FABER-CASTELL

INSTRUCTIONS

Precision Slide Rules

No. 2/83, 62/83

Contents

*** The inclusion of a divided 20 inch scale enables the most important calculations to be performed with a greatly increased measure of mathematical accuracy.**

The Scales of the Silde Rule

All graduations are marked with reference to the basic scales C and D; at the right-hand end of the slide rule they bear the mathematical formula based on the numbering of the basic scales.

The cursor, entirely surrounding the slide rule, enables each stage of the calculation to be connected up with the next, throughout all the scales on the front and back of the slide rule.

The scales on the **front** of the slide rule:	Cube scale	K	x^3	Upper body of slide rule
	1st tangent scale	T_1	$\sphericalangle \tan 0{\cdot}1\,x$ (cot)	
	2nd tangent scale	T_2	$\sphericalangle \tan x$ (cot)	
	Fixed π-scale	DF	πx	
	Movable π-scale	CF	πx	Slide
	Reciprocal π-scale	CIF	$1 : \pi x$	
	Reciprocal basic scale	CI	$1 : x$	
	Movable basic scale	C	x	
	Fixed basic scale	D	x	Lower body of slide rule
	Arc scale for small angles	ST	$\sphericalangle$ arc $0{\cdot}01\,x$	
	Sine scale	S	$\sphericalangle \sin 0{\cdot}1\,x$ (cos)	
	Pythagorean scale	P	$\sqrt{1-(0{\cdot}1\,x)^2}$	
The scales on the **back** of the slide rule:	Exponential scales for negative exponents	LL_{03}	e^{-x}	Upper body of slide rule
		LL_{02}	$e^{-0{\cdot}1\,x}$	
		LL_{01}	$e^{-0{\cdot}01\,x}$	
	2nd fixed root scale	W_2	$\sqrt{10\,x}$	
	2nd movable root scale	W_2'	$\sqrt{10\,x}$	Slide
	Mantissa scale	L	$\frac{1}{2}\log x$	
	Movable basic scale	C	x	
	1st movable root scale	W_1'	$\sqrt{x}$	
	1st fixed root scale	W_1	$\sqrt{x}$	Lower body of slide rule
	Exponential scales for positive exponents	LL_1	$e^{0{\cdot}01\,x}$	
		LL_2	$e^{0{\cdot}1\,x}$	
		LL_3	e^{x}	

How to read the Scales with graduation length of 10 inches, e.g. C, D, CF, DF, CIF

The following should be noted:

The slide rule does not show the actual place of decimals to which a number belongs. For example, the 6 shown on the slide rule may equally well denote 6, 0·6, 60, 600, 6000 or 0·006, and so forth.* The position of the decimal point is ascertained afterwards, by a rough calculation with round figures. In most practical calculations it is known in advance, so that no further rules for determining the decimal point are required. It is the basic scales C and D that give the clearest idea of the way in which the scales are sub-divided. Once familiar with the graduation of these two scales, we shall be able to understand the others likewise.

All scales marked in red run in the opposite direction (reciprocally) from right to left, the exceptions being the extended supplementary graduations which are provided to enable a calculation to be continued in the case of borderline values just below 1 (beginning of graduation) or just above 10 (end of graduation).

Let us now have a look at the basic scales C and D on the front of the slide rule, the reading- and setting-exercises being carried out by means of the long cursor line or the index-1 (beginning of scale) or index-10 (end of scale), as the case may be.

A section of the graduation-range from 1 to 2 (Scales C and D)

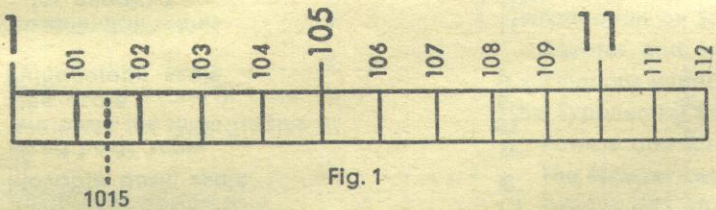


Fig. 1

From guide-number 1 to guide-number 1·1

10 sub-divisions of 10 intervals each (= 1/100 or 0·01 per graduation mark)

Here an accurate reading can be immediately taken to 3 places (e.g. 1-0-1). By **halving** the space between two graduation marks, 4 figures can be accurately set (e.g. 1-0-1-5). In all cases the final number must then be a 5.

A section of the graduation-range from 2 to 4 (Scales C and D)

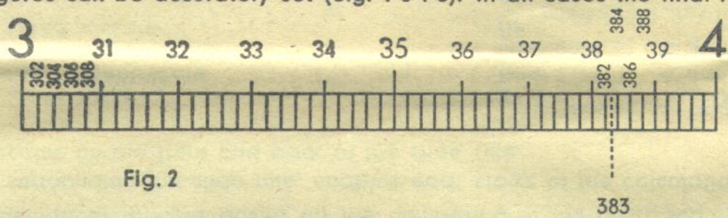


Fig. 2

From guide-number 3 to guide-number 4

10 sub-divisions of 5 intervals each (= 1/50 or 0·02 per graduation mark)

Here an accurate reading can be immediately taken to 3 places (e.g. 3-8-2). The last number is then always even (2, 4, 6, 8). If the intermediate spaces are halved, this provides the uneven numbers (1, 3, 5, 7, 9) as well (e.g. 3-8-3).

* An exception is provided by the exponential scales (see top of p. 21).

A section of the graduation-range from 4 to 10 (Scales C and D)

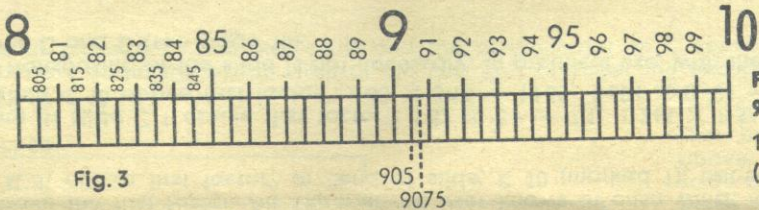


Fig. 3

From guide-number 8 to 9 and 9 to 10

10 sub-divisions of 2 intervals each (= 1/20 or 0·05 per graduation mark)

Here an accurate reading can be taken to 3 places, when the last number is a 5 (9-0-5). By halving the intermediate spaces it is even possible to take an accurate reading to 4 places. Here again the last number is always a 5 (9-0-7-5).

How to read the scales with graduation length of 20 inches: W_1 W'_1 W_2 W'_2

These scales are provided along the edges and adjacent edges of the slide, on the back of the slide rule, and run from 1 to 3·3 at the bottom and from 3 to 10 at the top. Their use results in doubled accuracy. They are sub-divided differently, however, from the scales on the 10 inch graduations.

Graduation-range from 1 to 2.

This section is first of all divided into **ten** sub-divisions, marked 1·1, 1·2, 1·3, 1·4 . . . 1·9. Each of these, in turn, is divided into **ten** further sub-divisions, but the latter are not numbered,owing to lack of space. Finally, a small stroke is also provided to show the exact centre between these graduation marks. Readings can be taken as follows: 1-1-2-5, 1-3-1-5, 1-4-4-5, 1-5-2-5, 1-7-1-5 . . . 1-9-7-5.

Graduation-range from 2 to 5.

Here again, this section is first of all divided into **tenths**, but they are not numbered, except for the graduation marks corresponding to the values 2, 2·5, 3, 3·5, 4, 4·5 and 5. The user must recognise the remaining tenths for himself, i.e. the values 2·1, 2·2, 2·3 . . . to 4·7, 4·8, 4·9.
Further tenths are entered in between these tenths but without "marked centres". We thus have the following values, starting with 2 and without using the decimal point: 2-0-0, 2-0-1, 2-0-2, 2-0-3, 2-0-4, 2-0-5, 2-0-6, etc., up to 4-9-7, 4-9-8, 4-9-9, 5-0-0.

In the case of the graduation-range from 5 to 10, the **tenths** first of all appear, as before, but in between these it is only the **fifths** that are provided. Starting with 5, we thus have the following graduation marks: 5-0-0, 5-0-2, 5-0-4, 5-0-6, 5-0-8, 5-1-0, 5-1-2, etc., up to 9-9-6, 9-9-8, 1-0-0.

Multiplication

a·b

This is chiefly carried out with the main scales C and D.

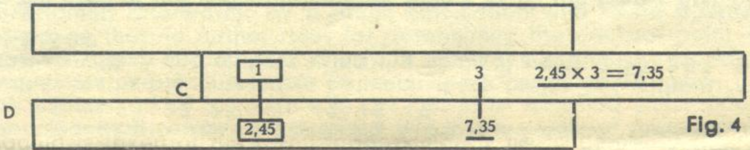


Fig. 4

Example: 2·45 × 3 = 7·35

Set the 1 of the slide of the rule (**C** 1) above the 2·45 on the lower scale of the slide rule (**D** 245), move the cursor line into position above the 3 of the lower slide scale (**C** 3) and read the product — 7·35 — underneath the cursor line on the lower scale of the slide rule (**D** 735).

Example: 2·04 × 3·18 = 6·487. Place C 1 above D 2·04, place the cursor line above D 3·18, and read the result — 6·49 — on D, likewise underneath the cursor line.

Example: 11·45 × 4·22 = 48·35. Place C 1 above D 11·45, place the cursor line above C 4·22, and read the result — 48·35 — on D, likewise underneath the cursor line.

In calculations on the lower scales C and D it sometimes happens that with the setting C 1 above the first factor on scale D the slide projects too far to the right, so that it is no longer possible to set the second factor on C.

Transposing of the slide

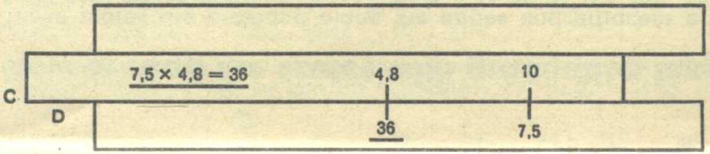


Fig. 5

Example: 7·5 × 4·8 = 36

In this case the slide is pushed to the left until the beginning **C** 1 is replaced by the end of the slide **C** 10 (marked 1) above the first factor on scale **D.**

This operation is termed transposing the slide. It can be avoided if, in case of need, **C** 10 (end of slide) is immediately placed above the first factor. An experienced user knows at once which method of setting to adopt, i.e. "beginning of slide, **C** 1, above first factor" or "end of slide, **C** 10 (marked 1), above first factor".

Examples for practice:

Setting: "Beginning of slide C 1 above first factor": 1·82 × 3·9 = 7·1; 0·246 × 0·37 = 0·091; 213 × 0·258 = 54·95
Setting: "End of slide C 10 above first factor": 4·63 × 3·17 = 14·68; 0·694 × 0·484 = 0·336

This operation of transposing of the slide is not necessary, in practical use, with the "π-displaced scales" CF, DF, in conjunction with scales C and D (see page 10).

Division

$\frac{a}{b}$

Use the cursor line to place the numerator on D and the denominator on C opposite each other; the result can then be found underneath the beginning of the slide of the rule, C 1, or under the end of the slide, C 10.

Example: 9·85 ÷ 2·5 = 3·94

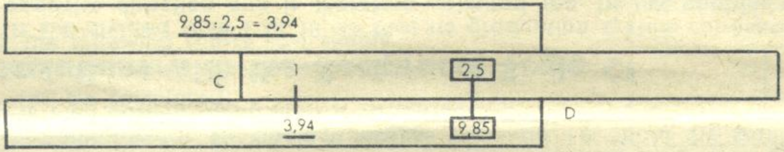


Fig. 6

First bring the cursor line into position above the numerator, 9·85, on the lower scale D of the body of the slide rule, then place the denominator 2·5 (on the C graduation) underneath the cursor line. The numerator and the denominator are now level with each other, and underneath the beginning of the slide, C 1, the result 3·94 can be found on scale D.

Examples for practice: 970 ÷ 26·8 = 36·2; 285 ÷ 3·14 = 90·7; 7500 ÷ 835 = 8·98;
0·685 ÷ 0·454 = 1·509; 68 ÷ 258 = 0·264

Combined Multiplication and Division

$\frac{a \cdot b \cdot c}{d \cdot e \cdot f}$

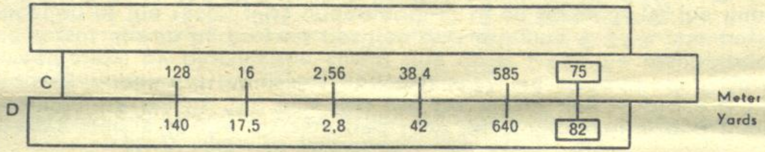
Example: $\frac{13{\cdot}8 \times 24{\cdot}5 \times 3{\cdot}75}{17{\cdot}6 \times 29{\cdot}6 \times 4{\cdot}96} = 0{\cdot}491$

Always start with the division and then let multiplication and division alternate. No readings need be taken of the intermediate results. We thus first use the cursor line to place D 1-3-8 and C 1-7-6 level with each other (division). No reading is taken of the result, approximately 0·8, underneath **C** 10 on **D**, and the multiplication by 24·5 is carried out immediately, by placing the cursor line on **C** 2-4-5. The result (about 1·9 on **D**) is then divided by 29·6, by leaving the cursor line in its present position and bringing **C** 2-9-6 into position underneath it. The next operation is the multiplication of the result (0·65 underneath **C** 10 on **D**) by 3-7-5, the final operation being the division by 4·96, in the same manner. The result 0·491 can then be found underneath **C** 10 on **D**.

Examples for practice: $\frac{38{\cdot}9 \times 1{\cdot}374 \times 16{\cdot}3}{141{\cdot}2 \times 2{\cdot}14} = 2{\cdot}883$; $\frac{1{\cdot}89 \times 7{\cdot}68 \times 8{\cdot}76}{0{\cdot}723 \times 4{\cdot}76} = 36{\cdot}95$

Formation of Tables

For forming a table, first set the slide rule to the required "equivalence"; mutual conversions of measurements, weights and other units can then be carried out. Once we establish that 1 inch = 25·4 mm, for example, C 1 is placed over the corresponding value; for 75 lbs = 34 kgs., the two values are placed opposite each other on C and D.
Example: To convert yards into metres. 82 yards = 75 metres.



Place **C** 75 above **D** 82. This produces a table, and reading may be taken as follows:
42 yards are 38·4 metres; 2·8 yards are 2·56 m; 640 yards are 585 m; 16 m are 17·5 yards; 128 m are 140 yards, and so forth.

$$\frac{a}{b} : \frac{c}{d} : \frac{e}{f}$$

Examples for practice:

1 inch = 25·4 mm ("Equivalence": 26" = 66 cm). Place C 1 (left-hand 1 of C) above D 2-5-4 (2-5-4 on D) and take the following readings by the aid of the cursor line:
17 inches = 43·2 cm
38 inches = 96·6 cm

1 metre of material costs DM 45. Place C 1 above D 45 and take the following readings with the cursor line:
3·20 metres of material costs DM 144.
2·40 metres of material costs DM 108.

Exchange rate: US$ 1·- = DM 4. Place C 10 above D 4-0-0 and take the following readings with the aid of the cursor line:
US$ 2·61 = DM 10·44
US$ 4·73 = DM 18·92

If, in the formation of tables, it is no longer possible to set and read off individual values, because the slide of the rule projects too far, we again transpose the slide, i.e. "hold the setting" by placing the cursor line over C. 1. The slide is then pushed along until C 10 replaces C 1.

Calculations with the Reciprocal Scale CI

This is sub-divided from 1 to 10, so that its graduation system corresponds to that of the scales C and D, but it takes the opposite direction and is therefore coloured red. Its use enables various types of calculation to be performed. (This manipulation can be arrived by using scales CF and DF, as explained on top of page 10).

$$\frac{1}{a}$$

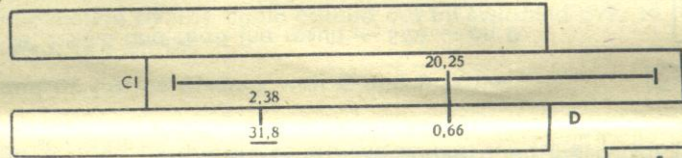
(1) If, for a given number a, the reciprocal 1 ÷ a is required, this is set on C or CI and the reciprocal read off, above it, on CI or on C. The reading can be taken without adjusting the slide, with the use of the cursor alone.
Example: 1 ÷ 8 = 0·125; 1 ÷ 2 = 0·5; 1 ÷ 4 = 0·25; 1 ÷ 3 = 0·333.

(2) Multiplication likewise can be carried out with the scales D and CI. (Division by reciprocal = multiplication). This method is popular with many slide rule users.

a × b

Example: 0·66 × 20·25 = 13·37.

Proceed as with division, i.e. first place the cursor line above 0·66 on D, then place the 20·25 on the CI underneath the cursor line; the product, 13·37, on D, can now be read off under C 1.



(3) The following shows how simply products with a number of factors can be calculated:

Example: 0·66 × 20·25 × 2·38 = 31·8.

a × b × c

Multiply the first two factors as above; with C 1 above 13·37 (intermediate results) we immediately have the first setting required for multiplication by the next factor (by the method learnt first at the top of p. 8). Consequently, we now bring the cursor line into position above C 2·38. The result, underneath it, on D, is 31·8. This could be followed immediately by a further multiplication, by bringing the next factor, on CI, into position underneath the cursor line and reading off the result, under C 1 (or C 10, as the case may be), on D. We thus multiply alternately with the aid of D and CI (see above, for method) and with the aid of C and D (first method — see page 6).

(4) **Combined Multiplication and Division.**

Example: $\frac{36 \cdot 4}{3 \cdot 2 \times 4 \cdot 6} = 2 \cdot 473$

$\frac{a}{b \times c}$

This likewise can be advantageously performed with the scale CI.

First the division, i.e. cursor line above D 3-6-4, then C 3-2 underneath the cursor line. (Intermediate result: 11·37, underneath C 1). With C 1 above D 11·37, we already have the first setting for the subsequent multiplication by $\frac{1}{6.4}$ which is performed by the aid of the scale CI ($\frac{1}{c}$). The cursor line is thus now brought into position above CI 4·6, the result — 2·472 — likewise being found beneath cursor line.

Example for pratice: $\frac{44}{4 \cdot 85 \times 3 \cdot 66} = 2 \cdot 48$; $\frac{4 \cdot 774}{0 \cdot 63 \times 1 \cdot 24} = 6 \cdot 11$; $\frac{23 \cdot 1}{2 \cdot 73 \times 17 \cdot 9} = 0 \cdot 473$

The trigonometric and exponential calculations provide further fields of use for the CI scale.

Calculations with the Scales CF, DF and CIF

(1) **Formation of Tables**

Since with the "π-folded" scales CF and DF, the value 1 is more or less in the centre, they provide an advantageous means of continuing the calculations, in the formation of tables, thus avoiding the necessity for transposing of the slide when calculating on C and D.

$\frac{a}{b}:\frac{c}{d}:\frac{e}{f}$

Example: 75 lbs = 34 kgs. Place **C** 3-4 above **D** 7-5; this provides the conversion from English lbs. into kilogrammes. Beyond 50 kgs. however (**C** 5), no further readings can be taken. Here we switch over to the upper scales CF and DF, and the slide rule can be set to the necessary values with the aid of the cursor line.

If the "equivalence" in question (e.g. 75 lbs = 34 kgs.) is not known, but the formula 1 lb = 0·454 kgs., for example, is known, then **CF** 1 is placed underneath **DF** 4-5-4, and this likewise provides the conversion from lbs into kgs.

(2) **Multiplication**

If, in multiplication on C and D, the second factor proves "unsettable", or if the method of transposing of the slide has to be adopted, this can be avoided by setting CF to the second factor and taking a reading of the result on DF.

$a \times b$

Example: 2·91 × 4 = 11·64. Place **C** 1 above **D** 2-9-1, and place the cursor line above **CF** 4. Above it, on **DF**, a reading may be taken of the result (11·64).

Examples for practice: 18·4 × 7·4 = 136·1; 42·25 × 3·7 = 156·3; 1·937 × 6 = 11·62.

(3) **Multiplication and Division by π**

The change-over from the C and D scales to the CF or DF scale can be carried out direct with the cursor and results in multiplication by π.

Example: 1·184 π = 3·72. With the slide in the zero position (**C** 1 above **D** 1 and **C** 10 above **D** 10) the cursor line is brought into position above **D** 1-1-8-4, and the result — 3·72 — is read on **DF**, likewise beneath the cursor line.

$a \times \pi$

The converse operation results in division by π.

Examples: $\frac{18{\cdot}65}{\pi}$ = 5·94. Place the cursor line above **DF** 1-8-6-5 and read the result — 5·94 — on **D**.

$\frac{a}{\pi}$

Examples for practice:

Area of an ellipse: $F = a \times b \times \pi$; $F = 5{\cdot}25 \times 2{\cdot}22 \times \pi = 36{\cdot}6$.

Place **C** 10 above **D** 5-2-5, bring the cursor line into position above **C** 2-2-2, and read the result — 36·6 — on **DF**, without any need to take a reading of the intermediate result — 11·65 — on **D**.

Length of a circular arc: $s = \frac{\alpha \, r \, \pi}{180}$ $\quad s = \frac{26{\cdot}2 \times 352 \times \pi}{180} = 161$

We start with the division, i.e. use the cursor line to bring **C** 1-8 and **D** 2-6-2 opposite each other. No reading need be taken of the intermediate result 0·1455 (under **C** 1). The multiplication by 352 is carried out by placing the cursor line under 3-5-2. (Intermediate result: 51·2, on D). The multiplication by π is again carried out by switching over to the top part and by finding the result 161 underneath the cursor line on DF.

(4) **The CIF graduation** operates in conjunction with CF and DF just as CI does with C and D scales.

Examples for multiplication by a number of factors:

2·23 × 16·7 × 1·175 × 24·2 = 1059. Solution: CI-2·23 placed above D-16·7 by the aid of the cursor line; the latter is placed above CF-1·175. CIF 24·3 under the cursor line. Read result, 1059, on DF, above CF 1.

0·53 × 0·73 × 39·1 × 0·732 = 11·07. Solution: CI-0·53 placed above D-0·73 with the aid of the cursor line; the latter is placed above CF-39·1. CIF 0·732 underneath cursor line. Read the result 11·07, on DF, above CF 1.

Squares and Square Roots

These are determined with the aid of the root-scalcs and will be explained later, on page 19.

Cubes and Cube Roots

For the cube scale K we have the formula $\log x^3 = 3 \log x$, i.e. it has 3 decimal places in the zone of that of the basic scale.

Numbers can be cubed by changing over from the C- or D-scale to the K-scale, using the cursor line. a^3

Examples for practice: $1{\cdot}54^3 = 3{\cdot}65$; $2{\cdot}34^3 = 12{\cdot}8$; $4{\cdot}2^3 = 74{\cdot}1$; $6{\cdot}14^3 = 232$; $8{\cdot}82^3 = 686$; $0{\cdot}256^3 = 0{\cdot}0168$; $8{\cdot}98^3 = 724$.

Cube roots can be obtained by changing over from the K-scale to the C- and D-scales, using the cursor line, in which connection it must be noted that single-digit numbers must be set on the left, two-digit numbers in the middle and three-digit numbers on the right.

Examples for practice: $\sqrt[3]{4{\cdot}66} = 1{\cdot}67$; $\sqrt[3]{29{\cdot}5} = 3{\cdot}09$; $\sqrt[3]{192} = 5{\cdot}77$; $\sqrt[3]{6{\cdot}8} = 1{\cdot}895$; $\sqrt[3]{0{\cdot}645} = 0{\cdot}864$; $\sqrt[3]{1953} = 12{\cdot}5$. $\sqrt[3]{a}$

Calculations with the Pythagoraean Scale P

This scale represents the function $y = \sqrt{1-(0{\cdot}1x)^2}$; it operates in conjunction with **D** (= x), of which the values must be read from 0·1 to 1. The graduation runs in the opposite direction and is therefore coloured red.

If D is set to the value x, then the corresponding value $y = \sqrt{1-x^2}$ can be read on P; conversely, if D is set to y, the value $x = \sqrt{1-y^2}$ can be read on P.

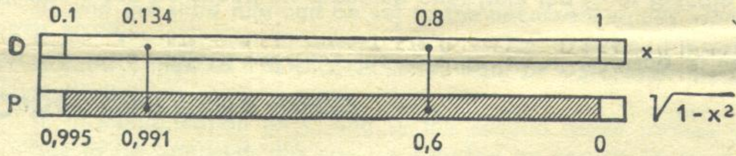


Fig. 9

Example: $y = \sqrt{1-0{\cdot}8^2} = 0{\cdot}6$; $x = \sqrt{1-0{\cdot}6^2} = 0{\cdot}8$
If, therefore, we set D to x = 0·8, we find the value y = 0·6 on P, and vice versa.
Example: sin α = 0·134; cos α = 0·991
If D is set to the sine, the cosine is obtained on P, and vice versa.

$$\sqrt{1-(0{\cdot}1x)^2}$$

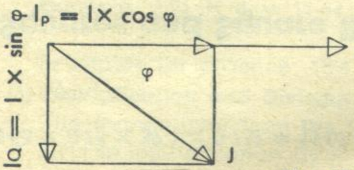


Fig. 10

Example:

Calculate the active current and the reactive current of a circuit absorbing 35 A with a cos φ of 0·8.

$I_p = I \times \cos\varphi = 35 \times 0{\cdot}8 = 28$ (A); $I_Q = I \times \sin\varphi = 35 \times 0{\cdot}6 = 21$ (A).

Place C 35 above D 10 and find at D 8 (for a cos φ of 0·8), on C, the active current 28; underneath D 8 the value 0·6 (i.e. sin φ = 0·6) is at the same time found on P. If the cursor is now moved into position onto D 6, the wattless current 21 is found above it on C.

Example: apparent power 530 kVA, effective power 428 KW. To find reactive power and cos φ:

Place C 530 above D 10, bring cursor into position above C 428, find the value (0·807) for the cos φ underneath it, on D; trace this value on P, by means of the cursor, and find the required reactive power — 313 kW — above it, on C.

Roots of numbers only just below 1 and 100 can be found by means of this scale with an increased degree of accuracy:

Example: $\sqrt{0{\cdot}925} = \sqrt{1-0{\cdot}075} = \sqrt{1-(0{\cdot}2739)^2} = 0{\cdot}9618$. We form the equation: $1-z = 1-0{\cdot}925 = 0{\cdot}075$.
Set the cursor on D 75 and find 0·2739 on W_1; the cursor is now placed on D 2739 and the reading 0·9618 is taken below it, on P.

Calculations with the Trigonometrical Scales S, T_1 and T_2

The trigonometric scales T_1, T_2 and S are sub-divided decimally; in conjunction with the basic scales C and D they indicate angular functions or, if read in the reverse direction, the angles.

When the T_1, T_2 and S scales are used in conjunction with the D, P and CI scales, as trigonometrical tables, the following must be noted:

sin *α*

The S scale, when the **black** figures are read, provides, in conjunction with the D scale (**black**), a **table of sines;** the same applies when the **red** figures are read, in conjunction with the P scale (**red**). In the case of small angles the first procedure is the more accurate, while in the case of wide angles preference is given to the second method.

cos *α*

The S scale, when the **red** figures are read, provides, in conjunction with D (**black**), a **table of cosines**; the same applies when the **black** figures are read, in conjunction with P (**red**). In the case of wide angles the former process is the more accurate, while with small angles preference is given to the latter method.

tan *α*

The two T scales, when the **black** figures are read, provide, in conjunction with the D scale (black), a **table of tangents;** the same applies when the **red** figures are read, in conjunction with CI (**red**).

cot *α*

The two T scales, when the **red** figures are read, provide, in conjunction with the D Scale (**black**), a **table of cotangents;** the same applies when the **black** figures are read, in conjunction with CI (**red**).

sin 13° = 0·225	/	S 13° (black) — D 0·225 (black)	
sin 76° = 0·97	/	S 76° (red) — P 0·97 (red)	
cos 11° = 0·982	/	S 11° (black) — P 0·982 (red)	
cos 78° = 0·208	/	S 78° (red) — D 0·208 (black)	
tan 32° = 0·625	/	T_1 32° (black) — D 0·625 (black)	
tan 57° = 1·54	/	T_2 57° (black) — D 1·54 (black)	
cot 18° = 3·08	/	T_2 18° (red) — D 3·08 (black)	or T_1 18° (black) — CI 3·08 (red)
cot 75° = 0·268	/	T_1 75° (red) — D 0·268 (black)	or T_2 75° (black) — CI 0·268 (red)

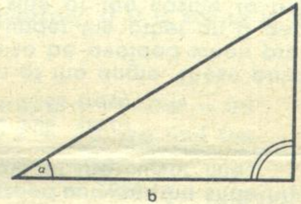
These settings are carried out with the slide rule set at zero, by the aid of the cursor line.

To proceed from the sine of angle to its cosine (or vice versa) no reading need be taken of the angle. These pairs of values are to be found one under the other on D and P. The reading of the angle can also be avoided when proceeding from the tangent to the cotangent, as these pairs of values are to be found one under the other on C and CI. An intermediate reading of the angle need only be taken when proceeding from the sine or the cosine to the tangent or cotangent. Since, when reading the functions, these can be obtained either on D or on CI, multiplications and divisions can in many cases follow immediately. It is only when the reading is taken on P that the value must be transferred to the main scales.

Further examples of the use of the trigonometrical and pythagoraean scales in a **right-angled triangle.**

(1) Example: Given: $a = 2$; $b = 3$. Find: c and α. Formula: $a \times \frac{1}{b} = \tan \alpha$; $a \times \frac{1}{c} = \sin \alpha$;

C 1 above D 2, cursor on CI 3; result for α (33·7) found on tan scale.
Cursor on 33·7 of the sine scale; value for c (3·6) found on CI.



(2) Example: Given $a = 8$; $b = 20$. Find c and α.
C 10 above D 8, cursor on CI 20; find α (21·8°) on tan scale.
Place cursor on 21·8 on sine scale; value for c (21·55) found on CI.

Fig. 11

(3) Example: Given: $a = 20$; $b = 8$. Find: c and α
C 1 above D 20, cursor on CI 8; value for α (68·2°) found on tan scale (T_2).
Cursor to 68·2 on sine scale; value for c (21·55) found on CI.

(4) Example: Given: $c = 5$; $\alpha = 36{\cdot}87^\circ$; Find: a and b. Formula: $a = c \times \sin \alpha$; $b = c \times \cos \alpha$.
C 5 above D 10, cursor on 36·87° on sine scale; value for a (3) found on C.
The value (0·8) for cos α is found on the P scale at the same time, and the cursor moved to D 8.
The value for b, i.e. 4, can now be read on the C scale.

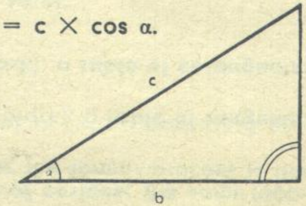


Fig. 12

(5) Example: Given: $c = 21{\cdot}54$; $b = 20$. Find: a and α.
C 2154 above D 10, cursor to be placed at C 2 (for $b = 20$) and find the value (21·8°) for α on the cos scale, but take the reading 0·372 at the same time from the P scale. Push the slide along to the left by one scale-length. Move cursor to D 0·372 and find the value for a, i.e. 8, on C.

For the **scalene triangle** we have the formula $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$

Example: Given: $a = 38{\cdot}3$; $\alpha = 52^\circ$; $\beta = 59^\circ$; $\gamma = 69^\circ$;

Find: b and c.

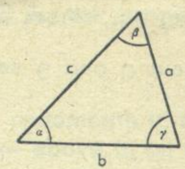


Fig. 13

Place C 383 above S 52°; above S 59° and S 69° we now find the results (41·7 and 45·4) on C.

The Small-Angle Scale ST and the ϱ Mark on Scale C and D

For the functions of small angles from 0·55 to 6° the lower part of the body of the slide rule bears the ST Scale ($\forall$ arc 0·01 x) with the formula $\sin\alpha \approx \tan\alpha \approx \text{arc}\,\alpha$

The ST scale operates in conjunction with scale D (or C).

All further calculations in this column are carried out solely with the aid of the cursor line.

The **functions of small angles** can also be determined with the mark $\varrho = \frac{\pi}{180} = 0{\cdot}01745$ in accordance with the formula: $\text{arc}\,\alpha = 0{\cdot}01745 \times \alpha = \varrho \times \alpha$.

In series calculations: set C 1 above ϱ on D; underneath the angle-value on C, read the result on D.

Examples for practice:

$\sin 2{\cdot}5^\circ \approx \tan 2{\cdot}5^\circ \approx \text{arc}\, 2{\cdot}5^\circ = 0{\cdot}0436$; $\sin 0{\cdot}4^\circ \approx \tan 0{\cdot}4^\circ \approx \text{arc}\, 0{\cdot}4^\circ = 0{\cdot}00698$; $\sin 0{\cdot}0052^\circ \approx \tan 0{\cdot}0052^\circ \approx \text{arc}\, 0{\cdot}0052^\circ = 0{\cdot}0000908$.

Set the angle-values on the arc-scale ST. **Read the functions from scale C (with zero setting) or from scale D (with the aid of the cursor line).**

Example: $\sin 3^\circ \approx \tan 3^\circ \approx \text{arc}\, 3^\circ = 0{\cdot}0524$.
Place beginning of slide, C 1, above D 3; the result (0·0524) can now be read on D, underneath ϱ on C.

For calculation of the cosine and cotangent functions of angles over 84·5°

Example: $\cos 88^\circ = \sin 2^\circ \approx \text{arc}\, 2^\circ = 0{\cdot}0349$
$\cot 86{\cdot}5^\circ = \tan 3{\cdot}5^\circ \approx \text{arc}\, 3{\cdot}5^\circ = 0{\cdot}0612$.

Place the cursor line above the angle-value on the **scale ST, and read the result from C (with zero setting) or on D, underneath the cursor line.**

Example: $\cos 88^\circ = \sin 2^\circ \approx \text{arc}\, 2^\circ \approx \varrho \times 2 = 0{\cdot}0349$
$\cot 86{\cdot}5^\circ = \tan 3{\cdot}5^\circ \approx \text{arc}\, 3{\cdot}5^\circ \approx \varrho \times 3{\cdot}5 = 0{\cdot}0612$

This is a simple multiplication; we thus place the beginning of the slide, C 1, above ϱ on D, then the cursor line above the second factor on C, and read the result underneath it, on D.

For conversion of arcs into angular degrees:

Examples for practice: $\overset{\frown}{6{\cdot}28} = 360^\circ$; $\overset{\frown}{1{\cdot}11} = 63{\cdot}5^\circ$; $\overset{\frown}{0{\cdot}04} = 2{\cdot}29^\circ$; $\overset{\frown}{0{\cdot}007} = 0{\cdot}402^\circ$; $\overset{\frown}{0{\cdot}64} = 36{\cdot}7^\circ$; $\overset{\frown}{0{\cdot}32} = 18{\cdot}35^\circ$.

Set the arc on the C or D Scale. The degrees can be read from the arc-scale ST (with the aid of the cursor line).

C 1 or C 10 above the ϱ mark on D, then place cursor line above the arc-measurement on D. The angular degrees can be read above it, on C.*

* Increased accuracy is achieved when using the ϱ marks on scales W_1 and W_1'.

Calculations with Complex Numbers

Two complex values $\mathfrak{x} = 7{\cdot}5\, e^{\frac{i\pi}{8}}$ and $\mathfrak{y} = 3{\cdot}4\, e^{\frac{i\pi}{10}}$ are to be added. They are first of all concerted, in accordance with the

Euler Equation $R \times e^{i\varphi} = R\,(\cos\varphi + i \sin\varphi)$ to the form $(a + i\, b)$.

For calculations on the slide rule it is best to express these magnitudes vectorially: as follows: $\mathfrak{x} = 7{\cdot}5 / 22{\cdot}5^0$ and $\mathfrak{y} = 3{\cdot}4 / 18^0$ we can now calculate as follows:

(1) Place C 75 above D 10, move cursor to S 22·5⁰ and read the value of b_1 (2·87) on C. At the same time find the value of $\cos\varphi$ (0·924) on the P scale. Mcve cursor to D 924 and find the value of a_1 (6·93) on C.

$(a_1 + i\, b_1) = 6{\cdot}93 + i\, 2{\cdot}87.$

(2) Place C 34 above D 1, move cursor to S 18⁰, and find the value for b_2 (1·05) on C. At the same time, find the value for $\cos\varphi$ (0·951) on P. Move cursor to D 951 (red extended graduation) and find the value for a_2 (3·24) on C.

$(a_2 + i\, b_2) = 3{\cdot}24 + i\, 1{\cdot}05$

$a_1 + a_2 = 6{\cdot}93 + 3{\cdot}24 = \mathbf{10{\cdot}17}$ and $i\,(b_1 + b_2) = i\,(2{\cdot}87 + 1{\cdot}05) = \mathbf{i\, 3{\cdot}92}.$

The result is thus: $\mathfrak{z} = (10{\cdot}17 + i\, 3{\cdot}92)$

If the result is to appear in vectorial form, we calculate thus:

Place C 10 above D 392, move cursor to CI 1017, and find the value of φ (21·07⁰) above it, on T_1 (Tangent scale). We then move the cursor to 21·07 on the sine scale S and find the value for $\mathfrak{z}$ (10·92) above it, on CI.

Thus $\mathfrak{z} = (10{\cdot}17 + i\, 3{\cdot}92) = 10{\cdot}92 / 21{\cdot}07^0$

and with $\varrho \times \varphi = \varphi$, $(\varphi = 0{\cdot}368)$ we have $\mathfrak{z} = 10{\cdot}92 / 21{\cdot}07^0 = 10{\cdot}92\, e^{i\, 0{\cdot}368}$

Example for the use of the T_2 scale: $\mathfrak{z} = 192 - i\, 256$

Place C 10 above D 256, move cursor to CI 192. On T_2 we obtain the angular value 53·1⁰. Then place C 1 above D 256 and move the cursor-line to S 53,1. On CI we obtain the result 320.

$\mathfrak{z} = 320 / \underline{-53{\cdot}1^0}$

As the number is in the IVth Quadrant, the angular value must be negative.

The **multiplication** of complex numbers is carried out in accordance with the equation:

$\mathfrak{x} \times \mathfrak{y} = X \times e^{i\varphi} \times Y \times e^{i\psi} = XY \times e^{i(\varphi+\psi)} = XY / \varphi + \psi$

Example: $(1 + 2i) \times (3 + 1i) = 2{\cdot}236 \times e^{i \times 1{\cdot}107} \times 3{\cdot}162\, e^{i \times 0{\cdot}316} = 2{\cdot}236 / \underline{63{,}45^0} \times 3{,}162 / \underline{18{,}42^0} = 7{,}07 / \underline{81{,}9^0} = 7{,}07\, e^{i \cdot 1{,}423}$

Calculations with the Root Scales W_1 W_1' W_2 W_2'

The advantage of these scales is the increased degree of accuracy with which calculations can be carried out on the usual handy 10" standard model. It is above all for the main calculations that they are used.
Operations with the root scales differ to some extent from the method to which we have so far become accustomed but can be mastered according to a basic rule and with a short period of practice. It should be noted that the root scales are sub-divided on the **20 inch graduation length.**

Multiplication

(I) **When the setting is carried out with the black Index-1 (or Index-10), the product is found on that scale of the body of the slide rule which is adjacent to the second factor.**

(II) **When the setting is carried out with the red index mark, the product is read on that scale of the body of the slide rule which is opposite to the second factor.**

Examples for (I): 1·635 × 5·365 = 8·77
Solution: Black Index-1 (W_1'-1) above W_1-1·635.
Cursor line above W_2'-5·365; find result (8·77) on W_2.

809 × 1·414 = 1144
Solution: Black Index-10 (W_2'-10) underneath W_2-809.
Cursor line above W_1'-1·414; find result (1144) on W_1.

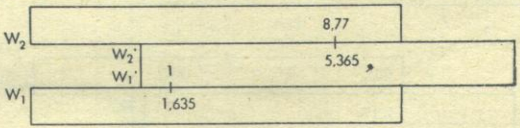
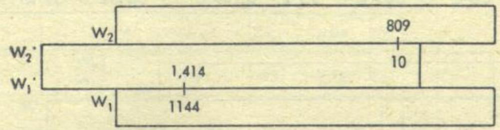


Fig. 14



Examples for practice: 236 × 4·06 = 958; 2·34 × 0·409 = 0·957; 7·77 × 66·3 = 515; 5·165 × 0·2265 = 1·1699

Examples for (II): 1·804 × 7·73 = 13·95
Solution: Red Index-mark above W_1-1·804; cursor line above W_2'-7·73; at the same time find the result (13·95) underneath the cursor line, on the opposite scale W_1.

50·45 × 4·64 = 234·1. Solution: Red index-mark under W_2 50·45; cursor line above W_2'-4·64; at the same time find the result (234·1) underneath the cursor line on the opposite scale W_1.

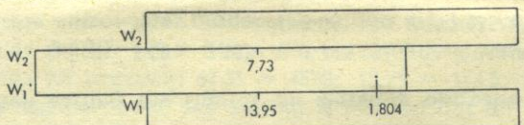
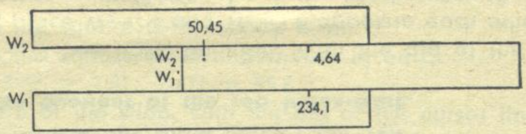


Fig 15



Examples for practice: 14·78 × 0·945 = 13·97; 29·4 × 123·6 = 3634; 0·395 × 0·562 = 0·222; 3·885 × 19·425 = 75·47

Division

(I) **When the figures are set on adjacent scales, the quotient is read at the black index-1 (or -10).**

(II) **When setting the figures on mutually opposite scales, find the quotient at the red index-mark.**

Examples for (I): 3·08 ÷ 2·135 = 1·443. Solution: With the aid of the cursor line, place W_1-3·08 and W_1'-2·135 opposite each other, and find the result (1·443) on W_1, underneath the black index-1.

42·3 ÷ 71·7 = 0·589. Solution: With the aid of the cursor line, place W_2-42·3 and W_2'-71 7 opposite each other, and find the result (0·589) above the black index-10, on W_2.

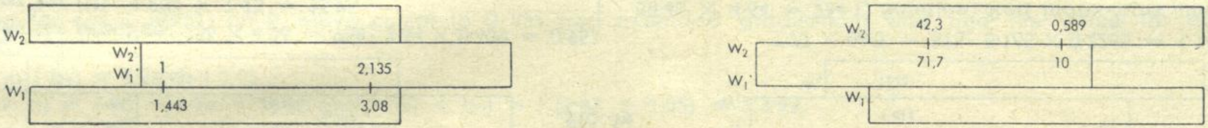


Fig. 16

Examples for practice: 2·975 ÷ 18·65 = 0·1595; 2·075 ÷ 148·25 = 0·014; 48·65 ÷ 79·05 = 0·6155; 5·55 ÷ 0·692 = 8·02

Examples for (II): 374·5 ÷ 1·5675 = 238. Solution: cursor line above W_2-374·5; W_1'-1·5675 under cursor line; the result, 238, is read on W_1, underneath the red index-mark.

23·77 ÷ 65·67 = 3·615. Solution: cursor line above W_2-23·77; W_2'-65·67 under cursor line; the result, (3·615) is found on W_2, above the red index-mark.

Fig. 17

Examples for practice: 689·5 ÷ 2·505 = 275·2; 432·5 ÷ 1·845 = 234·4; 1·965 ÷ 44·45 = 0·0442; 8·37 ÷ 1·1575 = 7·23

Formation of Tables

Set the "equivalence" or the "unit-value" and then take the reading in accordance with the basic rules given on the preceding pages.

Example: "Equivalence": 82 yards = 75 metres. By means of the cursor line place W_2-82 and W_2'-75 opposite each other. The following readings can now be taken, likewise with the cursor line: 42 yards = 38·4 metres; 136 yards = 124·4 metres.

This example corresponds to normal cases as in basic rule (I).

The following examples can only be dealt with in accordance with basic rule (II) and by the aid of the red index-mark.

Examples: "Unit Value": 1" = 2·54 cm. ("Equivalence": 26" = 66 mm); The red index-mark on W_1 is placed opposite the value 2·54 on W_1; the inches can now be read on W_1' and W_2' and the cm on W_1 on W_2.
20" = 50·8 cm; 40 cm = 15·75".

Exchange rate: \$ 1 = DM 4·00. The value 4·00 on W_2 is placed opposite the red index-mark; the DM can now be read on W_1 and W_2 and the \$ on W_1' and W_2':
\$ 1·50 = DM 6·00; \$ 2·26 = DM 9·04;
DM 7·40 = \$ 1·85; DM 10 = \$ 2·50.

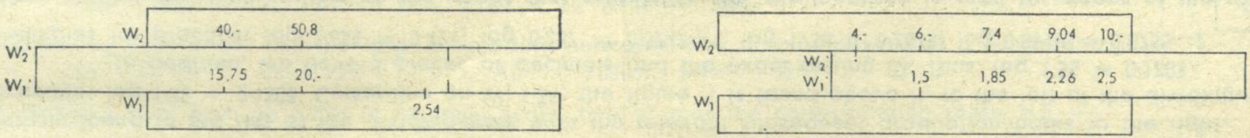


Fig. 18

Squares and Square Roots

a^2

Squares of numbers are found by changing over from the W scales to the C scale situated on the centre of the slide, using the cursor line.

Examples: $1·66^2$ = 2·76. Place the cursor line on W_1-1·66, and find the square (2·76) above it, on C.
$5·25^2$ = 27·6. Place the cursor line on W_2-5·25, and find the square (27·6) underneath it, on C.

$\sqrt{a}$

Examples for practice: $67·3^2$ = 4530; $10·7^2$ = 114·5; $2·3^2$ = 5·29; $1·345^2$ = 1·81; $7·47^2$ = 55·8

To find **square roots,** the setting is carried out on C, on the centre of the slide, with the aid of the cursor line, and the square-root is then found on scales W_2' or W_1', likewise underneath the cursor line. It should be noted that the roots of numbers from 1 to 10 will be found on the W_1 scale and those of numbers from 10 to 100 on the W_2 scale.

Examples: $\sqrt{4{\cdot}56} = 2{\cdot}135$. Cursor line on C-4·56; the result (2·135) will be found below it, on W_1' or W_1.

$\sqrt{56} = 7{\cdot}483$. Cursor line on C-56; the result will be found above it, on scale W_2' or W_2.

To extract roots of numbers under 1 (e.g. 0·76) or above 100 (e.g. 2375) with greater ease, suitable powers are separated from the radicand.

Example: $\sqrt{0{\cdot}76} = \sqrt{76 \div 100} = \sqrt{76} \div 10 = 8{\cdot}72 \div 10 = 0{\cdot}872$; $\sqrt{275} = \sqrt{100 \times 2{\cdot}75} = 10 \times \sqrt{2{\cdot}75} = 10 \times 1{\cdot}658 = 16{\cdot}58$

$\sqrt{2375} = \sqrt{100 \times 23{\cdot}75} = 10 \times \sqrt{23{\cdot}75} = 10 \times 4{\cdot}875 = 48{\cdot}75$; $\sqrt{0{\cdot}00378} = \sqrt{37{\cdot}8 \div 10000} = \sqrt{37{\cdot}8} \div 100 = 6{\cdot}15 : 100 = 0{\cdot}0615$

Calculations with the Mantissa Scale L

lg a

This scale operates in conjunction with the W scale. In this connection, attention should be paid to the zero position of the slide rule:

(I) When setting the antilogarithm on **the lower root-scales W_1', W_1,** the mantissa is read by means of the characteristic **found to the left of the dividing-mark** with the relevant subsequent graduation-marks to the right.
Example: log 1·35 = 0·1303. Cursor-line on W_1-1·35; the figure ·1 is found above it, to the left of the dividing-mark; in addition, we have 3 places of decimals and the exact setting 03. Thus, log 1·35 = 0·1303.
Examples for practice: log 2·655 = 0·424; log 0·237 = 0·3747—1; log 1938 = 3·2875; log 0·0119 = 0·0755—2

(II) When setting the antilogarithm on **the upper root scales W_2', W_2,** the mantissa is read by means of the characteristic **found to the right of the dividing-mark,** with the relevant subsequent graduation-marks to the right.
Example: log 57·3 = 1·758. Cursor line on W_2-57·3; the figure ·7 is found underneath it, on the right of the dividing mark; in addition we have 5 decimal places and the exact setting 8. Thus log 57·3 = 1·758.
Examples for practice: log 9·06 = 0·957; log 0·0636 = 0·8034—2; log 445 = 2·6484; log 66·5 = 1·823.

If the mantissa is known, the converse process provides the antilogarithm sought.

The logarithms enable the types of calculation to be reduced by one stage; multiplication and division thus become addition and subtraction, while the raising of numbers to powers or the extraction of roots becomes multiplication or division respectively.

For example: $245^{3{\cdot}24} = 3{\cdot}24 \times \log 245 = 3{\cdot}24 \times 2{\cdot}389 = 7{\cdot}74$; $245^{3{\cdot}24} = 55\,000\,000$

$$420^x = 10000;\ x \times \log 420 = \log 10000;\ x = \frac{\lg 10000}{\lg 420} = \frac{4{\cdot}0}{2{\cdot}623} = 1{\cdot}525$$

The Exponential Scales LL_1, LL_2 and LL_3 for positive exponents; LL_{01}, LL_{02} and LL_{03} for negative exponents

The back of the slide rule has two three-stage groups of scales for the exponential functions, which are based on the basic scale C. The scales for positive exponents (black) extend from 1·0095 to 60000, while those for negative exponents (red) extend from 0·00002 to 0·9905. The e^{-x} scales are reciprocal scales for the e^x scales. In this connection **it should be noted that the numerical values given on the exponential scales do not change where the decimal places are concerned**; thus, "1·04" invariably signifies 1·04 — not 10·4, 104, etc.

When changing over from an internal scale to the next one, proceeding outwards, the exponential scales give powers of ten, e.g. —

$0{\cdot}955^{10} = 0{\cdot}631$; $0{\cdot}631^{10} = 0{\cdot}01$; $0{\cdot}924^{10} = 0{\cdot}454$; $0{\cdot}454^{10} = 3{\cdot}7 \times 10^{-4} = 0{\cdot}00037$

$1{\cdot}0472^{10} = 1{\cdot}586$; $1{\cdot}586^{10} = 101$; $1{\cdot}08^{10} = 2{\cdot}16$; $2{\cdot}16^{10} = 2{\cdot}2 \times 10^3 = 2200$

a^{10}

a^{100}

When changing over to the next but one, proceeding outwards, one obtains powers of a hundred, e.g. —

$0{\cdot}955^{100} = 0{\cdot}01$; $1{\cdot}04715^{100} = 100$; $0{\cdot}924^{100} = 3{\cdot}7 \times 10^{-4} = 0{\cdot}00037$; $1{\cdot}8^{100} = 2200$

When changing over from an outer to an inner scale, one obtains the corresponding roots, e.g. —

$\sqrt[10]{0{\cdot}25} = 0{\cdot}8705$; $\sqrt[10]{0{\cdot}8705} = 0{\cdot}98623$; $\sqrt[100]{0{\cdot}25} = 0{\cdot}98623$; $\sqrt[10]{0{\cdot}00007} = \sqrt[10]{7 \times 10^{-5}} = 0{\cdot}384$; $\sqrt[10]{0{\cdot}384} = 0{\cdot}9087$; $\sqrt[100]{0{\cdot}00007} = 0{\cdot}9087$;

$\sqrt[10]{4} = 1{\cdot}1488$; $\sqrt[10]{1{\cdot}1488} = 1{\cdot}01396$; $\sqrt[100]{4} = 1{\cdot}01396$;

$\sqrt[10]{15000} = \sqrt[10]{1{\cdot}5 \times 10^4} = 2{\cdot}616$; $\sqrt[10]{2{\cdot}616} = 1{\cdot}1009$; $\sqrt[100]{15000} = 1{\cdot}1009$.

$\sqrt[10]{a}$; $\sqrt[100]{a}$

Note the following:

At 100 on the LL_3 Scale, the LL_{03} scale shows the value of $\frac{1}{100} = 0{\cdot}01$

At 1·25 on the LL_2 Scale, the LL_{02} scale shows the value of $\frac{1}{1{\cdot}25} = 0{\cdot}8$

since $e^{-x} = \frac{1}{e^x}$

Powers of e

e^n

The powers of e (base of natural logarithm e = 2·71828...) are obtained by setting the exponent on the C scale, with the aid of the cursor line (the slide rule being at zero). The e power is then found on the LL scale. In this connection the range for the C scale is 1 - 10 at LL_3, 0·1 - 1 at LL_2 and 0·01 - 0·1 at LL_1.

Examples: $e^{1\cdot61} = 5$; $e^{0\cdot161} = 1\cdot1747$; $e^{0\cdot0161} = 1\cdot01622$; $e^{6\cdot22} = 5 \times 10^2 = 500$; $e^{0\cdot622} = 1\cdot862$; $e^{0\cdot0622} = 1\cdot0642$;

$$e^{-1\cdot61} = \frac{1}{e^{1\cdot61}} = 0\cdot2;\quad e^{-0\cdot161} = 0\cdot8513;\quad e^{-0\cdot0161} = 0\cdot98402.$$

$$e^{-6\cdot22} = \frac{1}{e^{6\cdot22}} = 2 \times 10^{-3} = 0\cdot002;\quad e^{-0\cdot622} = 0\cdot537;\quad e^{-0\cdot0622} = 0\cdot9396.$$

$$e^{12\cdot5} = e^{10+2\cdot5} = e^{10} \times e^{2\cdot5} = 22000 \times 12\cdot2 = 268400$$

In forming the **hyperbolic functions,** the argument x is fixed on the C scale with the aid of the cursor.

On the e^x and e^{-x} scales, reading can then be taken of the e powers. Half the sum (or difference) then gives the cos h (or sin h), e.g. —

sinh x
cosh x

$$\cosh 35^0 = \cosh 0\cdot61 = \frac{e^x + e^{-x}}{2} = \frac{1\cdot84 + 0\cdot543}{2} = 1\cdot1915$$

$$\sinh 35^0 = \sinh 0\cdot61 = \frac{e^x - e^{-x}}{2} = \frac{1\cdot84 - 0\cdot543}{2} = 0\cdot6485$$

Roots of e

$\sqrt[n]{e}$

We express the root as a power with a reciprocal exponent and then proceed as above.

Example: $\sqrt[4]{e} = e^{0\cdot25} = 1\cdot284$; $\sqrt[0\cdot25]{e} = e^4 = 54\cdot6$; $\sqrt[8]{e} = e^{0\cdot125} = 1\cdot133$; $\sqrt[0\cdot125]{e} = e^8 = 2980$

$\sqrt[12\cdot5]{e} = e^{0\cdot08} = 1\cdot0834$; $\sqrt[0\cdot06]{e} = e^{16\cdot66} = e^{8\cdot33} \times e^{8\cdot33} = 4146 \times 4146 = 17189000$

The Natural Logarithms

$\ln a$

The natural logarithms are found by changing over from the LL scales to the central scale C of the slide. The foregoing applies, mutatis mutandis, to the "place-of-decimal ranges" of the basic scale.

Example: ln 25 = 3·22; ln 147 = 4·95; ln 1·3 = 0·262; ln 0·04 = —3·22; ln 0·66 = —0·416; ln 0·98 = —0·0202.

Powers of any numbers desired

a^n

Powers of the form a^n are obtained by moving C-1 into position above the basic value, a, of the corresponding LL scale, after which the cursor is moved to C-n. The value of a^n can then be found on LL, e.g. —

$3{\cdot}75^{2{\cdot}96} = 50$; place C-1 above LL_3-375 and find the value (50) at C-296 on LL_3;

Further examples:

$4{\cdot}2^{2{\cdot}16} = 22{\cdot}2$; $4{\cdot}2^{0{\cdot}216} = 1{\cdot}364$; $4{\cdot}2^{0{\cdot}0216} = 1{\cdot}0315$

$4{\cdot}2^{-2{\cdot}16} = 0{\cdot}045$; $4{\cdot}2^{-0{\cdot}216} = 0{\cdot}7335$; $4{\cdot}2^{-0{\cdot}0216} = 0{\cdot}96945$

By means of the LL_{03} scale:

$0{\cdot}05^{2{\cdot}16} = 1{\cdot}55 \times 10^{-3} = 0{\cdot}00155$

$0{\cdot}05^{0{\cdot}216} = 0{\cdot}524$; $0{\cdot}05^{0{\cdot}0216} = 0{\cdot}9374$

$0{\cdot}05^{-2{\cdot}16} = \dfrac{1}{0{\cdot}05^{2{\cdot}16}} = 646$ (reading to be taken on the LL_3 scale).

$0{\cdot}05^{-0{\cdot}0216} = \dfrac{1}{0{\cdot}05^{0{\cdot}216}} = 1{\cdot}91$ (reading to be taken on the LL_2 scale).

As regards the places of decimals, see remarks under "Powers of e."

Roots of any numbers desired

$\sqrt[n]{a}$

With the aid of the cursor line, place the root-exponent on C above the root-number on LL (first finding the root-number and placing the cursor line above it); the result is then found underneath C 1 or C 10.

Example:

$\sqrt[4{\cdot}4]{23} = 2{\cdot}04$; place C 4·4 above LL_3 23 and read the value (2·04) at C 10 on LL_2.

Examples: $\sqrt[2{\cdot}08]{1{\cdot}068} = 1{\cdot}03216$ (place C-2·08 above LL_1-1·068, take reading on LL_1).

$\sqrt[0{\cdot}6]{15{\cdot}2} = 93{\cdot}5$ (place C-0·6 above LL_3-15·2, take reading on LL_3).

$\sqrt[20]{4{\cdot}41} = 1{\cdot}077$ (place C-20 above LL_3-4·41, take reading on LL_1).

$\sqrt[5]{0{\cdot}5} = 0{\cdot}8705$ (place C-5 above LL_{02}-0·5, take reading on LL_{02}).

$\sqrt[50]{0{\cdot}5} = 0{\cdot}98622$ (place C-50 above LL_{02}-0·5, take reading on LL_{01}).

Further examples: $\sqrt[5]{2} = 1{\cdot}149$; $\sqrt[5]{20} = 1{\cdot}82$

$\sqrt[0{\cdot}06]{2{\cdot}42} = 2{\cdot}42^{16{\cdot}66} = 2{\cdot}42^{8{\cdot}33} \times 2{\cdot}42^{8{\cdot}33} = 1580 \times 1580 = 2496400$

The Logarithms to "Base 10"

lg a

Place the cursor line above LL_3-10 and C 1 of the central slide-scale underneath the cursor line. This provides a table of the logarithms to base 10. It can also be set, however, by placing C 10 above LL_3-10.
Settings and readings can now be carried out by the aid of the cursor.

log 10 = 1; log 100 = 2; log 1000 = 3; log 200 = 2·301
log 20 = 1·301; log 2 = 0·301; log 1·1 = 0·0414.

By means of the LL_{03} scale:
log 0·1 = -1; log 0·01 = -2; log 0·001 = -3.
log 0·2 = -0·699 = 0·301-1; log 0·05 = -1·301 = 0·699-2.

Production of Logarithmic Diagrams to any desired scale.*

When diagrams are produced with a logarithmic graduation, it is often necessary to solve y = a log x. (a = scale-factor = length of logarithmic unit). We cannot change over from C to L, since no further multiplications can be carried out on the linear L series. On the other hand, of course, the change-over from LL to D corresponds to the operation of forming the logarithm, and the C scale can then be used for further multiplication.
Example: a = 3·33; x = 2; 3; 4; 6.

* by a method devised by W. Rehwald, Dipl.-Phys. at the High Frequency Institute of the College of Darmstadt.

Place C 3·33 above LL_3 10 (logarithmic unit) and find, with LL_3 or LL_2 2; 3 . . ., the relevent y-values on the C scale.

y = 1·002; 1·591; 2·003; 2·593.

If necellary, push the slide through. A little thought will enable errors as regards the place of decimals to be avoided.

Logarithms to any desired base

$^{n}\log a$

Place the beginning of the C scale above the base on the LL scale; this provides a table of the relevant logarithms, e.g. $^{2}\log 200 = 7{\cdot}64$; $^{2}\log 22 = 4{\cdot}46$; place C 10 above LL_2-2; find the value 7·64, at LL_3-200, on C, and the value 4·46, at LL_3-22, on C.

Further examples:

$^{2}\log 1{\cdot}2 = 0{\cdot}263$; $^{0\cdot2}\log 10 = -1{\cdot}431$; $^{0\cdot8}\log 2 = -3{\cdot}11$; $^{5}\log 25 = 2$; $^{0\cdot5}\log 25 = -4{\cdot}64$;

Note the following: $^{a}\log a = 1$; e.g.: $^{2}\log 2 = 1$; $^{2}\log 4 = 2$; $^{2}\log 8 = 3$

$^{0\cdot5}\log 0{\cdot}5 = 1$; $^{0\cdot5}\log 4 = -2$; $^{0\cdot5}\log 8 = -3$

$^{0\cdot5}\log 0{\cdot}25 = 2$; $^{0\cdot5}\log 0{\cdot}125 = 3$

Exponential scale for $e^{0\cdot001\,x}$

Within the C scale on the back of the slide rule, just to the left of the numbers 4, 5, 6, 7, 8, 9, and 10, indented red marks are provided which give the values of an $e^{0\cdot001x}$ scale. Up to the value $e^{0\cdot003} = 1{\cdot}003$ these marks are not necessary, as the value $e^{0\cdot003}$ only differs from 1·003 by 0·000005.

The said red marks are very easy to use if one imagines them as bearing the references 1·004, 1·005, etc.

If the cursor is moved to C 7, then the same scale gives the value 1·00703 for $e^{0\cdot007}$. Within the interval this reading can also be taken by mentally adding on the interval-difference given by the red mark, e.g. $e^{0\cdot0074} = 1{\cdot}00743$.

To find the natural logarithm, the converse procedure is adopted. The cursor is placed on the mark, or the interval-difference is taken into account, and the values are read from the C scale, e.g. ln 1·008 = 0·00797 and ln 1·0063 = 0·00628.

If, for example, a table is to be formed of the logarithms to base 1·005, use is made of the D scale on the front of the slide rule together with the C scale on the back, by placing the red mark 1·005 of the C scale above the 1 of the D scale. Examples of the results of this procedure are as follows:

$^{1\cdot005}\log 1\cdot005 = 1$; $^{1\cdot005}\log 1\cdot009 = 1\cdot8$

If the table is to be extended beyond 1·01, the cursor is placed in the basic position; the cursor is then set to the 1·005 mark and C 10 placed underneath the cursor line. Readings can now be taken of the logarithms to base 1·005, for the values above 1·01, by changing over from LL_1 (or LL_2 or LL_3) to the C scale, e.g.

$^{1\cdot005}\log 1\cdot01 = 1\cdot996$; $^{1\cdot005}\log 1\cdot02 = 3\cdot97$

$^{1\cdot005}\log 1\cdot2 = 36\cdot5$; $^{1\cdot005}\log 4 = 278$

Meaning of the Scale Marks

The value $\pi = 3\cdot1416$ is specially marked on the scales C, D, CI, CF, DF, CIF, W_1, W_1', W_2, W_2'. This makes the value π far easier to find and set.

Mark $\varrho = 0{,}01745$ on the scales C, D, W_1, W_1' (see page 15).

The **ST scale for small angles** contains so-called **correction-marks** in the range 4^0-6^0, which give the correct functional values for sine and tangent.

Examples: $\tan 2\cdot5^0 \approx \sin 2\cdot5^0 = 0\cdot0436$; $\tan 4^0 \approx \sin 4^0 = 0\cdot0697$.

For an **exact** reading the tangent 4^0, the correction mark to the **right** of the graduation mark 4^0 is used. A reading is taken of 0·0699.

For the correction-marks of the tangent we thus have:

Tangent **greater** than arc, therefore correction-mark to the **right** of the graduation-mark.

Example: $\tan 5^0 = 0\cdot0875$.

If the angle is intermediate between full degrees having correction-marks, then the correction-interval must be transferred accordingly.

Example: $\tan 3\cdot5^0 = 0\cdot0612$; $\tan 4\cdot2^0 = 0\cdot0734$; $\tan 5\cdot33^0 = 0\cdot0934$.

If the functional value is known and the angle is required, the correction-interval to the **left** is taken into account.

For the sine, the correction-mark is provided to the **left** of graduation-mark 6^0. It applies to the range 5^0-6^0.

Proceed in this case as in the foregoing, only in the opposite direction.

Mark $e = 2{,}71828$ (base of the nat. log) on the scales LL_2 and LL_3.

The Cursor

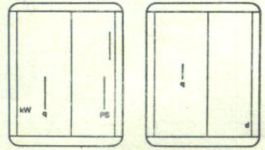


Fig. 19

The two-sided cursor has a long central line on the front and back, to enable settings to be carried out and readings to be taken in ordinary continuous calculations; it also has the red side-marks on the right-hand and left-hand edge, for reading values on the extended red supplementary graduations which the central cursor line cannot reach.

The fields of application for the remaining marks are as follows:

To calculate the areas of circles (d, s), scale W_1 or W_2 is set to the diameter, with the aid of the right-hand mark (d) on the back of the cursor; the relevant cross-section can be found underneath the cursor-mark s on scale C (rear side) or D (front side).

The conversion of kW into h.p. (and vice versa), can be carried out with the H.P. and kW marks on the C and D scales.

For direct calculation with the factor 3·6 the upper right-hand mark on the upper side of the cursor is used.

Examples: 150 km/h = 41·6 m/sec (mark 3·6 on DF 150 gives 41·6 under the main line on D).
Determine the interest on DM 2420 at 3·75% in 95 days. (Mark 3·6 on DF 2420; CI 3·75 under main line; the interest, i. e. DM 23·94, can be seen on DF, above CF 95).

The double cursor can be easily removed for cleaning, without interfering with the fine adjustment of the slide rule. Press open the two white cheeks in the notched part of the lower edge of the cursor. It is essential that the user's thumb should first pull the side portion of the cheek downwards, thus releasing the cursor-fastening. The two plexi-glass windows are then bent apart just as far as is necessary to enable the cursor to be slid off at the top. When replacing the cursor, see that the five index-marks of the window are on the front of the slide rule.

The Care of the Slide Rule:

CASTELL Slide Rules are valuable precision implements and require careful handling.
They are made of an ideal special plastic material. This is highly elastic and thus unbreakable provided it is competently handled. It will stand up to climatic conditions; it is moisture-proof and non-inflammable and will resist the majority of chemicals. These slide rules should nevertheless not be allowed to come in contact with corrosive liquids or powerful solvents, which are at all events liable to attack the colouring-agents applied to the graduation-marks, even if they do not actually harm the material itself. If necessary, the smooth movement of the slide can be improved by the use of vaseline or silicon oil. In order not to detract from the accuracy of the readings, the scales and the cursor should be protected from dirt and scratches and should be cleaned with the special cleaning agents CASTELL No. 211 (liquid), or No. 212 (cleaning paste).

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