THE RADIO ENGINEER'S SLIDE RULE

Designed by

J. F. MORRISON

Bell Telephone Laboratories

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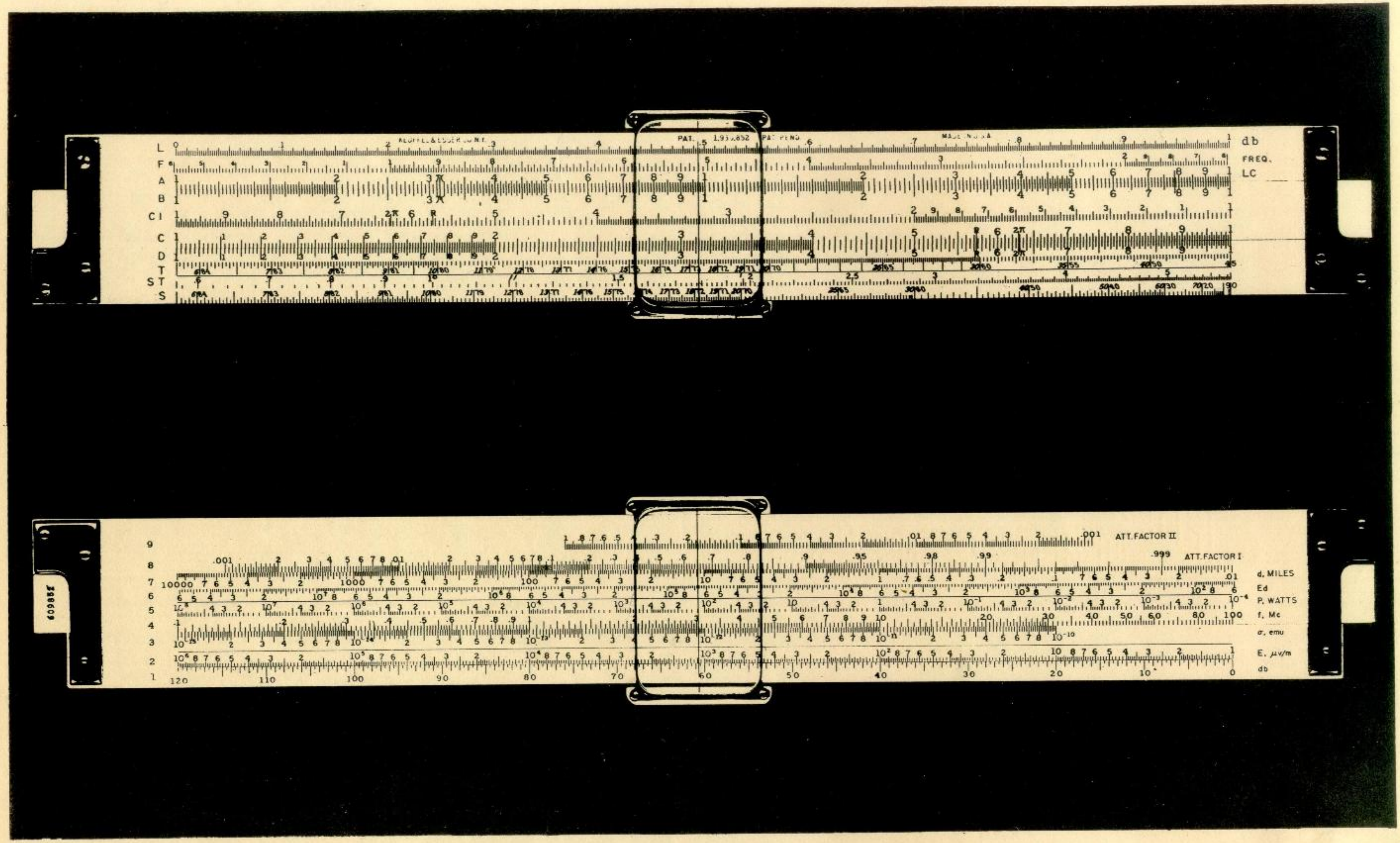

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Engineering practices have branched into many specialized fields each of which present problems of unique and specialized nature. In working with these problems engineers probably use the slide rule more frequently than any other tool, as it provides a convenient and rapid means for performing computations. While the conventional scales of the slide rule are convenient for many problems common to the various branches of engineering, its usefulness to a specialist can be greatly enhanced by the addition of special scales. Radio engineering is today being recognized as a specialty, and the Keuffel & Esser Company, in cooperation with the Bell Telephone Laboratories, have made available the Radio Engineer's Slide Rule.

A major difference between radio and other types of electrical communication is the means by which the signal energy is propagated. The transmission of signal energy is of primary importance and the radio engineer is often concerned with problems involving the propagation of electro-magnetic waves over the surface of the earth. Special scales for solving many practical problems of this nature are placed on one face of the rule. (scales 1-9). The conventional A, B, C, D, CI, L and complete trigonometric scales are all conveniently arranged upon the other face of the rule.

The rule facilitates the computation of:

(1) Radio propagation over a plane earth for the conductivity case. By two settings of the slide, corresponding values of field intensity for wide ranges of distance, frequency, power and soil conductivity are obtained.

(2) The LC product for a given frequency and also decibels for a given current, voltage, or power ratio may be read directly from the scales.

(3) The value of inductance or capacity required to resonate a reactive circuit, as well as the reactance of an inductance or capacity for a given frequency may be obtained with one setting of the slide.

(4) The transformation of vectors from rectangular to polar form or vice versa can be accomplished by one setting of the slide.

PART I

It may be briefly stated that many factors influence the intensity of a radio signal which has been transmitted over the earth's surface:

(1) The size, configuration and height above the earth of the transmitting antenna.

(2) The distance between the transmitting and receiving antennas.

(3) The frequency "f" and the operating power "P" of the transmitting station.

(4) The electrical conductivity "σ" and dielectric properties "ϵ" of the earth in the territories over which the signal is transmitted.

A close approximation of the signal intensity at a receiving point can be made if certain justifiable assumptions regarding these influencing factors are made. The attenuation factor (scale 8) has been computed for the conductivity case, assuming that vertical antennas erected above and very near to the surface of a plane earth, are used. The attenuation factor is the ratio of the received field intensity to that which would result if this plane surface had perfect conductivity. The equations upon which scale 8 was prepared are based on the Sommerfeld[1] solution as evaluated by Burrows[2] for the case where the ratio of the dielectric to the conduction currents flowing in the earth is very small, i. e., $\epsilon f/2\sigma$ * is small compared

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* Expressed in electrostatic units - See reference 2.

to unity. The computations for each division of this scale were carried out to the eighth and, in some cases, to the ninth decimal place. It was appreciated that this accuracy is not realized in practice, nevertheless, the divisions can be engraved with that accuracy and the solution is rigorous for the idealized assumptions mentioned above.

For convenience, the propagation scales are designated by standard abbreviations at the right hand end of the slide rule scales, i.e.:

Scale 1 - db - decibels

" 2 - E, μv/m - field intensity in microvolts per meter

" 3 - σ , emu = soil conductivity in electromagnetic units

" 4 - f, Mc = frequency in megacycles

" 5 - P, Watts - power

" 6 - Ed = the product of field intensity and distance

" 7 - d, Miles = distance

" 8 & 9 ATT. FACTOR = attenuation factor

The field intensity produced by a radio transmitter and vertical antenna is:

$$E = \frac{K \sqrt{P}}{d} \times \text{ATT. FACTOR}$$

where K is an antenna constant. (Close to the earth's surface K is numerically equal to 6170 for a vertical 1/4 wavelength antenna.)

With a given frequency (f, Mc) and soil conductivity (σ, emu), the ATT. FACTOR may be found by one setting of the rule for all distances within the limits of the scales. For example, opposite a soil conductivity of 2×10^{-14} emu on scale 3 set one megacycle on scale 4. Then opposite any distance in miles on scale 7, read the attenuation factor on scale 8. If then the field intensity at a given distance, say 30 miles, is desired, it will be found that the attenuation factor at this distance is .0401. Set 30 miles on scale 7 opposite .0401 on scale 9 (ATT. FACTOR II) and read the field intensity in μv/m on scale 2 or db above 1 μv/m on scale 1 opposite any given power in watts on scale 5. If the field intensity at one mile from the antenna in a given direction is known, use scale 6 (Ed) in place of the scale 5.

The position of scale 5 in relation to the other scales is based upon the field intensity produced at one mile by a vertical 1/4 wavelength antenna erected close to a perfect conducting plane earth. Therefore, by direct reference from scales 5 to 6, we have the unattenuated field intensity at one mile produced by a 1/4 wavelength antenna for any power within the range of the scales. Conversely, if we know the field intensity at one mile, we may read directly from scales 6 to 5 the "apparent" power of the station.

Scales 1 and 2 have an additional and very useful purpose. We have by direct reference to scale 1 and 2 a conversion from decibels (db) to either voltage or current ratios for all values up to 120 db. This feature will prove useful in many problems other than those involving radio propagation.

Typical Problems that May be Solved on the Radio Propagation Scales.

Example (1): A 1000-watt transmitting station is to operate on 1420 kc with an antenna 1/4 wavelength high. What is the field intensity produced by this station at a distance of 40 miles, if the average ground conductivity surrounding the station is 3.5×10^{-14} emu?

Solution:

Opposite 3.5×10^{-14} on scale 3, set 1.42 on scale 4. Opposite 40 miles on scale 7, read the ATT. FACTOR = .0249 on scale 8. Reset the indicator to .0249 on scale 9 and move the slide until 40 miles on scale 7 is coincident with the indicator line. Then under 10^3 watts on scale 5, read the field intensity 1.21×10^2 μv/m on scale 2 or 41.6 db above one μv/m on scale 1.

Example (2): Field measurements have been made up to a distance of 100 miles from a proposed location for a new transmitting station on a frequency of 1000kc. The data when plotted shows that the attenuation factor is .75 at 10 miles, .26 at 50 miles and .1 at 100 miles. What is the average soil conductivity in emu?

Solution:

Adjust scales 7 and 8 until the three observed attenuation factors on scale 8 are as nearly opposite to their corresponding distances on scale 7 as an average adjustment of the slide will permit. An average adjustment in this case is: .748 on scale 8 is opposite 10 miles, while .261 on scale 8 is opposite 50 miles, and .102 on scale 8 is opposite 100 miles. Below the given frequency 1 megacycle on scale 4 read the average soil conductivity 1.37×10^{-13} emu on scale 3.

Example (3): A radio telephone circuit is to be installed between Cape Cod and the mainland. The distance is 30 miles and transmission will take place over sea water on a frequency of 20 megacycles. It is known from measurements that 150 microvolts per meter signal intensity will be required to over-ride the prevailing noise when a single vertical antenna is used at the receiving location. What is the minimum power required at the transmitter to satisfactorily meet this requirement, if a directional antenna having 5 db gain is used at both the transmitter and receiver?

Solution:

The average conductivity of sea water is 10^{-11} emu. Opposite 10^{-11} on scale 3 set 20 on scale 4. Above 30 miles on scale 7, read ATT. FACTOR = .053 on scale 8. Set the indicator to .053 on scale 9 and set 30 miles on scale 7 to the indicator line. Reset the indicator to 1.5×10^2 μv/m on scale 2 and read 1.9×10^2 watts on scale 5 and also 43.5 db on scale 1.

The combined gain of the transmitting and receiving antennas is 10 db, so move the indicator to 43.5 - 10 = 33.5 db on scale 1 and read 19 watts on scale 5.

Example (4): A directional antenna which produces an hour-glass shaped field intensity pattern is used at a broadcasting station. The unattenuated field intensity at one mile north of the antenna is 2.5 volts per meter, 1 volt per meter northeast of the antenna and .2 volts per meter east of the antenna. What is the "apparent" power in these directions based upon 1/4 wavelength antenna elements?

Solution:

North, opposite 2.5×10^6 on scale 6 read 1.65×10^5 watts on scale 5.

N.E., opposite 1×10^6 on scale 6 read 2.6×10^4 watts on scale 5.

East, opposite 2×10^5 on scale 6 read 1.05×10^3 watts on scale 5.

Example (5): A broadcast station is advised that if they change their operating frequency from 890 kc. to 1350 kc., they will be permitted to increase their operating power from 1 to 5 kilowatts. It is known that the average soil conductivity surrounding the station is 3×10^{-14} emu. Will the station have a greater or lesser 500 μv/m service area if this suggested change is made?

Solution:

This problem can be most easily solved by plotting a field intensity versus distance curve for both frequencies and powers.

Opposite 3 x 10^{-14} emu on scale 3, set .89 on scale 4. Then tabulate the ATT. FACTOR as read on scale 8 opposite several distances on scale 7:

Distance Miles	ATT. FACTOR
10	.365
20	.16
30	.090
40	.062

Opposite each ATT. FACTOR on scale 9, set the corresponding distance on scale 7. Then opposite 10^3 watts on scale 5 read the field intensity on scale 2.

Distance Miles	Field Intensity μv/m
10	7.1 x 10^3
20	1.55 x 10^3
30	5.9 x 10^2
40	3 x 10^2

Repeat the process for the 1350 kc. frequency and 5 kw. power. Opposite 3 x 10^{-14} emu on scale 3, set 1.35 on scale 4. Then as before:

Distance Miles	ATT. FACTOR
10	.132
20	.0515
30	.0322
40	.0235

Opposite each ATT. FACTOR on scale 9 set the corresponding distance on scale 7. Then opposite 5 x 10^3 watts on scale 5 read the field intensity on scale 2.

Distance Miles	Field Intensity µv/m
10	5.7×10^3
20	1.12×10^3
30	4.7×10^2
40	2.55×10^2

It may be seen by an inspection of the tabulated data that operation on the 890 kc. frequency results in a greater service area although the power allowed for the 1350 kc. frequency is 5 times that used on 890 kc.

Accumulated Data on Soil Conductivity Measurements.

Tables I and II are typical of the information available on various types of terrain. The data in Table I is taken from Table IV of Burrow's(2) paper. The data in Table II and the map, Fig. 6, have been obtained from the Engineering Dept. of the Federal Communications Commission who in cooperation with licensees of broadcasting stations have collected and compiled the data.

The conventional A, B, C, D, CI, L and Trigonometric scales are all arranged upon one face of the rule. The Trigonometric scales are decimally divided and all functions are referred to the D scale. A special frequency scale, designated F is provided and located between the A and L scales. This scale is similar to the conventional CI scale with the exception that it is folded at $\frac{1}{2\pi}$ to facilitate the computation of tuned circuit problems.

Opposite a setting which represents a given frequency on the F scale the LC product may be read on the A scale and $\frac{1}{2\pi f}$ on the D scale.

Since:

$$LC = \left(\frac{1}{2\pi f}\right)^2$$

and $$X_L = 2\pi fL$$

and, $$X_C = \frac{1}{2\pi fC}$$

where L is inductance in henries, C capacity in farads, X reactance in ohms and f frequency in cycles, the inductance or capacity required for resonance at a given frequency or the reactance of an inductance or capacity may be obtained by one setting of the slide.

Example (6): What capacity is required to resonate an inductance of 300 uh at a frequency of 890 kc.?

Solution:

Opposite 890 on F scale set 300 on the B scale.

Opposite index of the B scale, read 106.5 μμf on the A scale.

Example (7): What inductance is required to resonate a capacity of .027 μf. at a frequency of 800 cycles?

Solution:

Opposite 800 on F scale set 27 on the B scale.

Opposite the index of the B scale, read 1.465 h on the A scale.

Example (8): What is the reactance of 252 microhenry at 340kc.?

Solution: Opposite 340 on the F scale set 252 on C scale.

Opposite the index of the D scale, read 538 ohms on the C scale.

Example (9): What is the reactance of 625 micro-microfarads at 4560 kc.?

Solution:

Opposite 4560 on the F scale set 625 on the C scale.

Opposite the index of the C scale, read 55.8 ohms on D scale.

The scales are not limited in range of frequency, capacity, inductance or reactance and the decimal point position may be determined by rough calculation or inspection. It may be more convenient, however, when working in an unfamiliar part of the frequency spectrum to use the attached chart (Fig. 1). By a glancing inspection of this chart the decimal point position may be readily determined.

Decibel Computations

In Radio or Communication engineering, power, current and voltage ratios are often expressed in decibels (db).

For power ratios:

$$db = 10 \log_{10} \frac{P_1}{P_2}$$

The solution of this equation is direct reading on the rule.

Example (10): How many db correspond to a power ratio $\frac{P_1}{P_2} = 4.36$?

Solution:

Opposite 436 on the D scale read 6.4 on the L scale.

Disregard the decimal point shown on the L scale and determine the decimal point position as follows. For power ratios between unity and 10:1 the first digit to the left of the decimal point in the number "db" is given by the numbers on the L scale. For power ratios greater than 10:1 the first digit in the number "db" is one less than the number of digits in the known power ratio.

Example (11): $\frac{P_1}{P_2} = 20,000$

Solution:

Opposite 2 on the D scale read 3.015 on the L scale.

Upon inspection there are 5 digits in the power ratio so the answer is 43.015 db.

Example (12): What is the power ratio corresponding to 28.6 db?

Solution:

Opposite 86 on the L scale read 724 on the D scale.

There are 3 digits to the left of the decimal point because 2 is the second digit to the left of the decimal point in 28.6 db.

For voltage or current ratios:

$$db = 20 \log_{10} \frac{V_1}{V_2}$$

and

$$db = 20 \log_{10} \frac{I_1}{I_2}$$

The solution of these equations are read directly between scales #1 and #2 for values between 0 and 120 db.

Example (13):

$$\frac{V_1}{V_2} \text{ or } \frac{I_1}{I_2} = 150$$

Solution:

Opposite 1.5×10^2 on scale #2 read 43.5 db on scale #1.

The decimal point is indicated on the rule.

Sine and Tangent Scales

The Sine and Tangent scales are placed on the body of the rule under the D scale. The operation of these scales in relation to the D and CI scales for solving ordinary trigonometric relations is explained in instruction books already published by the K. & E. Co. These scales are, however, conveniently arranged for solving problems frequently encountered by the Radio engineer.

<u>Resistance and Reactance in Series. Transformation from Rectangular to Polar Form or Vice Versa.</u>

$$R \pm jX = Z\underline{/\pm\theta} = \sqrt{R^2 + X^2}\underline{/\pm \tan^{-1}\frac{X}{R}}$$

<u>Example (14)</u>:

Given the impedance $R \pm jX = 4 \pm j\,3$ vector ohms what is this impedance expressed in the form $Z\underline{/\pm\theta}$?

<u>Solution:</u>

Set the smaller number of the expression, 3, on the CI scale over the right-hand index of the D scale.
Set the indicator line to 4 (the larger number of the complex expression) on the CI scale.
Read $\theta = \pm 36.85°$ (black numbers)* on the T scale.
Reset the indicator line to 36.85° (black numbers) or 53.15° (red numbers) on the S scale and read $Z = 5$ ohms under the indicator line on the CI scale.

The sign of the angle is the same as that preceding j in the rectangular form.

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* If the reactance X were larger than the resistance i.e: $3 \pm j4$, it would be obvious by inspection that the angle θ was greater than 45° in which case you read $\theta = 53.15°$ from the red numbers on the T scale.

Example (15): Given the impedance $Z/\pm\theta = 63/\pm 24.3°$ to be expressed in rectangular form:

Solution: Set the indicator line to 24.3° on the S scale. Then set 63 on the CI scale to the indicator line. Read the smaller number of the complex expression, 25.95, on the CI scale above the right hand index of the D scale.

Move the indicator line to 24.3° on the T scale and read 57.5 (the larger number of the complex expression) under the indicator line on the CI scale.

Since the angle in this case is less than 45° the resistance component is the larger and accordingly the answer is:

$$63/\pm 24.3° = 57.5 \pm j25.95 \text{ vector ohms.}$$

Since vectors can be added most conveniently in rectangular form and multiplied or divided most conveniently in polar form the impedance of any network of resistances and reactances can be solved by simple addition, multiplication and division by changing to the most convenient form of expression[3].

Example (16): Reference is made to Figure 2. This figure shows several reactances and resistances connected together to form a complete network and the problem presented is to solve for Z with the aid of the slide rule.

<u>Solution:</u>

The network is terminated by the impedance 43-j110 ohms connected in parallel with the impedance 12 + j62 ohms. This parallel combination gives a combined impedance:

$$Z_1 = \frac{Z_T \; Z_S}{Z_T + Z_S}$$

where

$$Z_T = 43 - j110 = 118 \angle -68.65^\circ$$
$$Z_S = 12 + j62 = 63.1 \angle +79.03^\circ$$

) Transformed to polar) form with slide rule as) explained in Example 1.

$$Z_1 = \frac{118 \angle -68.65^\circ \quad 63.1 \angle +79.03^\circ}{(43 - j110) + (12 + j62)}$$

$$= \frac{7450 \angle +10.38^\circ}{55 - j48}$$

$$= \frac{7450 \angle +10.38^\circ}{73 \angle -41.1^\circ}$$

$$= 102. \angle +51.48^\circ = 63.55 + j80$$

Connected in series with Z_1 is the impedance 75 + j315 so we add them in rectangular form.

$$(75 + j315) + (63.55 + j80) = 138.55 + j395$$
$$= 419 \angle +70.7^\circ$$

An impedance 0 - j280 is then connected in parallel with the impedance 419 $\angle +70.7^\circ$

$$Z = \frac{419 \angle 70.7 \quad 280 \angle -90°}{(138.55 + j395) + (0 - j280)}$$

$$= \frac{116700 \angle 19.3°}{138.55 + j115} = \frac{116700 \angle -19.3°}{180 \angle +39.7°}$$

$Z = 649 \angle -59.1°$ expressed in polar form or

$Z = 333 - j556$ expressed in rectangular form.

Resistance and Reactance in Parallel

Reference is made to Figure 3 where the resistance R_p is shown connected in parallel with the reactance X_p. The solution of this circuit is:

$$Z\angle\theta = R_s + j\,X_s = \frac{X^2_p\;R_p}{R_p^2 + X_p^2} + j\,\frac{X_p\;R^2_p}{R_p^2 + X_p^2}$$

the phase angle $\theta = \tan^{-1}\frac{X_s}{R_s} = \tan^{-1}\frac{R_p}{X_p}$

Example (17).

Given the parallel impedance R_p = 8.33 ohms, X_p = j6.26 ohms.

Solution:

To the right-hand index of the D scale set 8.33 (the larger number) on the C scale.

Set the indicator line to 6.26 (the smaller number) on the C scale and read θ = **53.1°** on the T scale.*

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*If the resistance is greater than the reactance, θ is greater than 45° so read θ from red numbers on the T and S scales. If the resistance is smaller than reactance θ is less than 45° so read θ from the black numbers on the T and S scales.

Reset the indicator to 53.1° on S scale and read

Z = 5 ohms on the C scale under the indicator line.

Example (18):

Given the parallel impedance $R_p = 10$, $X_p = -j30$.

Solution:

Proceed as before.

Set the larger number, 30, on the C scale to the index on the D scale. (For convenience use left-hand index of D scale in this example.) Under the smaller number, 10, on the C scale read $\theta = 18.43°$ on T scale. Reset hair line to 18.43° on S scale.

Read Z = 9.5 ohms on C scale.

Conversion from Series to Parallel Impedances

Any resistance and reactance in series has an equivalent resistance and reactance in parallel. It can be shown that they are related by the expression;

$$Z^2 = R_s R_p = X_s X_p$$

We have already explained how to solve for Z on the slide rule when given any resistance and reactance connected in series or parallel. Therefore, to obtain the equivalent circuit we square the impedance Z and divide the square by the resistance and reactance of the series or parallel circuit to obtain the equivalent circuit.

Example (19):

Reference is made to Figure 4.

The series circuit $R + jX = 3 + j4$.

Set the smaller number, 3, on the CI scale to the index on the D scale.

Then set the indicator line to 4 on the CI scale and read $\theta = 53.1°$ on the T scale.

Reset the indicator line to 53.1° on the S scale and read $Z = 5$ on the CI scale.

Reset the indicator line to 5 on the D scale and read $Z^2 = 25$ on the A scale.

Set 3 on B scale to the indicator line and read $R_p = 8.33$ on the A scale.

Set 4 on B scale to the indicator line and read $X_p = 6.26$ on A scale.

A Matching Network of Reactances

A simple network of reactances for matching between two resistances is shown on Figure 5. This type of network is often used for matching the impedance of an antenna to an unbalanced transmission line. The required series and shunt reactance may be solved by one setting of the slide rule.

Example (20):

What is series and shunt reactance required to match between a 150 and 65 ohms resistances?

Solution:

$R_L = 150$ ohms

$R_S = 65$ ohms

$a = R_L - R_S = 85$

Opposite $R_S = 65$ on the A scale set

$a = 85$ on the B scale

Opposite $a = 85$ on the C scale read the series reactance $X = 74.4$ on the D scale and opposite $R_L = 150$ on the C scale read 131.2 on the D scale.

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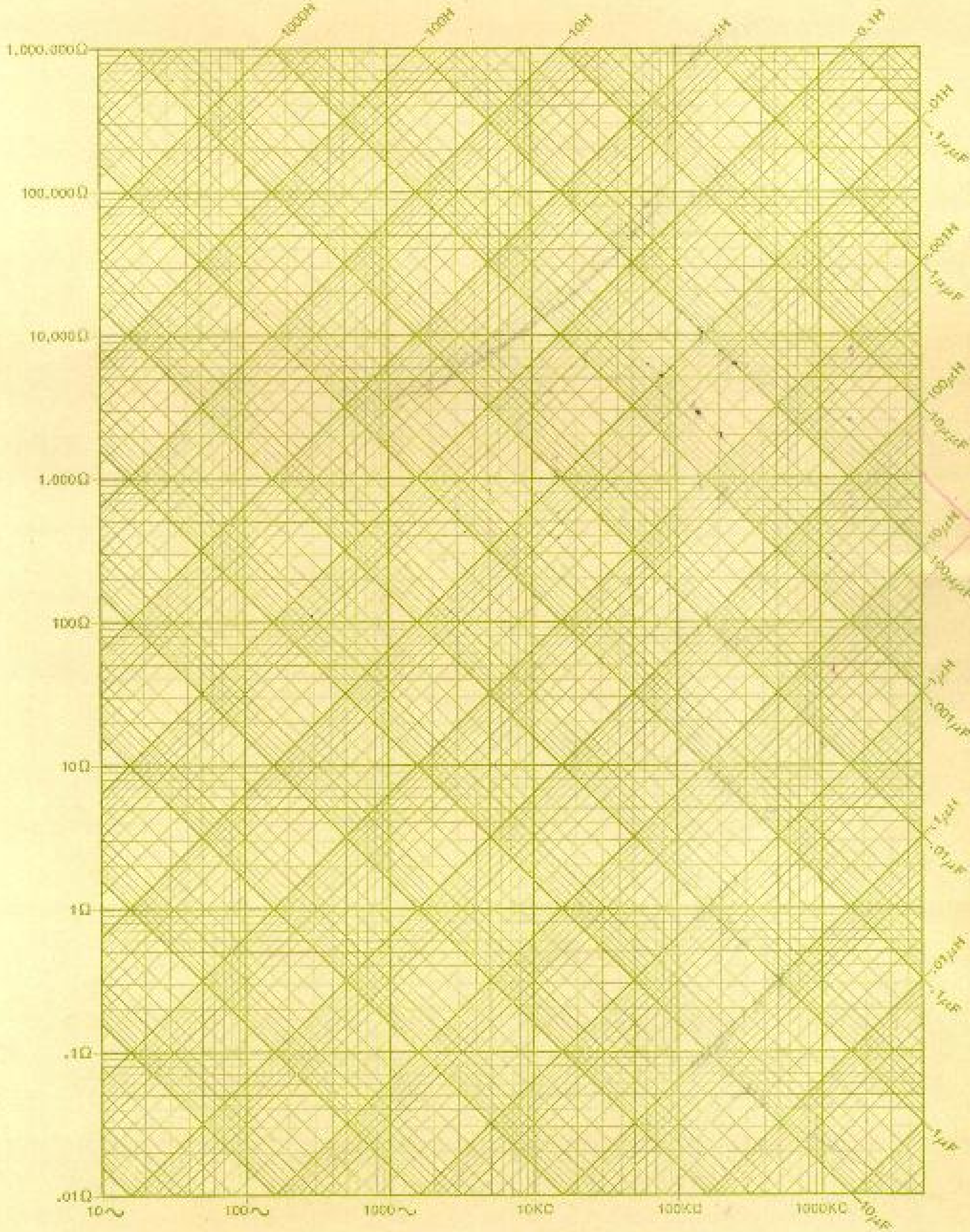

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