DEEVA
SLIDE RULES

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DEEVA

polylog

TRADE MARK

INSTRUCTIONS FOR USE

by

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The Deeva Slide Rule Polylog is the perfect slide rule. More eminently than other models it makes possible the solution of the most complicated technical problems in the simplest manner. In a way it is a slide rule for experts but it can also be used by anyone who will study the following instructions – and that with a minimum of trouble.

To one familiar with the ordinary slide rules, e. g. the **Deeva** slide rules model **Rietz, Darmstadt**, or **Electro**, it should be sufficient to study the special scales only. However, everyone is recommended to read the instructions throughout, particularly because the simple problems are dealt with briefly – yet sufficiently for a beginner – and more particularly because numerous examples are given for the purpose of obtaining complete grasp of the possibilities of the slide rule, among other things to attain the highest possible accuracy although the primary importance of the slide rule of course implies the quick attainment of approximate results.

The three parts of the slide rule are in the following designated: the stock, the slide, and the cursor, of which the two last mentioned may be displaced in proportion to the stock.

The **Polylog** slide rule is remarkable for having both the **Fundamental Scale** C on the slide and the identical **fundamental scale D** on the stock itself placed on both sides of the slide rule.

By the front of the slide rule we indicate the side which contains the scales: LL01, LL02, LL03, DF, CF, CIF, CI, C, D LL3, LL2, and LL1.

By the back of the slide rule we indicate the side which contains the scales: LL00, L, K, A, B, S, ST, T, C, D, DI, P, and LL0.

The fundamental scale on the front covers what may be called a logarithmic unit, i. e. the numbers from 1 to 10, supplied with a series of subdivisions.

Thus, subdivisions have been engraved between 1 and 2 for every hundredth part (1.00, 1.01, 1.02 etc.), between 2 and 4 for every fiftieth part (2.00, 2.02, 2.04 etc.), and between 4 and 10 for every twentieth part (4.00, 4.05, 4.10 etc.).

The **Fundamental Scales C** and **D** on the back are supplied outside the logarithmic unit with extra divisions going from 0.9 to 11.

By setting and reading the slide rule the tenth of the smallest subdivisions engraved is estimated. Thus numbers with four figures are always obtained, the accuracy of the last figure being, however, different.

Between 1 and 2, e. g., 1.743 with the accuracy ± 0.001,
between 2 and 4, e. g., 2.326 with the accuracy ± 0.002, and
between 4 and 10, e. g., 4.055 with the accuracy ± 0.005.

Thus the setting or reading of a single number may be performed with an accuracy of about 1 ‰.

As the intervals are of rather different length it is of course somewhat easier to judge tenth of the larger intervals than of the smaller, and after some practice a few may be able to judge twentieth of the larger intervals while others have

difficulty in judging more than fifth of the smaller intervals. It is recommended, however, always to judge tenth.

The displaced or **folded scales CF** (on the slide) and **DF** (on the stock) are closely connected with the fundamental scales C and D. They are only found on the front of the slide rule, they are displaced the quantity π and indicate numbers from π to π or, more correctly, from π to 10π. The scales are supplied with some extra divisions so that they cover the region from 3 to 3.6 or, more correctly, from 3 to 36.

Multiplication ab of two factors a and b is performed in the following way:

1) The left index of C (the extreme left mark engraved) is set on the first factor a (e. g. 1.743) on the scale D,

2) the hairline of the cursor is set on the other factor b (e. g. 2.326) on C, and

3) the product ab is read under the hairline on D.

$1.743 \times 2.326 = 4.055$ Ex. No. 1.

The example shown may, however, illustrate other multiplications too, e. g.

$174.3 \times 0.2326 = 40.55$,

where the figures are the same as in ex. no. 1, but where the decimal points have been moved.

Matters become simpler if only numbers between 1 and 10 are dealt with, and this may be achieved by implying multiplication by 10^n where n is an integer positive or negative.

$174.3 = 1.743 \times 10^2$

$0.2326 = 2.326 \times 10^{-1}$

$40.55 = 4.055 \times 10$

The decimal point in the result is most easily placed by means of an estimated calculation.

174.3 is rounded up to 200,

0.2326 is rounded down to 0.2, and therefore

the product is about $200 \times 0.2 = 40$.

It is immediately seen that the product $1.743 \times b$ may be calculated in this manner if only b is smaller than $1 \div 1.743 = 5.735 \times 10^{-1}$. If b is still larger, numbers cannot be found on D opposite the numbers b on C. This difficulty may be managed by moving the slide to the left, so that not the left but the right index on C is set on the factor a (here 1.743).

The Polylog slide rule manages this more easily as the folded scales CF and DF supplement the fundamental scales C and D.

Thus opposite 7.305 on CF is read 1.273 on DF, wherefore

$1.743 \times 0.7305 = 1.273$ Ex. No. 2.

It is, moreover, seen that for a certain interval of b the multiplication may be performed either on the fundamental scales or on the folded scales. In the example above this holds good for all values of b that do not lie in the interval from $\pi \div 1.743 = 1.802$ to π or in the interval from $10 \div 1.743 = 5.735$ to 10. By setting of the factor a it is important not to move the slide more than half of its length corresponding to $\sqrt{10} = 3.162$. Therefore it should be most rationally to displace the folded scales by this quantity, but as this value does not differ very much from $\pi = 3.142$ (approximated) and as multiplication by π is often used (see below) this displacement has been chosen. For a durable slide rule of as high a quality as the **DEEVA** slide rule it is less important where the calculation is made, otherwise the scales that give the result in the middle of the slide rule should normally be chosen. By multiplications of more factors (see below) the place should be selected in consideration of the position of these factors on the slide rule.

Multiplication by π.

The folded scale CF directly gives the result of multiplication by π of the opposite numbers on the fundamental scale C.

$2.326 \times \pi = 7.305$ Ex. No. 3.

Correspondingly, the folded scale DF gives the result of multiplication by π of the corresponding numbers on the fundamental scale D.

Division is performed correspondingly, the examples above given being interpreted as problems of division.

$4.055 \div 2.326 = 1.743$ Ex. No. 4.

$12.73 \div 7.305 = 1.743$ Ex. No. 5.

$7.305 \div \pi = 2.326$ Ex. No. 6.

Reciprocal values. The reciprocal value of a number a is the number $1 \div a$ which may be calculated by ordinary division, but the result may be directly read on the **Reciprocal** scale **CI** which gives the reciprocal values of the corresponding numbers on the fundamental scale C.

$1 \div 0.2326 = 4.300$ Ex. No. 7.

Correspondingly the **Reciprocal scale DI** on the back of the slide rule may be used, it gives the reciprocal values of the corresponding numbers on the fundamental scale D.

A folded reciprocal scale CIF is also found, corresponding to the folded scale CF.

$1 \div 0.7305 = 1.369$ Ex. No. 8.

By means of CF and CI we may calculate

$\pi \div 0.7305 = 4.300$ Ex. No. 9.

or

$\pi \div 0.4300 = 7.305$ Ex. No. 10.

By means of CIF and C we may calculate

$1 \div 0.2326\pi = 1.369$ Ex. No. 11.

or

$1 \div 0.1369\pi = 2.326$ Ex. No. 12.

By means of the reciprocal scale multiplication of two factors ab may be replaced by division $a \div c$ where $c = 1 \div b$ and vice versa.

$0.4055 \times 4.300 = 0.4055 \div (1 \div 4.300) = 1.743$ Ex. No. 13.

$17.43 \div 4.300 = 17.43 \times (1 \div 4.300) = 4.055$ Ex. No. 14.

Multiplication of three factors abc may be performed by multiplication ab of the first two factors, after which the result is multiplied by the third factor c. By means of the reciprocal scale the calculation may, however, be performed much more simply

$abc = a \div (1 \div b) \times c$

$0.4055 \times 4.300 \times 3.118 =$

$0.4055 \div (1 \div 4.300) \times 3.118 = 5.435$ Ex. No. 15.

This calculation may thus be performed with only one setting of the slide and 2 settings of the cursor.

Multiplication of more factors $abcd$ is performed correspondingly

$abcd = a \div (1 \div b) \times c \div (1 \div d)$

$0.4055 \times 4.300 \times 0.3118 \times 5.185 =$

$0.4055 \div (1 \div 4.300) \times 0.3118 \div (1 \div 5.185) = 2.818$ Ex. No. 16.

This rather complicated calculation requires only 2 settings of the slide and two settings of the cursor.

Solution of the equation of second degree. Suppose the equation given is

$x^2 - ax + b = 0$

or

$x + b \div x = a$

or

$x + y = a$

where

$x \times y = b.$

Thus, the problem is to find two numbers the sum and product of which are given.

$x^2 - 4.700\,x + 1.743 = 0$ Ex. No. 17.

In ex. no. 1 opposite numbers on D and CI (or on DF and CIF) have the product $b = 1.743$.

Therefore, by means of the hairline of the cursor numbers are sought out lying opposite on D and CI with the sum $a = 4.700$.

It is here necessary to determine the position of the decimal point,

1) E. g., try to put $x = 0.30$ on the scale D, whereupon $y = 5.82$ corresponds on the scale CI. The sum is 6.12.

2) A new attempt gives $x = 0.40$, whereupon $y = 4.36$ corresponds. The sum is now 4.76.

3) A new attempt gives $x = 0.4055$, whereupon $y = 4.300$ corresponds. The sum is 4.706.

4) A new attempt gives $x = 0.4060$, whereupon $y = 4.295$ corresponds. The sum is 4.701.

5) The accuracy of the numbers being $\pm$ 0.0005 and $\pm$ 0.005 respectively, the value of y may be corrected to 4.294 without any alteration of x.

Thus we have the solution

$x = 0.406$ and 4.294.

Another example.

$x^2 - 8.642\,x + 17.43 = 0$ Ex. No. 18.

1) $x = 3.210$
   $y = 5.430$
   sum 8.640

2) $x = 3.208$
   $y = 5.435$
   sum 8.643

3) $x = 3.208$
   $y = 5.434$
   sum 8.642

Thus, we have the solution

$x = 3.208$ and 5.434.

If b is negative, the roots x and y have opposite signs, and if a is negative, the sum of the roots is negative.

Squares a^2 may be calculated by ordinary multiplication aa by means of the fundamental scales C and D (con i igently CF and DF), but it is easier to read the square directly on the **Square scale A** (on the back of the stock) or the identical **Square scale B** (on the back of the slide). The reading is performed in relation to the fundamental scales D and C which are also found on the back of the slide rule.

The square scale contains two logarithmic units in prolongation of each other, together corresponding to the logarithmic unit of the fundamental scale. The two halves are figured respectively from 1 to 10 and from 10 to 100, supplied with a series of subdivisions. Moreover, the square scales are supplied outside the normal region with extra divisions going from 0.8 to 120.

Subdivisions have been engraved between 1 and 2 for every fiftieth part (1.00, 1.02, 1.04 etc.), between 2 and 5 for every twentieth part (2.00, 2.05, 2.10 etc.), and between 5 and 10 for every tenth part (5.0, 5.1, 5.2 etc.).

Thus, it is evident that by the employment of the square scales A and B less accuracy is obtained than by ordinary multiplication on the fundamental scales C and D, but, in return, time is saved by the simpler handling.

$1.743^2 = 3.040$ (A and D) Ex. No. 19.

$1.743 \times 1.743 = 3.038$ (C and D) Ex. No. 20.

Square-Roots $\sqrt{a}$ are calculated correspondingly, the example no. 19, being interpreted as extraction of square root.

$\sqrt{3.040} = 1.743$ Ex. No. 21.

Attention must be paid to the question whether the number a is to be set on the left or the right half of A.

Thus ex. no. 21 is calculated on the left half whilst

$\sqrt{30.40} = 5.515$ Ex. No. 22.

is calculated on the right half of A.

Problems of placing the decimal point are illustrated by the following examples

$\sqrt{304.0} = 10\,\sqrt{3.040} = 17.43$ Ex. No. 23.

$\sqrt{0.3040} = \sqrt{30.40} \div 10 = 0.5515$ Ex. No. 24.

Extraction of square root may also be done by means of the fundamental scales by seeking out a position of the hairline of the cursor where the readings on CI and D (or on CIF and DF) are equal. In our example (look at the figure) we get

$\sqrt{1.743} = 1.320$ (CIF and DF) Ex. No. 25.

$\sqrt{17.43} = 4.175$ (CI and D) Ex. No. 26.

In this manner, of course, greater accuracy is attained than by the square scales.

Ordinary multiplication and division may be performed by means of the square scales A and B; it is only to be remembered that the accuracy of the result is less. Folded square scales are not found, not being necessary because each position of the slide also gives opposite numbers on the scales A and B for all values between 1 and 10.

The area of a circle is computed by means of the formula πr^2. Of course the square scale can be used for this calculation but the easiest and most accurate method is to use the scales C, D, and DF the formula being re-written as follows:

$r \times r \times \pi = r \div (1 \div r) \times \pi$

in which the first multiplication is performed as a division by means of the scales D and CI, as in example no. 13 or example no. 26, and in which the subsequent multiplication by π is performed by means of the scales D and DF in a similar way as in example no. 3.

$r = 4.175$

$\pi r^2 = 4.175 \div (1 \div 4.175) \times \pi = 17.43 \times \pi = 54.75$ Ex. No. 27.

The first multiplication may, however, also be performed on the scales DF and CIF as in example no. 25.

$r = 1.320$

$\pi r^2 = 1.320 \div (1 \div 1.320) \times \pi = 1.743 \times \pi = 5.475$ Ex. No. 28.

However, the calculation may also be performed by using the scales D and CIF. The value of r is set to the CIF scale off the same value on D, whereafter the arae of the circle may be read on D off the index 1 on C.

$r = 2.355$

$\pi r^2 = 17.43$ Ex. No. 29.

The problem may also be solved by means of the square scales and the left side hairline of the cursor. The hairline is set on the diameter d on scale D and the result is read under the left side hairline on scale A giving the square of the diameter d divided by a number corresponding to the distance between the hairline and the left extra hairline of the cursor. This number is $4 \div \pi = 1.273$. The formula is

$\pi r^2 = d^2 \div (4 \div \pi)$.

The radius of a circular area is determined in a similar way. The index π on scale CF is set off the area of the circle on scale DF, whereafter the place giving identical readings on the scales D and CI, or DF and CIF is sought with the cursor. See examples nos. 27 and 28. The index 1 on scale C may also be set off the area of the circle on scale D, and with the cursor the place is sought giving the same reading on the scales D and CIF. See example no. 29.

Naturally the square scale and the left side hairline of the cursor may be used also. The left side hairline is set on the area on scale A and the diameter is read under the hairline on scale D.

Solution of the equation of third degree. Suppose the equation given is

$z^3 + pz^2 + qz + r = 0$.

By putting

$$z = x - \frac{p}{3}$$

the equation given is brought into the form

$x^3 - ax + b = 0$

We now get

$x^2 + b \div x = a$

or

$x^2 + y = a$

where

$x \times y = b$.

The procedure is now clear. Opposite numbers on D and CI have a constant product b (here 1.743). Such a pair of numbers x on D and y on CI are now sought out so that the number x^2 on the square scale A corresponding to the number x on D is equal to $a - y$.

$x^3 - 16.87\,x + 1.743 = 0$ Ex. No. 30.

Attention must at once be paid to the position of the decimal point.

1) First try $x = 4.00$, whence $x^2 = 16.00$ and $y = 0.44$ correspond. $a - y = 16.43$.

2) Second try $x = 4.05$, whence $x^2 = 16.40$ and $y = 0.43$ correspond. $a - y = 16.44$.

3) Further try $x = 4.055$, whence $x^2 = 16.44$ and $y = 0.430$ correspond. $a - y = 16.440$.

The solution is now

$x = 4.055$.

Just as by the solution of the equation of second degree, a different degree of accuracy is attained of the numbers x^2 and y, viz. $\pm$ 0.01 and $\pm$ 0.0005 respectively. If necessary, x^2 might have been suitably corrected.

Now one root is found in the equation of third degree, and *a priori* it is known that this equation always has one real root. A complete discussion of the different cases where there is only one root or three roots will be omitted here.

The other roots may be correspondingly calculated, or the equation given may be reduced to an equation of second degree by means of the root already found.

Cubes a^3 may be calculated by ordinary multiplication aaa on the fundamental scales C and D, or better – as mentioned at multiplication of three factors – in the following manner

$aaa = a \div (1 \div a) \times a$

but may be read directly on the **cube scale** K on the back of the stock itself.

The cube scale contains three logarithmic units in prolongation of each other, together corresponding to the logarithmic unit on the fundamental scale D. The three thirds are figured from 1 to 10, from 10 to 100, and from 100 to 1000. The cube scale, too, is supplied with some subdivisions.

Subdivisions have been engraved between 1 and 3 for every twentieth part (1.00, 1.05, 1.10 etc.), between 3 and 6 for every tenth part (3.0, 3.1, 3.2 etc.), and between 6 and 10 for every fifth part (6.0, 6.2, 6.4 etc.). The subdivisions from 10 to 1000 correspond to the subdivisions from 1 to 10.

Thus, it is evident that the employment of the cube scale gives less accuracy, but in return time is saved and simpler handling gained.

$1.743^3 = 5.30$ (D, CI, CF and DF) Ex. No. 31.

$1.743 \times 1.743 \times 1.743 = 5.300$ (D and K) Ex. No. 32.

Extraction of **cube-roots** $\sqrt[3]{a}$ may be calculated correspondingly, the example no. 32 being interpreted as extraction of cube-root.

$\sqrt[3]{5.30} = 1.743$ Ex. No. 33.

Attention must be paid to the question whether the number a is to be set on the left, middle, or right third of K.

Ex. no. 30 i thus calculated on the left third, whilst

$\sqrt[3]{53.0} = 3.756$ Ex. No. 34.

is calculated on the middle third, and

$\sqrt[3]{530} = 8{,}095$ Ex. No. 35.

is calculated on the right third of K.

The extraction of cube-root may, however, also be calculated be means of the square scale A (on the back of the stock) and the reciprocal scale CI (on the front of the slide), seeking out by means of the cursor equal readings on these scales while the index is set on the number a (on A) of which the cube-root is to be extracted.

It is, however, preferable to seek out equal readings on the square scale B (on the back of the slide) and the reciprocal scale DI (on the back of the stock), so that the whole calculation may be performed on the back of the slide rule. Here the index of the stock is set on the number a on B.

$\sqrt[3]{3.290} = 1.488$ Ex. No. 36.

The reason is now seen why the reciprocal scale – as mentioned before – is also found on the back and only just on the stock itself.

Brigg's logarithms *log a* may be read on a scale of equidistance L found on the back of the slide rule. It goes from 0-1 and is throughout divided into five-hundredth parts (0.000, 0.002, 0.004 etc.).

The scale is read in relation to the fundamental scale D and gives Briggian logarithms to the base 10.

$log\ 1.743 = 0.2412$ Ex. No. 37.

The scale L may be used by calculation of powers and roots.

$y = 1.743^{2.4}$ Ex. No. 38.

$log\ y = 2.4 \times log\ 1.743 = 2.4 \times 0.2412 = 0.5789$

$y = 3.792$

The multiplication may be performed by ordinary arithmetic or by simple multiplication on the fundamental scales C and D.

$y = \sqrt[5]{1.743}$ Ex. No. 39.

$log\ y = 0.2412 \div 5 = 0.4824$

$y = 1.117$

These calculations are much more easily performed by means of the log log scales LL0, LL1, LL2, LL3, LL00, LL01, LL02, and LL03 as will be shown later.

LL3, LL2, and LL1 are placed under one another on the lower part of the front of the rule, while LL0 is placed lowest on the back of the rule. They are all used in connection with the fundamental scale D. The four scales are folded or displaced and should really be imagined as lying in extension of one another. They give the exponential function e^x where x goes from 0.001 to 0.01 on LL0,

from 0.01 to 0.1 on LL1, from 0.1 to 1 on LL2, and from 1 to 10 on LL3. The decimal points should therefore at once be taken into consideration. The scales are to a certain extent supplied with extra subdivisions outside the normal region.

LL0 gives values from 1.001 to 1.011 and is figured by means of the numbers 1.001, 1.0015, 1.002, 1.003, 1.004, 1.005, 1.006, 1.007, 1.008, 1.009, 1.010, and 1.011.

A series of subdivisions have been engraved between 1.001 and 1.002 for every hundredthousandth part (1.00100, 1.00101, 1.00102 etc.), between 1.002 and 1.004 for every fiftythousandth part (1.00200, 1.00202, 1.00204 etc.) ,and between 1.004 and 1.011 for every twentythousandth part (1.00400, 1.00405, 1.00410 etc.).

LL1 gives values from 1.01 to 1.11 and is figured by means of the numbers 1.01, 1.015, 1.02, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, 1.10, and 1.11.

A series of subdivisions have been engraved between 1.01 and 1.02 for every tenthousandth part (1.0100, 1.0101, 1.0102 etc.), between 1.02 and 1.04 for every fivethousandth part (1.0200, 1.0202, 1.0204 etc.), and between 1.04 and 1.11 for every twothousandth part (1.0400, 1.0405, 1.0410 etc.).

LL2 gives values from 1.1 to 3 and is figured by means of the numbers 1.1, 1.12, 1.14, 1.16, 1.18, 1.2, 1,25, 1.3, 1.35, 1.4, 1,5, 1.6, 1.7, 1.8, 1.9, 2, 2.5, e, and 3.

A series of subdivisions have been engraved between 1.1 and 1.2 for every thousandth part (1.100, 1.101, 1.102 etc.), between 1.2 and 1.3 for every fivehundredth part (1.200, 1.202, 1.204 etc.), between 1.3 and 1.5 for every twohundredth part (1.300, 1.305, 1.310 etc.), between 1.5 and 2 for every hundredth part (1.50, 1.51, 1.52 etc.), and between 2 and 3 for every fiftieth part (2.00, 2.02, 2.04 etc.).

LL3 gives values from 2.5 to 10^5 and is figured by means of the numbers 2.5, e, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000, 2000, 3000, 4000, 5000, 10000, 20000, 50000, and 100000.

A series of subdivisions have been engraved between 2.5 and 3 for every fiftieth part (2.50, 2.52, 2.54 etc.), between 3 and 5 for every twentieth part (3.00, 3.05, 3.10 etc.), between 5 and 10 for every tenth part (5.0, 5.1, 5.2 etc.), between 10 and 20 for every half (10.0, 10.5, 11.0 etc.), between 20 and 50 for every whole (20, 21, 22 etc.), between 50 and 100 for every two (50, 52, 54 etc.), between 100 and 200 for every five (100, 105, 110 etc.), between 200 and 300 for every ten (200, 210, 220 etc.), between 300 and 500 for every twenty (300, 320, 340 etc.), between 500 and 1000 for every fifty (500, 550, 600 etc.), between 1000 and 2000 for every hundred (1000, 1100, 1200 etc.), between 2000 and 5000 for every twohundred (2000, 2200, 2400 etc.), between 5000 and 10000 for every fivehundred (5000, 5500, 6000 etc.), between 10000 and 20000 for every thousand (10000, 11000, 12000 etc.), between 20000 and 50000 for every twothousand (20000, 22000, 24000 etc.), and between 50000 and 100000 for every fivethousand (50000, 55000, 60000 etc.).

The following examples no. 40–57 are all calculated by means of LL1, LL2, and LL3, but the methods may easily be transfered to LL0.

Characteristic for LL0 is e. g.

$e^{0.001743} = 1.001743$ (LL0 and D),

small deviations from this peculiarity are only found in the rigth extremity of the scale.

$e^{1.743} = 5.72$ (LL3 and D) Ex. No. 40.

$e^{0.1743} = 1.1904$ (LL2 and D) Ex. No. 41.

$e^{0.01743} = 1.01758$ (LL1 and D) Ex. No. 42.

Natural logarithms $log_e x$ are found in the same manner by interpretation of the examples no. 40–42.

$log_e\ 5.72 = 1.743$ Ex. No. 43.

$log_e\ 1.1904 = 0.1743$ Ex. No. 44.

$log_e\ 1.01758 = 0.01743$ Ex. No. 45.

The scales, as mentioned, are used for the calculation of powers and roots.

$5.72^{2.326} = 57.6$ (LL3 and C) Ex. No. 46.

1) The left index of C is set on the number 5.72 on LL3,
2) the hairline of the cursor is set on the power 2.326, and
3) the result 57.6 is read on LL3.

$1.1904^{2.326} = 1.500$ (LL2 and C) Ex. No. 47.

1) The left index of C is set on the number 1.1904 on LL2,
2) the hairline of the cursor is set on the power 2.326, and
3) the result 1.500 is read on LL2.

$1.01758^{2.326} = 1.04138$ (LL1 and C) Ex. No. 48.

1) The left index of C is set on the number 1.01758 on LL1,
2) the hairline of the cursor is set on the power 2.326, and
3) the result 1.04138 is read on LL1.

If the power is above a certain amount (in the above example 5.735) the slide is displaced to the left, so that the right index is set on the number, whereupon the hairline of the cursor is set on the power, and the result is read on LL2 if the index is set on LL1, and on LL3 if the index is set on LL2.

$1.1904^{6.6} = 3.160$ (LL2, C, and LL3) Ex. No. 49.

$1.01758^{6.6} = 1.1219$ (LL1, C, and LL2) Ex. No. 50.

If we want to calculate

$5.715^{6.6}$

this method fails, but we may succeed by means of a circumscription.

$5.715 = 3 \times 1.905$

$5.715^{6.6} = 3^{6.6} \times 1.905^{6.6} = 1410 \times 70.36 = 99200$ Ex. No. 51.

Extraction of root is performed analogously, the examples no. 43–47 being interpreted as extraction of root.

$\sqrt[2.326]{57.6} = 5.72$ Ex. No. 52.

$\sqrt[2.326]{1.500} = 1.1904$ Ex. No. 53.

$\sqrt[2.326]{1.04138} = 1.01758$ Ex. No. 54.

$\sqrt[6.6]{3.160} = 1.1904$ Ex. No. 55.

$\sqrt[6.6]{1.1219} = 1.01758$ Ex. No. 56.

The method of dividing the number into factors may also be employed

$\sqrt[6.6]{99200} = \sqrt[6.6]{992} \times \sqrt[6.6]{100} = 2.844 \times 2.010 = 5.715$ Ex. No. 57.

Correspondingly LL00 is placed highest on the back of the rule, while LL01, LL02, and LL03 are placed under one another on the upper part of the front of the rule. They are all used in connection with the fundamental scale D. These scales are also folded or displaced and should really be imagined as lying in extension of one another. They give the exponential function e^{-x} where x goes from 0.001 to 0.01 on LL00, from 0.01 to 0.1 on LL01, from 0.1 to 1 on LL02, and from 1 to 10 on LL03. The decimal points should therefore also here at once be taken into consideration. The scales are to a certain extent supplied with extra subdivisions outside the normal region.

LL00 gives values from 0.999 to 0.989 and is figured by means of the numbers .999, .9985, .998, .997, .996, .995, .994, .993, .992, .991, .990, and .989.

A series of subdivisions have been engraved in part (.99900, .99899, .99898 etc.), between .998
between .999 and .998 for every hundredthousandth part (.99800, .99798, .99796 etc.), and between .996 and .989 for
and .996 for every fiftythousandth part (.99800, .99798, .99796 etc.), and between .996 and .989 for
every twentythousandth part (.99600, .99595, 99590 etc.).

LL01 gives values from 0.99 to 0.9 and is figured by means of the numbers .99, .985, .98, .975, .97, .965, .96, .955, .95, .94, .93, .92, .91, and .9.

A series of subdivisions have been engraved
between .99 and .98 for every tenthousandth part (.9900, .9899, .9898 etc.), between .98 and .96 for
every fivethousandth part (.9800, .9798, .9796 etc.), and between .96 and .9 for every twothousandth
part (.9600, .9595, 9590 etc.).

LL02 gives values from 0.91 to 0.35 and is figured by means of the numbers .91, .9, .85, .8, .75, .7, .65, .6, .55, .5, .45, .4, and .35.

A series of subdivisions have been engraved
between .91 and .8 for every thousandth part (.910, .909, .908 etc.) and between .8 and .35 for every
fivehundredth part (.800, .798, .796 etc.).

LL03 gives values from 0.4 to 0.00001 and is figured by means of the numbers .4, .35, .3, .25, .2, .15, .1, .08, .06, .04, .02, .01, .005, .002, .001, .0005, .0002, .0001, .00005, .00002, and .00001.

A series of subdivisions have been engraved
between .4 and .1 for every fivehundredth part (.400, .398, .396 etc.), between .1 and .01 for every
thousandth part (.100, .099, .098 etc.), between .01 and .001 for every fivethousandth part (.0100,
.0098, 0096 etc.), between .001 and .0005 for every twentythousandth part (.00100, .00095, .00090
etc.), between .0005 and .0001 for every fiftythousandth part (.00050, .00048, .00046 etc.), between
.0001 and 0002 for every twohundredthousandth part (.000100, 000095, 000090 etc.), and be-
tween .00002 and .00001 for every fivehundredthousandth part (.000020, .000018, .000016 etc.).

The scales LL00, LL01, LL02, and LL03 are used exactly as the scales LL0, LL1, LL2, and LL3. The following examples should therefore be easily understandable.

$e^{-0.01743} = 0.98272$ (LL01 and D) Ex. No. 58.

$e^{-0.1743} = 0.8400$ (LL02 and D) Ex. No. 59.

$e^{-1.743} = 0.1750$ (LL03 and D) Ex. No. 60.

$l_n\, 0.98272 = -0.01743$ Ex. No. 61.

$l_n\, 0.8400 = -0.1743$ Ex. No. 62.

$l_n\, 0.1750 = -1.743$ Ex. No. 63.

$0.98272^{2.326} = 0.96025$ Ex. No. 64.

$0.8400^{2.326} = 0.6666$ Ex. No. 65.

$0.1750^{2.326} = 0.01735$ Ex. No. 66.

$\sqrt[2.326]{0.96025} = 0.98272$ Ex. No. 67.

$\sqrt[2.326]{0.6666} = 0.8400$ Ex. No. 68.

$\sqrt[2.326]{0.01735} = 0.1750$ Ex. No. 69.

Solution of the exponential equation $a^x = b$. The examples no. 46–50 may be interpreted as examples of this problem.

$5.72^x = 57.6$ $x = 2.326$ Ex. No. 70.

$1.1904^x = 1.500$ $x = 2.326$ Ex. No. 71.

$1.01758^x = 1.04138$ $x = 2.326$ Ex. No. 72.

$1.1904^x = 3.160$ $x = 6.6$ Ex. No. 73.

$1.01758^x = 1.1219$ $x = 6.6$ Ex. No. 74.

The index of C is set on the number with the unknown exponent. This is done on one of the scales LL1, LL2, or LL3. The hairline of the cursor is set on the result, viz. the right side of the equation on the same scale or the nearest higher scale. The required exponent is read on the scale C.

$0.98272^x = 0.96025 \quad x = 2.326$ Ex. No. 75.

$0.8400^x = 0.6666 \quad x = 2.326$ Ex. No. 76.

$0.1750^x = 0.01735 \quad x = 2.326$ Ex. No. 77.

These examples correspond to the examples nos. 64–66 and are correspondingly solved by means of the scales LL01, LL02, LL03, and C.

Calculation of Interest.

Calculation of simple interest on a capital a at a rate of interest r (corresponding to $100r$ per cent. pro anno) after d days is done by means of the formula

$ard \div 360.$

This calculation is made according to the rules for multiplication of more factors.

★ The division by 360 can be made by means of the right side hairline of the cursor as the distance between the hairline and the side hairline is equal to the distance between the index 1 on scale C or D and 3.6 on scale CF or DF.

Calculation of compound interest is made according to the formula

$a(1 + r)^x,$

where n is the number of settling periods.

The calculation can be made by means of the scale LL1 or LL2 for interest between 1 and 11 per cent. or more than 10 per cent. respectively. Reference is made to the rules for calculation of powers.

The **trigonometrical scales** S, ST, and T are found on the back of the slide. The **sine scale** S gives values of sine corresponding to angles between 5.°5 and 90°. The scale is figured by means of the numbers 5.5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 25, 30, 35, 40, 50, 60, 70, 80, and 90, where ° is understood. These figures are all black. The scale is, moreover, figured by means of red numbers (complemental numbers to the black figures), which are used when the scale is used as a **cosine scale.**

A series of subdivisions have been engraved between 5.5 and 20 for every tenth degree (5.5, 5.6, 5.7 etc.), between 20 and 40 for every fifth degree (20.0, 20.2, 20.4 etc.), between 40 and 60 for every half degree (40.0, 40.5, 41.0 etc.), between 60 and 80 for every whole degree (60, 61, 62 etc.), together with 85 and 90.

The **tangent scale** T gives values of tangent corresponding to angles between 5.°5 and 45°. The scale is figured by means of the numbers 5.5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 25, 30, 35, 40, and 45, where ° is understood. These figures are all black. The scale moreover, is figured by means of red numbers (complemental numbers to the black ones), which are used when the scale is used as a **cotangent scale.**

A series of subdivisions have been engraved between 5.5 and 20 for every tenth degree (5.5, 5.6, 5.7 etc.) and between 20 and 45 for every fifth degree (20.0, 20.2, 20.4 etc.).

The **joint sine and tangent scale ST** gives joint values corresponding to angles between 0.°55 and 6°. In fact the scale is a folded fundamental scale which has been displaced the quantity $\pi \div 180$. The scale is figured by means of the numbers .55, .6, .7, .8, .9, 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2, 2.5, 3, 3.5, 4, 5, and 6, where ° is understood.

★ **Use extreme right red line for 365 days.**

A series of subdivisions have been engraved between .55 and 2 for every hundredth degree (.55, .56, .57 etc.), between 2 and 4 for every fiftieth degree (2.00, 2.02, 2.04 etc.), and between 4 and 6 for every twentieth degree (4.00, 4.05, 4.10 etc.).

$\sin\ 10°.04 = 0.1743 = \cos 79°.96$ Ex. No. 78.

$\tan\ 9°.89 = 0.1743 \doteq \cot 80°.11$ Ex. No. 79.

$\sin\ 1°.00 = 0.01743 = \cos 89°.00$ Ex. No. 80.

$\tan\ 1°.00 = 0.01743 = \cot 89°.00$ Ex. No. 81.

Sin and *tan* of angles smaller than 0°.55 are also calculated by means of the scale ST only the results shall be divided by 10 or 100.

$\sin 0°.10 = \sin 6'.0 = 0.001743$ Ex. No. 82.

$\sin 0°.01 = \sin 36'' = 0.0001743$ Ex. No. 83.

These values may also be calculated by division by the numbers 3438 and 206300 respectively.

$\sin 6'.0 = 6.0 : 3438 = 0.001743$

$\sin 36'' = 36 : 206300 = 0.0001743$

By multiplication and division of ordinary numbers and trigonomentrical functions the scales S, T, and ST are used in connection with the fundamental scale D or the reciprocal scale DI, which are all found on the back of the slide rule. Examples are not given.

Most frequently the trigonometrical functions are employed in connection with **calculation of triangles. Therefore a short survey of these problems will** be given.

Given: *2 angles and 1 side.*

The third angle may be calculated, whereupon the law of sines is employed.

Ex. $A = 13°.45$ Ex. No. 84.

$B = 20°.00$

Sum $A + B = 33°.45$

Now we get $C = 146°.55$

The sine law gives

$$\frac{a}{\sin 13.45} = \frac{b}{\sin 20.00} = \frac{c}{\sin 33.45}$$

Herd once of the sides is known. E. g. $a = 4.055$.

By only one setting of the slide is now directly found

$b = 5.960$ and $c = 9.610$.

Given: *1 angle and 2 sides.*

1) If the angle given lies opposite one of the sides, the angle opposite the second side given is first calculated by means of the sine law. Then the third angle is calculated, and finally the third side by means of the sine law.

2) If the angle given lies between the 2 sides given, we may proceed as follows.

Ex. $A = 13°.45 \quad b = 5.960 \quad c = 9.610$ Ex. No. 85.

The sine law gives

$$\frac{a}{\sin 13.45} = \frac{5.960}{\sin B} = \frac{9.610}{\sin C}$$

We may now displace the slide of the slide rule until the sum of the angles which on the sine scale lie opposite to the numbers 5.960 and 9.610 on the fundamental scale D, is equal to $180° - 13°.45 = 166°.55$.

Thus $B = 20°.00$ and $C = 146°.55$ are found, whereupon $a = 4.055$ is read opposite 13°.45 on the sine scale.

The relation may also be formed

$$\frac{sin\ B}{sin\ C} = \frac{5.960}{9.610} = 0.6200$$

The hairline of the cursor is set on 0.6200 on the fundamental scale D, whereupon the slide is displaced until the angles which lie opposite 1 and 0.6200 respectively, have the sum 166°.55.

Then we get again

$B = 20°.00$ and $C = 146°.55$,

whereupon a may be calculated by means of the sine law.

Given: *3 sides.*

The problem may be solved by displacing the slide until the sum of the three angles opposite to the three numbers a, b, and c is 180°.

This problem may, however, be solved in a simpler way by means of the common logarithmic formulas

$$tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad \text{etc.}$$

The scale of Pythagoras P is placed at the bottom of the back of the slide rule. It is connected with the fundamental scale D, and it could also be designated $\sqrt{1-x^2}$, as readings at the same place on scale D and scale P give the square sum of 1.

$\sqrt{1-0.1743^2} = 0.98470$ Ex. No. 86.

Readings on scale D must consequently be imagined as lying between 1 and 0. The scale has been figured by means of the numbers 0.995, 0.99, 0.98, 0.97, 0.96, 0.95, 0.94, 0.93, 0.92, 0.91, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, and 0, and with a number of subdivisions.

Thus subdivisions have been engraved from 0.995 to 0.99 for every tenthousandth part (0.9950, 0.9949, 0.9948, etc.), from 0.99 to 0.98 for every fivethousandth part (0.9900, 0.9898, 0.9896, etc.), from 0.98 to 0.97 for every twothousandth part (0.9800, 0.9795, 0.9790, etc.), from 0.97 to 0.9 for every thousandth part (0.970, 0.969, 0.968, etc.), from 0.9 to 0.7 for every twohundredth part (0.900, 0.895, 0.890, etc.), from 0.7 to 0.3 for every hundredth part (0.70, 0.69, 0.68, etc.), and from 0.3 to 0.1 for every twentieth part (0.30, 0.25, 0.20, etc.).

As $\sqrt{1-\sin^2 v} = \cos v$, and $\sqrt{1-\cos^2 v} = \sin v$

scale P may also be used as a very valuable supplement to scale S. The fact is that this scale gives sufficiently accurate values for sin angles from 0°—45° but quite unsatisfactory values for sin from 45°—90°. For cos it is of course reversely. As sin and cos to the same angle have the square sum 1 it is evident that scale P may be used as an extension of scale S. If therefore sin 79°.96 is to be found scale S will give an exceedingly unsatisfactory reading and instead setting is made to cos 79°.96 but reading is made not of S but of P whereupon

$\sin 79°.96 = 0.9847 = \cos 10°.04$ Ex. No. 87.

is obtained.

Improvement of the results gained by calculation on the slide rule.

As mentioned above the accuracy by a single setting or reading on the fundamental scales of the slide rule is about 1 ‰. If n settings or readings are performed by a calculation on these scales, the accuracy of the result is about $\sqrt{n}$ ‰. If the square scales and the cube scale are included in the calculation, the accuracy is less. When the *log log* scales are used, a very different percentage of accuracy is attained in the different places of the scales.

Sometimes, an improvement on a result obtained by slide rule calculation is desired, and some examples of the manner of proceeding are given below.

$$1.743 \times 2.326 = 1.7 \times 2.3 + 0.043 \times 2.3 + 1.7 \times 0.026 + 0.043 \times 0.026$$ Ex. No. 1a.

$= 3.91$
$\ 0.0989$
$\ 0.0442$
$\ 0.001118$
$\overline{\ 4.054218}$

Each product is calculated on the slide rule. If any uncertainty arises by the reading of the last figures of these products, the correct value of these figures is known *a priori*, as e. g. 1.7×2.3 is to end in a 1-figure because $7 \times 3 = 21$ which ends in 1.

Thus we have succeeded in calculating a product with 7 correct figures by means of a slide rule only.

$4.055 \div 2.326 = 1.743 + \Delta$ Ex. No. 4a.
$4.055 = 2.326 \times 1.743 + 2.326\,\Delta$
$\quad = 4.054218 + 2.326\,\Delta$
$2.326\,\Delta = 0.000782$
$\Delta = 0.0003362$
$4.055 \div 2.326 = 1.7433362$

The result of the intervening calculation is found in ex. no. 1a. The division is thus performed with 8 correct figures.

$x^2 - 4.700x + 1.743 = 0$ Ex. No. 17a.
$x = 0.406 + \Delta$
$(0.406 + \Delta)^2 - 4.700\,(0.406 + \Delta) + 1.743 = 0$
$0.164836 + 0.812\,\Delta + \Delta^2 - 1.9082 - 4.700\,\Delta + 1.743 = 0$
$3.888\,\Delta = -0.000364 + \Delta^2$
$\Delta = (-0.000364 + \Delta^2) \div 3.888$
As a first approximation we put $\Delta^2 = 0$
$\Delta = -0.00009360$
$\Delta^2 = 0.0000000088$
$\Delta^2 \div 3.888$ therefore has no influence until the ninth decimal.
The improved root is now obtained
$x = 0.406 - 0.00009360 = 0.40590640$
The second root is then
$4.700 - 0.40590640 = 4.29409360$

$\sqrt{1.743} = 1.320 + \Delta$ Ex. No. 25a.

$1.743 = 1.742400 + 2.640\,\Delta + \Delta^2$
$2.640\,\Delta = 0.000600 - \Delta^2$
$\Delta = (0.000600 - \Delta^2) \div 2.640$
As a first approximation we put $\Delta^2 = 0$
$\Delta = 0.0002272$
$\Delta^2 = 0.0000000516$
$\Delta^2 \div 2.640$, therefore, has no influence until the eighth decimal.
The improved root is thus
$1.320 + 0.0002272 = 1.3202272$

In these examples no regard has been paid to the question of the real value of these many figures. This depends, of course, on the accuracy of the given numbers.

Important for all users of DEEVA Duplo Slide Rules.

The DUPLO Slide Rule is a delicate precision-instrument and should be treated as such.

Above all protect the rule from heat over 50° C. (120° F.) and from direct exposure to glaring sunlight.

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TRADE MARK

INSTRUCTIONS FOR USE

by

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