents between −0.001 and −11.

The exponential scales, in calculating e^n, can be most reliably used in conjunction with the fixed Scale D. **Changes must not then be made to the "place value" given on the scales.** Everything else is explained by the designations on the right-hand edge of the scale.

Examples:	Change-over	Result
$e^{1.8}$	D | LL_3	6.05
e^{8}	D | LL_3	2.98×10^3
$e^{0.54}$	D | LL_2	1.716
$e^{0.076}$	D | LL_1	1.079
$e^{-3.32}$	D | LL_{03}	0.0362
$e^{-0.1565}$	D | LL_{02}	0.8551
$e^{-0.0435}$	D | LL_{01}	0.9574
$e^{-0.0096}$	D | LL_{00}	0.99045

The scale LL_0 for $e^{0.001x}$ is incorporated in Scale D as follows:
For a sufficient small value of x,

$$e^{0.001x} \approx 1 + 0.001\,x.$$

The desired value $e^{0.001x}$ is thus obtained by placing "0.00" in front of "x" and adding 1. The two operations are performed simultaneously, by finding the value "x" on Scale D and then including, in the reading, the italic figures shown **in front of** the normal text on the scale.

Examples: $e^{0.003} = 1.003.$
$e^{0.00243} = 1.00243.$

Up to $e^{0.003}$ the deviation of the value thus found from the true value is less than 0.000 005. From $e^{0.004}$ onwards, the deviation can be corrected by incorporating in the reading the italic figures shown **after** the normal text of the scale.

Examples: $e^{0.004} = 1.00401.$
$e^{0.008} = 1.00803.$

With the above approximation

$$e^n \approx 1 + n \text{ (for a sufficiently small value of } |n|)$$

the seldom used values between $e^{-0.001}$ and $e^{0.001}$ can likewise be calculated with sufficient accuracy.

Examples: $e^{0.000\,75} = 1.00075.$
$e^{-0.000\,28} = 1 - 0.000\,28 = 0.999\,72.$

Exercises will be found on Working Sheet 17.

Working Sheet No. 17 Complete the following table:

x	e^x	e^{-x}
2.24		
0.675		
0.143		
0.0306		
5.95		
0.094		
1.39		
0.003 72		
0.008 85		
0.000 64		
9.2		
7.9		

Answers:

x	e^x	e^{-x}
2.24	9.39	0.1065
0.675	1.964	0.509
0.143	1.1537	0.8668
0.0306	1.031 05	0.969 85
5.95	384	0.002 61
0.094	1.0986	0.9103
1.39	4.015	0.249
0.003 72	1.003 72	0.996 29
0.008 85	1.008 89	0.991 18
0.000 64	1.000 64	0.999 36
9.2	9.9×10^3	0.000 10
7.9	2.7×10^3	0.000 37

12.2 Hyperbolic functions.

From the powers of "e" we obtain, after simple subsidiary calculations, the hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

and

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

To calculate the above, place the cursor line on D x, read e^x and e^{-x} and calculate half the difference or sum.

For a sufficiently high value of x (where the slide rule is accurate for an x value of at least 3) we have

$$\sinh x \approx \cosh x \approx \frac{e^x}{2}$$

Examples:

x	0.003	0.03	0.3	3.0	6.0
e^x	1.003	1.030 45	1.350	20.1	405
e^{-x}	0.997	0.970 45	0.741	0.0498	0.002 48
sinh x	0.003	0.030 00	0.305	10.02	202
cosh x	1.000	1.000 45	1.045	10.07	202

Where $|x| \leq 0.003$, $\sinh x \approx x$ and $\cosh x \approx 1$.

Furthermore:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}} \left(= \frac{e^{2x} - 1}{e^{2x} + 1} \right)$$

and

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{1 + e^{-2x}}{1 - e^{-2x}} \left(= \frac{e^{2x} + 1}{e^{2x} - 1} \right)$$

For the calculation of functional values with the slide rule, the expressions on the right are particularly suitable. (The formulae in brackets are more convenient but give inaccurate readings for greater values of x).

Examples:

x	0.003	0.03	0.3	3
e^{-2x}	0.994 02	0.9418	0.549	0.002 48
tanh x	0.0030	0.029 99	0.2913	0.995
coth x	333.3	3.433	34.33	1.005

For $x > 4$, $\tanh x = \coth x = 1$ applies accurately, while for $|x| \leq 0.04$ $\tanh x = x$.

Working Sheet No. 18

x	0.745	0.0196	3.94	0.002 82	0.000 805	−0.915
e^x						
e^{-x}						
sinh x						
cosh x						
e^{-2x}						
tanh x						
coth x						

Results:

x	0.745	0.0196	3.94	0.002 82	0.000 805	−0.915
e^x	2.106	1.0198	51.5	1.002 82	1.000 805	0.400
e^{-x}	0.4745	0.9806	0.0194	0.997 18	0.999 195	2.497
sinh x	0.816	0.0196	25.7	1.000 00	1.000 000	−1.048
cosh x	1.291	1.0019	25.7	1.000 00	1.000 000	1.449
e^{-2x}	0.225	0.961 55	0.000 375	0.994 37	0.998 39	0.160
tanh x	0.632	0.0196	1.00	0.002 82	0.000 805	0.724
coth x	1.582	51.0	1.00	354.8	1242	1.382

12.3 Roots of "e"

$\sqrt[n]{e}$ | $\frac{1}{\sqrt[n]{e}}$

As roots are powers with fractional indices, any desired roots of "e", and their reciprocals, can be calculated with the LL scales. The calculation can be carried out with one single cursor setting if the reciprocal scale DI (on the front of the slide rule) is used for the calculation of the reciprocal of the root exponent.

Examples: $\sqrt[3]{e} = e^{1/3} = 1.396$ Setting: DI 3 | LL_2: 1.396

$\frac{1}{\sqrt[4.6]{e}} = e^{-\frac{1}{4.6}} = 0.845$ Setting: DI 4 6 | LL_{02}: 0.845

12.4 Natural logarithms

ln a

Since $y = e^x$, it follows (conversely) that $x = \log_n y$.
In other words, $\log_n y$ is the power to which "e" must be raised to obtain "y". It follows that the change-over from one of the LL scales to the D scale (taking the "place value" into account) provides the natural logarithm of the number to which LL has been set.

Examples:		Setting:
	ln 12 = 2.485	LL_3 12 | D: 2.485
	ln 1.635 = 0.4915	LL_2 1.635 | D: 0.4915
	ln 1.015 = 0.014 89	LL_1 1.015 | D: 0.014 89
	ln 1.003 28 = 0.003 28	LL_0 1.003 28 | D: 0.003 28
	ln 0.998 12 = −0.001 88	LL_{00} 0.998 12 | D: −0.001 88
	ln 0.938 = −0.0640	LL_{01} 0.938 | D: −0.0640
	ln 0.626 = −0.4685	LL_{02} 0.626 | D: −0.4685
	ln 0.003 = −5.81	LL_{03} 0.003 | D: −5.81

Exercises:

a	1.643	0.947	32	0.991	1.028	0.773	1.001 49	0.002
ln a								

Results:
0.4965; −0.054 45; 3.465; −0.009 04; 0.0276; −0.2575; 0.001 49; −6.215.

12.5 Powers of any desired numbers.

a^n

To calculate a^n the mark C 1 is placed above or below the number "a" on one of the LL scales. For a > 1.001 it will be found on one of the black scales LL_0, LL_1, LL_2 or LL_3, while for a < 0.9991 it will be found on one of the red scales LL_{00}, LL_{01}, LL_{02} or LL_{03}. By this setting C 1 | LL a the scale in question becomes a table for the function a^x.

If, therefore, the cursor line is placed above a number "x" on Scale C, the value a^x will be found on the LL scale concerned, underneath the cursor line.

Examples:		Setting:
	$3.1^{2.72} = 21.7$	LL_3 3.1 | C 1; C 2.72 | LL_3: 21.7
	$1.31^{3.48} = 2.53$	LL_2 1.31 | C 1; C 3.48 | LL_2: 2.53
	$1.0191^{5.35} = 1.1065$	LL_1 1.0191 | C 1; C 5.35 | LL_1: 1.1065
	$1.0013^{1.97} = 1.002\ 56$	LL_0 1.0013 | C 1; C 1.97 | LL_0: 1.002 56
	$0.9976^{4.05} = 0.990\ 32$	LL_{00} 0.9976 | C 1; C 4.05 | LL_{00}: 0.990 32
	$0.9632^{2.5} = 0.9105$	LL_{01} 0.9632 | C 1; C 2.5 | LL_{01}: 0.9105
	$0.814^{3.22} = 0.5155$	LL_{02} 0.814 | C 1; C 3.22 | LL_{02}: 0.5155
	$0.218^{1.65} = 0.081$	LL_{03} 0.218 | C 1; C 1.65 | LL_{03}: 0.081

This, however, by no means exhausts the facilities provided by the LL scales. As already explained, the scale on which the basic number "a" has been set becomes the a^x scale, no matter whether it belongs to the black or to the red group. The meaning of the remaining scales will then consequently be changed, on the following system:

The next scale outwards becomes the a^{10x} scale and the one after it (if any) the a^{100x} scale, and so forth.

The next scale inwards becomes the $a^{0.1x}$ scale, the one after it the $a^{0.01x}$ scale, and so forth. By analogy, the meaning of the other colour group is likewise changed. This will be clarified by the following examples:

1st example:

To calculate 1.35^n, C 1 is placed above LL_2 1.35. This scale thus becomes, in conjunction with the "displaced" Scale C, a table for the function 1.35^x. The corresponding scale of the red group, i.e. the scale LL_{02}, becomes a table for 1.35^{-x}. For the remaining scales we have the following scheme:

Scale:	Provides, in conjunction with scale D:	Provides, in conjunction with the "displaced" scale C:
LL_{03}	e^{-x}	1.35^{-10x}
LL_{02}	$e^{-0.1x}$	1.35^{-x}
LL_{01}	$e^{-0.01x}$	$1.35^{-0.1x}$
LL_{00}	$e^{-0.001x}$	$1.35^{-0.01x}$
LL_0	$e^{0.001x}$	$1.35^{0.01x}$
LL_1	$e^{0.01x}$	$1.35^{0.1x}$
LL_2	$e^{0.1x}$	1.35^{x}
LL_3	e^{x}	1.35^{10x}

2nd example:

To calculate 0.95^n, C 1 is placed under LL_{01} 0.95. LL_{01} thus becomes a table for 0.95^x, while the scale LL_1 corresponding to it becomes a table for 0.95^{-x}. For the remaining scales we have:

Scale:	Provides, in conjunction with scale D:	Provides, in conjunction with the "displaced" scale C:
LL_{03}	e^{-x}	0.95^{100x}
LL_{02}	$e^{-0.1x}$	0.95^{10x}
LL_{01}	$e^{-0.01x}$	0.95^{x}
LL_{00}	$e^{-0.001x}$	$0.95^{0.1x}$
LL_{0}	$e^{0.001x}$	$0.95^{-0.1x}$
LL_{1}	$e^{0.01x}$	0.95^{-x}
LL_{2}	$e^{0.1x}$	0.95^{-10x}
LL_{3}	e^{x}	0.95^{-100x}

If, in calculations with the exponential scales, it becomes necessary to move the slide through, i.e. if the base "a" has to be set with the mark C 10 instead of with the mark C 1, the mark used also assumes the "place value" of C 1, i.e. the significance of "1", while the figures on the scale C, to the left of the said mark, denote 0.9, 0.8 etc., in succession. The value of a^9, for instance, is accordingly to be found on the next scale above.

Examples:		Setting:
1.	$2.1^{0.6} = 1.561$	LL_2 2.1 | C 10; C 6 | LL_2: 1.561
2.	$2.1^{6} = 85.8$	LL_2 2.1 | C 10; C 6 | LL_3: 85.8
3.	$2.1^{-4} = 0.0514$	LL_2 2.1 | C 10; C 4 | LL_{03}: 0.0514
4.	$0.45^{-7} = 268$	LL_{02} 0.45 | C 10; C 7 | LL_3: 268

Working Sheet No. 19

1. $3.75^{2.6}$ = ...	15. $\frac{1}{0.000\,065^{0.7}}$ = ...
2. $3.75^{0.65}$ = ...	16. $170^{0.037}$ = ...
3. $3.75^{0.92}$ = ...	17. $62^{-0.084}$ = ...
4. $3.75^{-0.092}$ = ...	18. $0.000\,28^{0.012}$ = ...
5. 0.65^{22} = ...	19. $0.9485^{-0.64}$ = ...
6. $0.65^{7.5}$ = ...	20. $0.997\,82^{23}$ = ...
7. $0.65^{-0.4}$ = ...	21. $2600^{0.25}$ = ...
8. $1.36^{0.85}$ = ...	22. $13\,500^{-0.18}$ = ...
9. 1.08^{-30} = ...	23. $0.000\,46^{0.059}$ = ...
10. 1.0028^{138} = ...	24. $(6 \times 10^{-5})^{-0.075}$ = ...
11. $0.0032^{1.3}$ = ...	25. 0.847^{-9} = ...
12. $0.102^{-0.05}$ = ...	26. $(1/e)^{3.9}$ = ...
13. $1.0165^{0.085}$ = ...	27. 1.0037^{2} = ...
14. $\frac{1}{27.5^{1.35}}$ = ...	28. 1.045^{200} = ...

Answers: 1. 31.1; 2. 2.361; 3. 3.37; 4. 0.8855; 5. 7.65×10^{-5}; 6. 0.0395; 7. 1.188; 8. 1.2985; 9. 0.0994; 10. 1.471; 11. 5.7×10^{-4}; 12. 1.121; 13. 1.001 39; 14. 0.0114; 15. 850; 16. 1.209; 17. 0.707; 18. 0.9065; 19. 1.0344; 20. 0.9509; 21. 7.14; 22. 0.1805; 23. 0.6355; 24. 2.073; 25. 4.46; 26. 0.0202; 27. 1.0074; 28. 6.7×10^{3}.

12.6 Roots of any desired numbers.

$\sqrt[n]{a}$

1. Method

According to the equation $\sqrt[n]{a} = a^{1/n}$, the root is regarded as a power with a fractional index, the procedure in Sect. 12.5 then being adopted. The reciprocal 1/n is simple to find by setting the reciprocal Scale CI to "n".

Examples:		Setting:
1.	$\sqrt[3.5]{40} = 2.87$	LL_3 40 | C 10; CI 3 5 | LL_3: 2.87
2.	$\sqrt[7]{1.24} = 1.0312$	LL_2 1.24 | C 1; CI 7 | LL_1 1.0312
3.	$\sqrt[2.8]{0.004} = 0.139$	LL_{03} 0.004 | C 10; CI 2.8 | LL_{03}: 0.139
4.	$\frac{1}{\sqrt[6.3]{2.35}} = 2.35^{-\frac{1}{6.3}} = 0.873$	LL_2 2.35 | C 10; CI 6.3 | LL_{02}: 0.873

2. Method

From $\sqrt[n]{a} = b$ it follows that $b^n = a$. To find the value "b" of which the n^{th} power will be "a":

Examples:		Setting:
1.	$\sqrt[3.2]{35} = 3.04$	LL_3 35 | C 3.2; C 1 | LL_3: 3.04
2.	$\sqrt[6]{9.4} = 1.453$	LL_3 9.4 | C 6; C 10 | LL_2: 1.453
3.	$\sqrt[0.45]{0.724} = 0.488$	LL_{02} 0.724 | C 0.45; C 10 | LL_{02}: 0.488
4.	$\frac{1}{\sqrt[3.8]{15.5}} = 0.486$	LL_3 15.5 | C 3.8; C 10 | LL_{02}: 0.486

Exercises:

1. $\sqrt[7.5]{420}$ = ...
2. $\sqrt[1.26]{1.074}$ = ...
3. $\sqrt[2.6]{1.19}$ = ...
4. $\sqrt[0.073]{1.0032}$ = ...
5. $\sqrt[1.6]{0.9982}$ = ...
6. $\sqrt[5.8]{0.954}$ = ...
7. $\sqrt[25]{0.79}$ = ...
8. $\sqrt[18]{0.034}$ = ...
9. $\sqrt[6.9]{4 \times 10^{-5}}$ = ...
10. $\sqrt[0.43]{0.582}$ = ...
11. $\frac{1}{\sqrt[3.8]{9.4}}$ = ...
12. $\frac{1}{\sqrt[9.3]{0.971}}$ = ...

Answers: 1. 2.238; 2. 1.0583; 3. 1.0692; 4. 1.0448; 5. 0.998 874; 6. 0.9919; 7. 0.990 61; 8. 0.8287; 9. 0.23; 10. 0.284; 11. 0.554; 12. 1.003 17.

12.7 Logarithms to any desired base.

$\log_b a$

From $b^n = a$, it follows that $n = \log_b a$.

${}^b\log a$ is thus the number which must be raised to the b^{th} power to obtain "a". It follows that there is a simple method of calculating logarithms to any desired base by means of the exponential scales.

Examples:		Setting:
1.	$\log_3 26 = 2.965$	LL_3 3 | C 1; LL_3 26 | C: 2.965
2.	$\log_{2.5} 1.3 = 0.2865$	LL_2 2.5 | C 10; LL_2 1.3 | C: 0.2865
3.	$\log_{1.7} 0.85 = -0.306$	LL_2 1.7 | C 10; LL_{02} 0.85 | C: −0.306
4.	$\log_{0.6} 8 = -4.07$	LL_{02} 0.6 | C 10; LL_3 8 | C: −4.07

Exercises:

1. $\log_{3.5} 120$ = ...
2. $\log_{1.2} 52$ = ...
3. $\log_{1.08} 6.8$ = ...
4. $\log_{1.002} 2400$ = ...
5. $\log_{30} 4.5$ = ...
6. $\log_{9.5} 1.04$ = ...
7. $\log_{1.85} 0.54$ = ...
8. $\log_{0.7} 1.95$ = ...

Results: 1. 3.82; 2. 21.65; 3. 24.9; 4. 3895; 5. 0.442; 6. 0.0174; 7. −1.001; 8. −1.872.

12.8 Common logarithms.

$\log a$

The process described in 12.7 can naturally also be used for determining logarithms to base 10. By comparison with the method explained in 11, it offers the advantage that it provides not only the mantissa but the characteristic at the same time. In addition, the process is more accurate – sometimes considerably more so – if the antilogarithm is between 1/e (= 0.368) and e (= 2.718); in other words, if the antilogarithm is to be found on one of the scales LL_{02}, LL_{01}, LL_{00}, LL_0, LL_1, LL_2 (i.e. all the exponential scales except the uppermost and the bottom one).

Examples: 1. log 1.94 = 0.288   2. log 1.0515 = 0.0218

3. log 0.9526 = −0.0211   4. log 0.462 = −0.335

The advantage of the greater accuracy, moreover, can also be extended to all the numbers of which the figure sequences (disregarding the "place value") are to be found on the six scales mentioned, i.e. of which the figure sequences are between 1-0-0-1 and 2-7-1-8 or between 3-6-8 and 9-9-9-1 – although the user must then determine the characteristic himself.

Examples:

(1) To find log 1058.5. The logarithm sought has the same mantissa as log 1.0585, but its characteristic is greater by 3. We find log 1.0585 = 0.0247, from which log 1058.5 = 3.0247 is then found.

(2) To find log 236. log 2.36 = 0.373, so that log 236 = 2.373.

(3) To find log 694. log 0.694 = −0.1586, and log 694 = log 0.694 + 3 = 2.8414.

(4) To find log 9928 5. log 0.99285 = −0.003115, and log 9928 5 = −0.003115 + 4 = 3.996885.

In favourable cases, therefore, the same accuracy can be obtained as with a table extending to five or six digits.

Exercises:

1. log 19.7 = ...
2. log 102.4 = ...
3. log 1570 = ...
4. log 67 = ...
5. log 9235 = ...
6. log 998.37 = ...

Results: 1. 1.2945; 2. 2.010 30; 3. 3.1959; 4. 1.8261; 5. 3.9654; 6. 2.999 291.

12.9 Logarithms to base 2.

lb a

As the left-hand and right-hand side lines on the back of the cursor are at the same distance from each other as mark 2 and mark "e" on Scale LL_2, the logarithm of number to base 2 can be found very rapidly with these two cursor lines: Place the left-hand side line above the antilogarithm on one of the LL scales; the logarithm to base 2 will then immediately be found on Scale D, underneath the right-hand line.

Examples:

1. lb 8 = 3 — Setting: Left-hand line on LL_3 8. Underneath right-hand line on D: 3.
2. lb 1.48 = 0.566 — Left-hand line on LL_2 1.48. Underneath right-hand line on D: 0.566.
3. lb 0.638 = −0.648 — Left-hand line on LL_{02} 0.638. Underneath right-hand side line on D: −0.648.

12.10 Production of logarithmic diagrams to any desired scale.

1st example: To produce a diagram, a distance of 9 cm is to be subdivided logarithmically from 1 to 15. The initial point of the distance is thus to correspond to log 1 = 0 and the final point to log 15. As the logarithms of different systems are proportional to one another, the subdivision is independent of the system selected; we will therefore select the most convenient system, i.e. that of natural logarithms.

Statement (set-up): ln 15 ≙ 9 cm,

$$\frac{\ln 15}{9\text{ cm}} = \frac{\ln a}{x\text{ cm}}$$

In the above, x cm is the distance of the division point, a, from the beginning of the distance concerned.

Setting: LL_3 15 | C 9
LL a | C: x

With this setting we find the following (if necessary, after moving the slide through):

Division point a	1	2	3	4	5	6	7	8
x	0	2.3	3.65	4.61	5.35	5.95	6.46	6.91

Division point a	9	10	11	12	13	14	15
x	7.3	7.65	7.97	8.26	8.52	8.77	9

Fig. 30

2nd example: A distance of 16 cm in length is to be subdivided logarithmically from 0.005 to 10. In this case the graduation extends from zero point (log 1 = 0) to the right, as far as log 10 = 2.3, and to the left, as far as log 0.005 = −5.29.

The sum of these two amounts, i.e. 2.3 + 5.29 = 7.59, corresponds to the distance s = 16 cm. From the proportion $\frac{7.59}{16\text{ cm}}$ we obtain the distance of the zero point (division point 1)

from the left-hand edge, i.e. $5.29 \times \frac{16\text{ cm}}{7.59} = 11.15$ cm, and

from the right-hand edge, i.e. $2.3 \times \frac{16\text{ cm}}{7.59} = 4.85$ cm (to verifiy: 11.15 + 4.85 = 16).

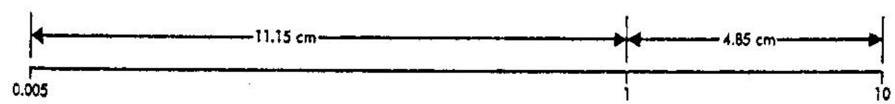


Fig. 31

The distances of the individual graduation points from point 1 are now calculated:

Setting: LL_3 10 | C 4.85
LL a | C: x

provides:

Graduation point a	2	3	4	5	6	7	8	9	10
x	1.46	2.31	2.92	3.39	3.77	4.1	4.38	4.53	4.85

Setting: LL_{03} 0.005 | C 11.15
LL a | C: x

provides:

Graduation point a	0.005	0.01	0.02	0.03	0.04	0.05	0.1	0.2
−x	11.15	9.69	8.23	7.38	6.77	6.3	4.85	3.38

Graduation point a	0.3	0.4	0.5
−x	2.53	1.93	1.46

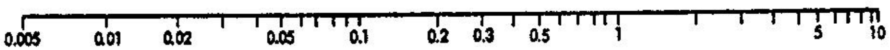


Fig. 32

Fundamentally, it would suffice to calculate one "decade" (e.g. from 1 to 10) and to subdivide the others in the same manner with a pair of dividers. (This has been done in Fig. 32 for the ranges between 0.05 and 0.1 and between 0.5 and 1).

Exercises: A distance of 8 cm in length is to be subdivided logarithmically from 5 to 100.

Graduation point a	5	6	7	8	9	10	20	30	40	50	100
Distance from grad. point 1											

Result:

Statement (set-up): The distance 8 cm etc. corresponds to the difference between log 100 and log 5 (natural logarithms).

Graduation point a	5	6	7	8	9	10	20	30	40	50	100
Distance from grad. point 1	4.3	4.78	5.2	5.55	5.87	6.15	8.0	9.09	9.85	10.45	12.3

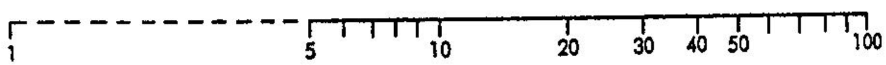


Fig. 33

12.11 Formation of reciprocal values.

The exponential scales are eminently suitable for the convenient calculation of reciprocals, because, in contradistinction to the reciprocal scales, they also provide the correct "place-value". The calculation of the reciprocal is carried out in a simple manner, by the change-over from a LL scale to the corresponding scale of the other group of scales, e.g. from LL_2 to LL_{02} or from LL_{03} to LL_3 etc.

Examples: 1. $1/23.5 = 0.0426$ Setting: LL_3 23.5 | LL_{03}: 0.0426
2. $1/0.9435 = 1.0599$ LL_{01} 0.9435 | LL_1: 1.0599

As regards the degree of accuracy obtained, the remarks made under 12·8 apply here again: the method is more accurate – in some cases far more so – than the use of the reciprocal scales, if the number in question does not happen to be situated on one of the outer scales (LL_3 and LL_{03}).

Exercises:

a	34	1.252	1.068	0.9966	0.0022	1002.14	107.4
1/a							

Results: 0.0294; 0.7985; 0.9396; 0.001 481; 450; 0.997 86; 0.009 311.

12.12 Tenth and hundredth powers; tenth and hundredth roots.

a^{10}
a^{100}
$\sqrt[10]{a}$
$\sqrt[100]{a}$

In the change-over from an exponential scale to the next scale outwards, the number selected is raised to the power of ten, while in the change-over to the next scale beyond that it is raised to the power of a hundred, and so forth.

Examples: 1. $1.02^{10} = 1.219$ Setting: LL_1 1.02 | LL_2: 1.219
2. $\sqrt[10]{0.029} = 0.702$ LL_{03} 0.029 | LL_{02}: 0.702

13. Conversions of complex numbers

A complex number $z = a + jb$ (normal form or algebraical form) can, as is known, be represented as follows:

$$z = r(\cos\varphi + j\sin\varphi) \qquad \text{(trigonometrical form),}$$

$$z = re^{j\varphi} \qquad \text{(exponential form),}$$

$$\text{and} \quad z = r\underline{/\varphi} \qquad \text{(vectorial form).}$$

The magnitudes occuring are designated as follows:

a = real part, jb = imaginary part, r = amount, φ = argument of complex number z.

The relationships between them are as follows:

$$r = \sqrt{a^2 + b^2}, \quad \varphi = \arctan\frac{b}{a}$$

$$a = r\cos\varphi \quad \text{and} \quad b = r\sin\varphi$$

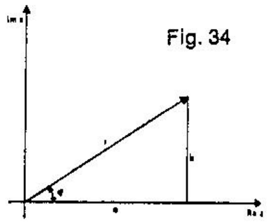


Fig. 34

As for additions and subtractions of complex numbers the normal form is the most favourable, whereas for all other calculations the vectorial form (or the equivalent exponential form) is the most suitable, calculations with complex numbers often involve recourse to conversions. Each of them can be carried out with one movement of the slide only.

1. Conversion of normal form $a + jb$ into the vectorial form $\underline{r/\varphi}$.

To calculate φ from a and b, we use the equation $\tan\varphi = b/a$ and replace the division b/a by a multiplication by the reciprocal: $\tan\varphi = b \times \frac{1}{a}$. From the angle φ and the equation $\sin\varphi = b/r = b \times \frac{1}{r}$ we can then find the value of "r" with one movement of the cursor only.

Setting: D b | CI 10 (CI 1)

CI a | T: φ { T_1 if $b/a < 1$; T_2 if $b/a > 1$ }

S φ | CI: r

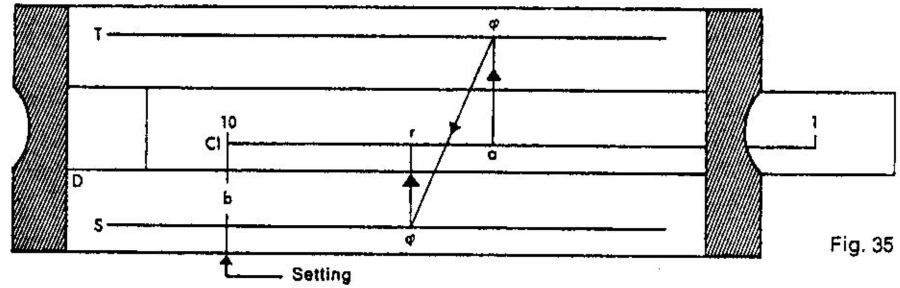


Fig. 35

Examples: (1) $z = 23 + j18$ Setting: D 18 | CI 10
CI 23 | T_1: 38.05°
S 38.05° | CI: 29.2

Answer: $z = 23 + j18 = \underline{29.2/38.5°}$

(2) $z = 16 + j24$ Setting: D 24 | CI 1
CI 16 | T_2: 56.3°
S 56.3° | CI: 28.8

Answer: $z = 16 + j24 = \underline{28.8/56.3°}$

In the above, it was first assumed that "a" and "b" were both positive, so that the number "z" would be situated in the 1st quadrant of the Gauss plane. If this is not the case, we calculate with the absolute values of "a" and "b" and determine the angle $\varphi' = \arctan\frac{|b|}{|a|}$ in the manner described, after which the argument φ then provides, according to the position of z, one of the following:

Fig. 36.1 z in 2nd quadrant

$a < 0;\ b > 0$

$\varphi = 180° - \varphi'$

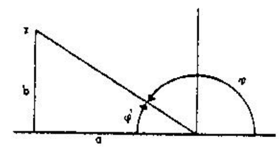


Fig. 36.2 z in 3rd quadrant

$a < 0;\ b < 0$

$\varphi = 180° + \varphi'$

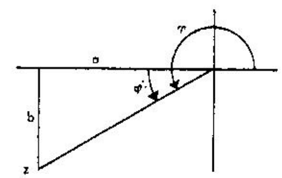
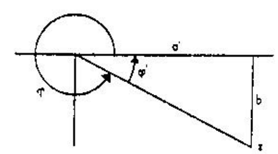


Fig. 36.3 z in 4th quadrant

$a > 0;\ b < 0$

$\varphi = 360° - \varphi'$



Examples: (1) $z = -5 + j\,4$ $\tan\varphi' = 4/5$ Setting: D 4 | CI 10
CI 5 | T_1: 38.65° $(= \varphi')$
S 38.65° | CI: 6.4
$\varphi = 180^\circ - \varphi' = 141.35^\circ$

Result: $z = -5 + j\,4 = 6.4\underline{/141.35^\circ}$

(2) $z = -5 - j\,6$ $\tan\varphi' = 6/5$ Setting: D 6 | CI 1
CI 5 | T_2: 50.2°
S 50.2° | CI: 7.8
$\varphi = 180^\circ + \varphi' = 230.2^\circ$

Result: $z = -5 - j\,6 = 7.8\underline{/230.2^\circ}$

Exercises:

$z_1 = 7 - j\,13 =$ .... $z_3 = -34 - j\,29 =$ ....

$z_2 = -5.6 + j\,3.8 =$ .... $z_4 = 19 + j\,25.4 =$ ....

Results: $z_1 = 14.76\underline{/298.3^\circ}$; $z_2 = 6.77\underline{/145.85^\circ}$; $z_3 = 44.7\underline{/220.5^\circ}$; $z_4 = 31.7\underline{/53.2^\circ}$.

2. Conversion from vectorial form $r\underline{/\varphi}$ to normal form $a + j\,b$.

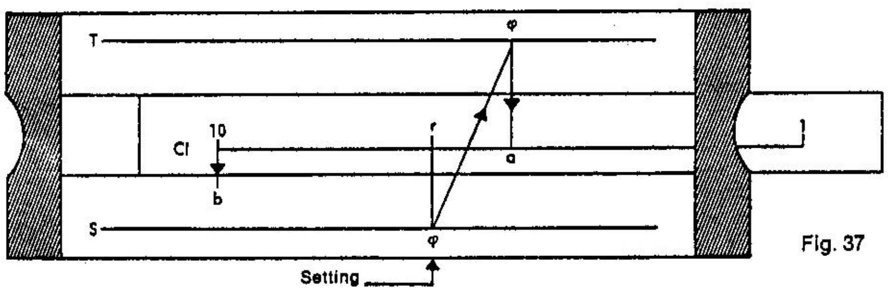


Fig. 37

Setting: S φ | CI r
CI 10 (CI 1) | D: b
T φ | CI: a

Examples: (1) $z = 4.4/32^\circ$ Setting: S 32° | CI 4.4
CI 10 | D: 2.33 = b
T_1 32° | CI: 3.73 = a

Result: $z = 4.4\underline{/32^\circ} = 3.73 + j\,2.33$

(2) $z = 26/59^\circ$ Setting: S 59° | CI 26
CI 10 | D: 22.3 = b
After moving slide through: T_2 59° | CI: 13.4 = a

Result: $z = 26\underline{/59^\circ} = 13.4 + j\,22.3$

If "z" is not situated in the 1st quadrant, we first of all calculate φ' (see Fig. 36) and then calculate the values of "a" and "b" therefrom. The signs for "a" and "b" are evident from the position of "z".

Examples: (1) $z = 47\underline{/163^\circ}$ $\varphi' = 180^\circ - \varphi = 17^\circ$; Setting: S 17° | CI 47
CI 10 | D: 13.74 = |b|
T_1 17° | CI: 44.9 = |a|

As z is in the 2nd quadrant, $a < 0$, $b > 0$.

Result: $z = 47\underline{/163^\circ} = -44.9 + j\,13.74$

(2) $z = 15.6\underline{/218^\circ}$ $\varphi' = \varphi - 180^\circ = 38^\circ$; Setting: S 38° | CI 15.6
CI 1 | D: 9.60 = |b|
T_1 38° | CI: 12.29 = |a|

As z is in the 3rd quadrant, $a < 0$, $b < 0$.

Result: $z = 15.6\underline{/218^\circ} = -12.29 - j\,9.60$

Exercises: $z_1 = 27.4\underline{/64^\circ} =$ .... $z_3 = 32\underline{/229^\circ} =$ ....

$z_2 = 9.3\underline{/112^\circ} =$ .... $z_4 = 56\underline{/286^\circ} =$ ....

Answers: $z_1 = 12.01 + j\,24.65$; $z_2 = -3.485 + j\,8.625$; $z_3 = -21 - j\,24.15$; $z_4 = 53.8 - j\,15.43$.

14. The special scales of the CASTELL 2/82 N

14.1 The Scale BI

The reciprocal square scale BI is a simplified means of calculating expressions such as $a\sqrt{b}$ (without moving the slide through to the end), $\sqrt{a \times b \times c}$ and other compound expressions with roots and squares such as occur in particular in constructional engineering.

Examples:

1. $4.1\sqrt{39}$ $\left(= 4.1 \div \frac{1}{\sqrt{39}}\right)$ Setting: D 4.1 | BI 39
   C 1 | D: 25.6
2. $\sqrt{2.85 \times 7.4 \times 53}$ $\left(= \sqrt{2.85 \div \frac{1}{7.4} \times 53}\right)$ Setting: A 2.85 | BI 7.4
   B 53 | D: 33.4
3. $1.95 \times \sqrt{62} \times \sqrt{14.5}$ $\left(= 1.95 \div \frac{1}{\sqrt{62}} \times \sqrt{14.5}\right)$ Setting: D 1.95 | BI 62
   B 14.5 | D: 58.5
4. $\frac{1}{(1.4 \times 4.8)^2}$ Setting: D 1 | C 1.4 (multiplication in reversed order)
   D 4.8 | BI: 0.022 15
5. $\frac{2.92^2}{16.4 \times 0.36}$ $\left(= \frac{2.92^2}{16.4} \cdot \frac{1}{0.36}\right)$ Setting: D 2.92 | B 16.4
   BI 0.36 | A: 1.445
6. $56 \times 2.8 \times 0.47^2$ $\left(= 56 \div \frac{1}{2.8} \times 0.47^2\right)$ Setting: A 56 | BI 2.8
   C 0.47 | A: 34.6
7. Table of the function $\frac{21.2}{\sqrt{x}}$ Setting: D 21.2 | C 1
   BI x | D: $\frac{21.2}{\sqrt{x}}$

x	1.2	4	7	12	82	850	2400
$\frac{21.2}{\sqrt{x}}$	19.35	10.6	8.01	6.12	2.34	0.727	0.433

For exercises, see Working Sheet 16 (page 82).

14.2 The Scale K'

The movable cube scale K' enables compound expressions with 3rd powers and cube roots of the following form to be calculated, in conjunction with Scale K:

$$\frac{a^3}{b}, \quad \frac{a \times b^3}{c}, \quad \frac{a \times b}{c^3}; \quad \sqrt[3]{a \times b}; \quad \sqrt[3]{\frac{a}{b}} \text{ etc.}$$

In this process, multiplication and division are carried out with the pair of scales K/K' exactly as with pairs A/B and C/D, although with reduced accuracy.

Examples:

1. $\frac{4.85^3}{62} = 1.84$; Setting: D 4.85 | K' 62
   K' 1 | K: 1.84
2. $\frac{305 \times 1.86^3}{245} = 8.0$; Setting: K 305 | K' 245
   C 1.86 | K: 8.0
3. $\frac{78 \cdot 46}{5.2^3} = 25.5$; Setting: K 78 | C 5.2
   K' 46 | K: 25.5
4. $\sqrt[3]{16.2 \times 510} = 20.2$; Setting: K 16.2 | K' 1000
   K' 510 | D: 20.2

As always when extracting roots, attention must be paid to the correct "decade" of the cube scale for the setting of the radicand. To ensure that in this exercise the radicand (16.2 x 510) will always be in the correct "decade", the two factors must be set in the "decade" of the correct "place-value" in each case, 16.2 thus being set in the second and 510 in the third "decade". In addition, only the mark K' 1 or K' 1000 – but not K' 100 – must be used for setting the slide to the first factor.

If a factor is not between 1 and 1000, it will once again be necessary to "split off" a suitable power of ten.

5. $\sqrt[3]{0.072 \times 1800} = \sqrt[3]{72 \times 10^{-3} \times 1.8 \times 10^3} = \sqrt[3]{72 \times 1.8} = 5.06$
   Setting: K 72 | K' 1
   K' 18 | D: 5.06
6. $\sqrt[3]{\frac{48.5}{126}} = 0.7275$; Setting: K 48.5 | K' 126
   K' 1000 | D: 0.7275
7. $\frac{\left(\frac{7.4}{13.6}\right)^{2/3}}{0.375} = 1.78$; Setting: K 7.4 | K' 13.6
   K' 1000 | (A: 0.666) | B 0.375
   B 1 | A: 1.78
8. $\frac{\left(\frac{57}{8.3}\right)^{2/3} \cdot 48.5}{2.9} = 60.4$; Setting: K 57 | K' 8.3
   K' 1 | (A: 3.61) | B 2.9
   B 48.5 | A: 60.4
9. $\left(\frac{19.4}{61}\right)^{3/2} \times 235 = 42.1$; Setting: A 19.4 | B 61
   K' 235 | K: 42.1
10. $\frac{2.62^4}{1.86^3} = \frac{2.62^3 \times 2.62}{1.86^3} = 7.32$; Setting: D 2.62 | C 1.86
    K' 2.62 | K: 7.32

Exercises will be found on Working Sheet No. 20 (Problems 1-12).

14.3 Scale S'

The movable sine scale S' simplifies the calculation of products and quotients of trigonometric functions if at least one of these is a sine or cosine function.

Examples:

1. $\sin 54.5° \times \sin 18.3° = 0.2555$; Setting: S 54.5° | C 10; S' 18.3° | D: 0.2555
2. $\sin 9.6° \times \sin 63.2° = 0.0752$; Setting: S 9.6° | C 1; S' (red) 63.2° | D: 0.0752
3. $\sin 2.6° \times \sin 38° = 0.0279$; Setting: ST 2.6° | C 10; S 38° | D: 0.0279
4. $\sin 21° \times \tan 34.2° = 0.244$; Setting: T_1 34.2° | C 10; S' 21° | D: 0.244
5. $\tan 68.5° \times \cos 55° = 1.456$; Setting: T_2 68.5° | C 10; S' (red) 55° | D: 1.456
6. $\sin 14.5° \div \sin 43° = 0.367$; Setting: S 14.5° | S' 43°; C 10 | D: 0.367
7. $\tan 54° \div \sin 28° = 2.93$; Setting: T_2 54° | S' 28°; C 10 | D: 2.93
8. $\sin 33° \div \tan 23° = 1.283$; Setting: T_1 23° | S' 33°; D 1 | C: 1.283
9. $\dfrac{\sin 4.1° \times \cos 79.6°}{\sin 36°} = 0.021\,95$; Setting: ST 4.1° | S' 36°; S' (red) 79.6° | D: 0.021 95
10. $\dfrac{\tan 26.5° \times \cos 39°}{\cos 62°} = 0.825$; Setting: T_1 26.5° | S' (red) 62°; S'(red) 39° | D: 0.825
11. $\sin^2 63° \times \cos^2 71° = 0.0841$; Setting: S 63° | C 10; S' (red) 71° | A: 0.0841
12. $\dfrac{\cot^2 11°}{\sin^2 38°} = 69.8$; Setting: T_2 (red) 11° | S' 38°; B 100 | A: 69.8

Now solve the remaining problems on Working Sheet No. 20.

Working Sheet No. 20

1. $\dfrac{0.041^3}{18.2} =$ ....
2. $\dfrac{52.5^3}{26\,000} =$ ....
3. $\dfrac{17.9^3 \times 7.6}{480} =$ ....
4. $\dfrac{0.23 \times 11.5^3}{2850} =$ ....
5. $\dfrac{335 \cdot 760}{61^3} =$ ....
6. $\dfrac{8.2^3 \times 0.76}{4.4^3} =$ ....
7. $\dfrac{12.4^3 \times 0.62^3}{1420} =$ ....
8. $\dfrac{14.2}{2.94^3 \times 1.67^3} =$ ....
9. $\sqrt[3]{19.6 \times 0.65} =$ ....
10. $\sqrt[3]{\dfrac{128 \times 21.5}{3.6}} =$ ....
11. $\sqrt[3]{\dfrac{0.46 \times 630}{5.7}} =$ ....
12. $\sqrt[3]{\dfrac{2900 \times 31\,000}{0.048}} =$ ....
13. $\sqrt[3]{92} \times \sin 30.4° =$ ....
14. $\dfrac{\sqrt[3]{255}}{\cos 72°} =$ ....
15. $\dfrac{7.4}{\sin^3 46.5°} =$ ....
16. $285 \times \cos^3 53.2° =$ ....
17. $\cot 24.5° \times \cos 62.2° =$ ....
18. $\sin^2 43.5° \times \cos 58.4° =$ ....
19. $\dfrac{1}{\sin^2 26.8°} =$ ....
20. $\dfrac{\tan 38.7° \times \sin 29.2°}{\cos 47°} =$ ....

Results: 1. 3.79×10^{-6}; 2. 5.57; 3. 90.8; 4. 0.1225; 5. 1.12; 6. 4.92; 7. 0.320; 8. 0.120; 9. 2.335; 10. 9.14; 11. 3.70; 12. 1.23×10^3; 13. 2.285; 14. 20.5; 15. 19.4; 16. 61.3; 17. 1.025; 18. 0.2485; 19. 4.92; 20. 0.573.

15. Summary
Illustration of most important calculations

15.1 Explanatory remarks

The following summary shows how the most important types of calculation can be carried out with only one movement of the slide and of the cursor in each case (and possibly with the cursor only). The symbols used for the purpose will once again be explained below:

(1) Calculations with only one movement of the cursor.

Example: For the calculation of $a^{3/2} = (\sqrt{a})^3$ the central cursor line is placed at the value "a" on Scale B (Symbol: A a), after which the result is read underneath the cursor line, on Scale K (Symbol: K: $a^{3/2}$). This is summarized as follows:

A a | K: $a^{3/2}$

The vertical stroke denotes that the two values, "a" and "$a^{3/2}$", are vertically opposite each other.

If, as in the following table, the operation in question has already been mentioned before, the repetition thereof can be dispensed with. One then uses the following notation:

$a^{3/2}$	A a | K:

(2) Operations with one movement of the slide.

Example: To find the product a × b:

Illustration:

a × b	D a | C 1 C b | D:	i.e. place mark C 1 vertically above D a. i.e. the result is then shown on Scale D, underneath C b.

15.2 Powers

		With 2/83 N likewise (more exact):	Explanation of calculation method:
a^2	D a | A:	W a | D:	
a^3	D a | K:		
	D a | BI a B 1 | A:	W a | CI a C 1 | D:	$(a \cdot \sqrt{a})^2$
a^4	D a | CI a B 1 | A:		$(a \div \frac{1}{a})^2$
	K a | CI a C 1 | K:		$(\sqrt[3]{a} \div \frac{1}{a})^3$
		W a | A:	$(a^2)^2$
		W a | W' 1 W' a | D:	$(a \cdot a)^2$
a^5	D a | CI a B a | A:		$(a \div \frac{1}{a})^2 \cdot a$
a^6	D a | C 1 C a | K:		$(a \cdot a)^3$
		W a | K:	$(a^2)^3$
$a b^2$	A a | B 1 C b | A:	D a | C 1 W b | D:	
$a^2 b$	D a | C 1 B b | A:	W a | W' 1 C b | D:	
$a b^3$	K a | B 1 C b | A:		
$a b^4$	A a | CI b C b | A:		$(a \div \frac{1}{b^2}) \cdot b^2$
	D b | BI a C b | A:	W b | CI a W' b | D:	$(b^2 \div \frac{1}{a}) \cdot b^2$
$a^2 b^2$	D a | C 1 C b | A:	W a | W' 1 W' a | D:	$(a \cdot b)^2$
	D a | CI b C 1 | A:		$(a \div \frac{1}{b})^2$

		With 2/83 N likewise (more exact):	Explanation of calculation method:
$a^2 b^3$	D a | Cl b B b | A:		$(a \div \frac{1}{b})^2 \cdot b$
$a^2 b^2 c$	A c | Cl a C b | A:		$(c \div \frac{1}{a^2}) \cdot b^2$
	D a | Bl c C b | A:	W a | Cl c W' b | D:	$(a \div \frac{1}{c}) \cdot b^2$
$a^2 b^3 c$	K c | Cl a C b | K:		$(c \div \frac{1}{a^2}) \cdot b^3$
$\frac{a}{b^2}$	A a | C b C 1 | A:	D a | W' b W' 1 | D:	
	A a | B 1 Cl b | A:		$a \cdot \frac{1}{b^2}$
$\frac{a}{b^3}$	K a | C b C 1 | K:		
$\frac{a^2}{b^3}$	D a | C b Bl b | A:	W a | W' b Cl b | D:	$\left(\frac{a}{b}\right)^2 \cdot \frac{1}{b}$
	D a | B b Cl b | A:		$\frac{a^2}{b} \cdot \frac{1}{b^2}$
$\frac{a^2}{b}$	D a | B b B 1 | A:	W a | C b C 1 | A:	
$\frac{a^3}{b}$	D a | B b B a | A:	W a | C b C a | D:	$\left(\frac{a^2}{b}\right) \cdot a$
	D a | K' b K' 1 | K:		
$\frac{a^3}{b^2}$	A a | Cl a Cl b | A:		$a \div \frac{1}{a^2} \cdot \frac{1}{b^2}$
$\frac{1}{a^2}$	C a | Bl:	W a | Dl:	
	Dl a | A:		
	Cl a | B:		

		With 2/83 N likewise (more exact):	Explanation of calculation method:
$\frac{1}{a^3}$	Dl a | K:		
	Cl a | K':		
$\frac{1}{a^4}$	D 1 | C a Cl a | A:		$\left(\frac{1}{a} \cdot \frac{1}{a}\right)^2$
	D 1 | C a D a | Bl:	W 1 | W' a W a | Cl:	$(a \cdot a)^{-2}$
		W 1 | W' a W' 1 | A:	$\left(\frac{1}{a}\right)^4$
$\frac{1}{a \cdot b}$	D 1 | C a Cl b | D:		$\frac{1}{a} \cdot \frac{1}{b}$
$\frac{1}{ab^2}$	A 1 | B a Cl b | A:		$\frac{1}{a} \cdot \frac{1}{b^2}$
		Dl a | W' b W' 1 | D:	$\frac{1}{a} : b^2$
$\frac{1}{a^2b^2}$	D 1 | C a Cl b | A:		$\left(\frac{1}{a} \cdot \frac{1}{b}\right)^2$
		W a | W' 1 W' b | Dl:	$(a \cdot b)^{-2}$
$\frac{1}{a^3b^3}$	K a | C b Cl a | K:		$\frac{a}{b^3} \cdot \frac{1}{a^3}$

15.3 Roots

<table>
<tr><th></th><th></th><th>With 2/83 N likewise (more exact):</th><th>Explanation of calculation method:</th></tr>
<tr><td>$\sqrt{a}$</td><td>A a | D:</td><td>D a | W:</td><td></td></tr>
<tr><td>$\sqrt{a^2}$</td><td>A a | K:</td><td></td><td></td></tr>
<tr><td>$\sqrt[3]{a}$</td><td>K a | D:</td><td></td><td></td></tr>
<tr><td>$\sqrt[3]{a^2}$</td><td>K a | A:</td><td></td><td></td></tr>
<tr><td>$\sqrt[4]{a}$</td><td></td><td>A a | W:</td><td></td></tr>
<tr><td>$\sqrt{a \cdot b}$</td><td>A a | B 1
B b | D:</td><td>D a | C 1
C b | W:</td><td></td></tr>
<tr><td>$\sqrt{a \cdot b \cdot c}$</td><td>A a | Bl b
B c | D:</td><td>D a | Cl b
C b | W:</td><td></td></tr>
<tr><td>$a\sqrt{b}$</td><td>D a | C 1
B b | D:</td><td>W a | W' 1
C b | W:</td><td></td></tr>
<tr><td rowspan="3">$a^2\sqrt{b}$</td><td>A b | Cl a
C a | D:</td><td></td><td>$(\sqrt{b} \div \frac{1}{a}) \cdot a$</td></tr>
<tr><td></td><td>W a | W' 1
B b | D:</td><td></td></tr>
<tr><td></td><td>W a | Cl b
W' a | W:</td><td>$(a \div \frac{1}{\sqrt{b}}) \cdot a$</td></tr>
<tr><td>$\sqrt{\frac{a}{b}}$</td><td>A a | B b
B 1 | D:</td><td>D a | C b
C 1 | W:</td><td></td></tr>
<tr><td>$\frac{\sqrt{a}}{b}$</td><td>A a | C b
C 1 | D:</td><td>D a | W' b
W' 1 | W:</td><td></td></tr>
<tr><td>$\frac{a}{\sqrt{b}}$</td><td>D a | B b
C 1 | D:</td><td>W a | C b
W' 1 | W:</td><td></td></tr>
<tr><td>$\sqrt{\frac{a \cdot b}{c}}$</td><td>A a | B c
B b | D:</td><td>D a | C c
C b | W:</td><td></td></tr>
<tr><td>$\sqrt{\frac{a}{b \cdot c}}$</td><td>A a | B b
Bl c | D:</td><td>D a | C b
Cl c | W:</td><td></td></tr>
</table>

<table>
<tr><th></th><th></th><th>With 2/83 N likewise (more exact):</th><th>Explanation of calculation method:</th></tr>
<tr><td rowspan="3">$\frac{1}{\sqrt{a}}$</td><td>Bl a | C:</td><td>Dl a | W:</td><td></td></tr>
<tr><td>B a | Cl:</td><td></td><td></td></tr>
<tr><td>A a | Dl:</td><td></td><td></td></tr>
<tr><td rowspan="2">$\frac{1}{\sqrt{a \cdot b}}$</td><td>A 1 | B a
A b | Cl:</td><td></td><td rowspan="2">$\sqrt{\frac{1}{a} \cdot \frac{1}{b}}$</td></tr>
<tr><td>A a | B 1
B b | Dl:</td><td>Dl a | Cl 1
Cl b | W:</td></tr>
<tr><td rowspan="2">$\frac{1}{a\sqrt{b}}$</td><td>A b | Cl a
D 1 | C:</td><td></td><td>$(b : \frac{1}{a})$</td></tr>
<tr><td>D 1 | C a
Bl b | D:</td><td>W 1 | W' a
Cl b | W:</td><td>$\frac{1}{a} \cdot \sqrt{\frac{1}{b}}$</td></tr>
</table>

15.4 Operations with trigonometric functions

Note: All the following calculations with Scale S′ can be carried out, in the same manner, with cos α instead of with sin α; all operations for which the Scale S′ is not required, can be performed, analogously, with cos α, tan α and cot α likewise, instead of with sin α or sin β.

<table>
<tr><th></th><th></th><th>With 2/83 N only</th><th>With 2/83 N only</th></tr>
<tr><td>$a \cdot \sin\alpha$</td><td>S α | C 1
C a | D:</td><td></td><td></td></tr>
<tr><td>$(a \cdot \sin\alpha)^2$</td><td>S α | C 1
C a | A:</td><td></td><td></td></tr>
<tr><td>$\sqrt{a \cdot \sin\alpha}$</td><td></td><td>S α | C 1
C a | W:</td><td></td></tr>
<tr><td>$\frac{a}{\sin\alpha}$</td><td>S α | C a
D 1 | C:</td><td></td><td>C a | S′ α
C 1 | D:</td></tr>
<tr><td>$\frac{\sin\alpha}{a}$</td><td>S α | C a
C 1 | D:</td><td></td><td></td></tr>
<tr><td>$\frac{a}{b}\sin\alpha$</td><td>S α | C b
C a | D:</td><td></td><td></td></tr>
<tr><td>$\sin\alpha \ \sin\beta$</td><td></td><td></td><td>S α | C 1
S′ β | D:</td></tr>
<tr><td>$\frac{\sin\alpha}{\sin\beta}$</td><td></td><td></td><td>S α | S′ β
C 1 | D:</td></tr>
<tr><td>$\frac{1}{\sin\alpha}$</td><td></td><td>S α | DI:</td><td>S′ α | CI:</td></tr>
<tr><td>$\frac{1}{\sin^2\alpha}$</td><td>S α | C 1
A 1 | B:</td><td></td><td>S α | BI:</td></tr>
<tr><td>$(\sin\alpha \sin\beta)^2$</td><td></td><td></td><td>S α | C 1
S′ β | A:</td></tr>
<tr><td>$\left(\frac{\sin\alpha}{\sin\beta}\right)^2$</td><td></td><td></td><td>S α | S′ β
C 1 | A:</td></tr>
</table>

16. Subject Index: