FABER-CASTELL

INSTRUCTIONS

CASTELL-Biplex

PRECISION SLIDE RULE

Nr. 2/82 62/82

$\sqrt[3]{527}$ = 7,32

$\frac{3750}{0,25 \cdot 4[illegible]}$ $2\frac{1}{2}$ $D = \frac{\pi \, 32^2}{4}$

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Contents

Instructions for the Precision Slide Rule CASTELL-Biplex

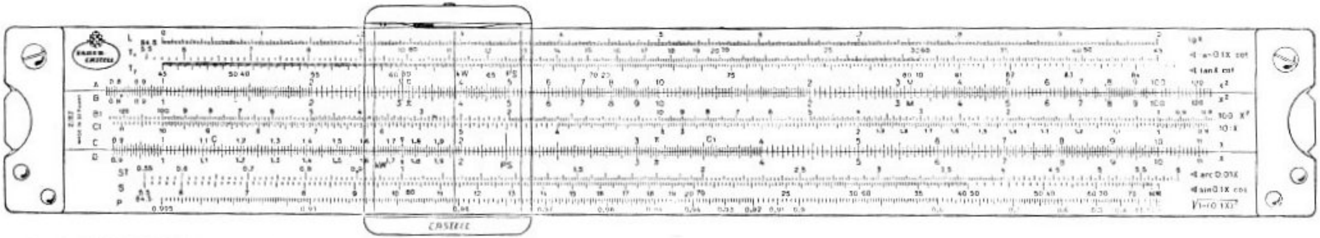
The double-side type slide rule represents a practical extension of the "Darmstadt" model slide rule. The structure of the principal scales has been left the same as it is on this well-known and proven model. The supplementary scales have all been devised so as to fit in with them in the best possible manner.

Scales of the Slide Rule

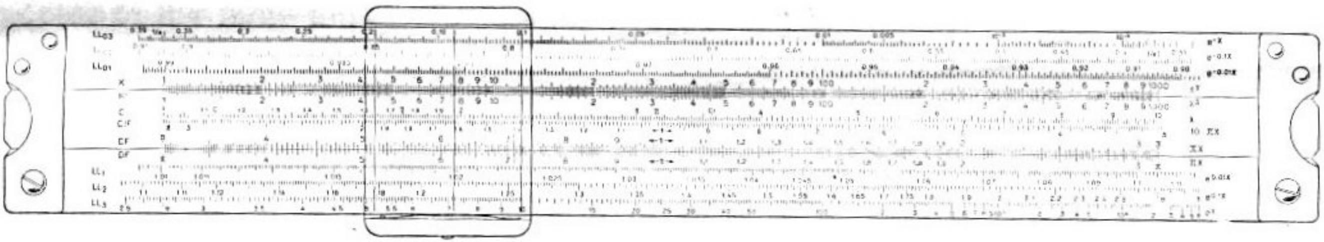
The **front of the slide rule** contains the following scales:

Mantissa scale	L	$\lg x$	Upper stock
1st scale of tangents	T_1	$\sphericalangle \tan 0{\cdot}1\,x$	
2nd scale of tangents	T_2	$\sphericalangle \tan x$	
fixed scale of squares	A	x^2	
movable scale of squares	B	x^2	Slide
Scale of reciprocals to B	BI	$100 \div x^2$	
Scale of reciprocals to C	CI	$10 \div x$	
Movable base scale	C	x	
Fixed base scale	D	x	Lower stock
Radian scale	ST	$\sphericalangle \mathrm{arc}\ 0{\cdot}01\,x$	
Scales of sines	S	$\sphericalangle \sin 0{\cdot}1\,x$	
Pythagorean scale	P	$\sqrt{1-(0{\cdot}1x)^2}$	

Front of the 10" slide rule



Back of the 10" slide rule



The **back of the slide rule** contains the following scales:

Log log scales for negative exponents	LL_{03}	e^{-x}	Upper stock
	LL_{02}	$e^{-0.1x}$	
	LL_{01}	$e^{-0.01x}$	
Fixed scale of cubes	K	x^3	
Movable scale of cubes	K'	x^3	Slide
Movable base scale	C	x	
Scale of reciprocals to CF	CIF	$1 \div \pi x$	
Base scale folded at π	CF	πx	
Base scale folded at π	DF	πx	Lower stock
Log log scales for positive exponents	LL_1	$e^{0.01x}$	
	LL_2	$e^{0.1x}$	
	LL_3	e^{x}	

All the scales are graduated in reference to the base scales C and D and at the right-hand end of the rule are to be found the mathematical formulae linking them with the values on the base scale.

The cursor encompasses the slide rule completely and makes it possible to connect up the calculation stages of all the scales on both the front and back of the Rule.

The base scales CF and DF which are 'folded' by the value of π; simplify calculations involving the use of π and eliminate a movement of the slide when carrying out multiplications on the base scale. With the help of the adjacent cube scales it is possible to make a direct computation of formulae and equations containing cubes and cube roots. The extended scale of tangents T_2 makes it possible to carry out direct computations up to an angle of 84°.

The decimal point

As the upper scales, **A** and **B**, run from 1 to 100, and the lower from 1 to 10, a novice is inclined to think that it is only possible to use the slide rule for numbers within these limits. This is not so, since the position of the decimal point is ignored in slide rule working. For instance, to multiply 320 by 580, it would be possible to multiply 3·2 by 5·8 and increase the answer ten thousand times, or in other words, move the decimal point four places to the right. The graduation 3 on any of the scales may be taken to represent 30, 300, 3000, etc., or 0·3, 0·03, 0·003, etc. In slide rule working significant figures only are considered, and the position of the decimal point is found from a rough estimate of the size of the answer. In practical problems the number of figures is obvious.

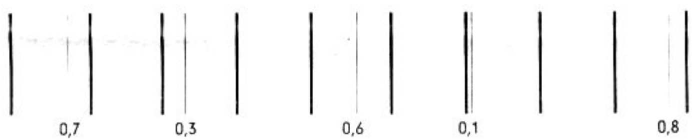
Reading the scales with graduated length 10 inch.

Subdivisions 1 to 2

Let us first turn our attention to the lower scales, **C** and **D**. Here it should be noted that the tenths are shown and numbered between **1 and 2,** these tenths in their turn being subdivided in a like manner (hundredths). The division-marks are thus read off as follows from the start: 100-101-102-103 . . . . 109-110-111-112-113 . . . . 198-199-200.
Exercises: Set the cursor with its hair line over the following values: 175, 163, 157, 130, 103, 170, 107, 111, 191.
When calculating with the slide rule, however, we are not merely concerned with numbers shown by a mark on the scales. We must also be able to set the cursor-line correctly to further imaginary subdivisions in between the hundredths (that is, to thousandths). This is by no means so difficult as it first appears. Firstly practice finding the exact centre.

Exercises: Set the cursor-line to 1075, 1355, 1675, 1425, 1985, 1705, 1075.
For all other values it is advisable to work from the centre-position outwards, so that if the cursor has to be set, for example, to 1074, it should first be placed so that its line is in the right position for 1075 and then moved slightly to the left. If the value desired is 1688, one starts at 1690 and then goes back a little way. After a certain amount of practice the position of the tenths can be satisfactorily estimated.

The setting of the cursor-line between two adjacent division-marks is an operation which the user must practice regularly. The best way of doing this is to draw two vertical lines at a distance of about 10 mm. and then placing a thread on each tenth of the distance in turn, judging the position with the eye in each case. One can then use a ruler to check ones accuracy of judgment.



Exercises: Place cursor-line over 1172, 1784, 1098, 1346, 1777, 1007, 1703.
Finally, set the cursor-line somewhere between 1 and 2 and read off the number denoted by that position.

Subdivision from 2 to 4.

Now let us consider our next division, that extending from **2 to 4.** Here we first find the tenths marked, as before, but the only further divisions marked are the fifths. The values from 2 onwards are: 200-202-204-206-208-210-212 . . . . 396-398-400. In setting the odd hundredths, therefore, the position must be judged with the eye.
Exercises: Place cursor-line over the values 207, 347, 277, 209, 315, 373.
In this section it is recommended that the beginner should not for the moment attempt to set the cursor to thousandths. If he requires 2358, for instance, he should round it off to 236, 2073 being rounded off to 207, and so forth.
Finally, the cursor is placed at any desired point between 2 and 4, the user endeavouring to take an accurate reading of the result.

Subdivision from 4 to 10.

From **4** onwards to the end of the lower scales, only the halves are marked between the tenths. After 4, therefore, readings are taken as follows: 405-410-415 etc., up to 995-1000. All the other hundredths must be judged with the eye. First place the cursor-line over the following easier numbers: 4225; 7875; 9175; 6025, etc. If the setting required is 423, the best method is to begin with 4225 and then to move the cursor-line a little towards the right. For 787 one starts with 7875, then moving a little to the left.
Exercises: Set to 633; 752; 927; 538; 467.
For 444 and 446 one starts with 445 and then moves to the left or to the right respectively. On the same principle one starts at 790 for 789 or 791.
Exercises: Set to 908; 426; 709; 627; 517.
The user is also advised to select numbers of his own for setting and reading.

Reading the scales with graduated length 5 inch.

On the small slide rules with a graduated length of 5 inch. only the tenths, with the fifths of the latter, i.e. the fiftieths, are marked between 1 and 2. Readings are thus taken from 102—104—106 to 196—198 and 200. From 2 to 5, we find only the tenths, with their respective halves, so that readings are taken from 205—210—215 etc. to 490—495 and 500. From 5 to 10 only the tenths are marked. On the upper scales the subdivision in the interval from 1 to 3 is the same as that between 2 and 5 on the lower scales, whilst that in the interval between 3 and 6 is the same as that between 5 and 10 below. In the interval from 6 to 10 only the fifths are marked. The reading in this case is thus from 62—64 to 96—98 and 100.

Multiplication

a×b

Example: 2·5 × 3 = 7·5 (Fig. 1).

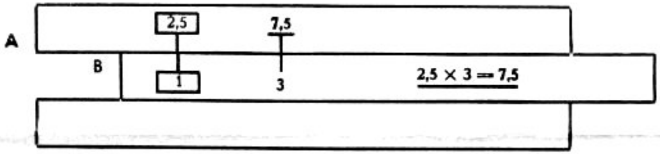


Fig. 1

The left index line 1 of the slide (Scale **B**) is placed under the 2·5 of the upper "body" scale (**A** 25), the cursor line then being placed over the 3 of the upper slide scale (**B** 3). The product (7·5) can then be read off beneath the cursor line on the upper "body" scale (**A** 75). Exactly the same procedure can be adopted on the lower scales, and here you get more accurate results.

Example: 2·45 × 3 = 7·35 (Fig. 2).

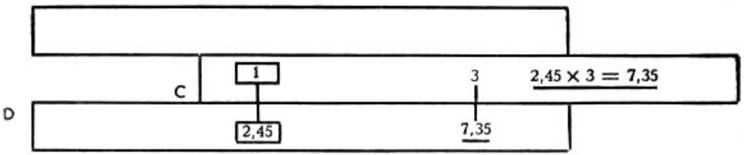


Fig. 2

The 1 on the slide (**C** 1) is placed above the 2·45 on the lower "body" scale (**D** 245), the cursor line then being placed above the 3 on the lower slide scale (**C** 3). The product (7·35) can then be read off underneath the cursor line on the lower "body" scale (**D** 735).

Example: 7·5 × 4·8 = 36 (Fig. 3)

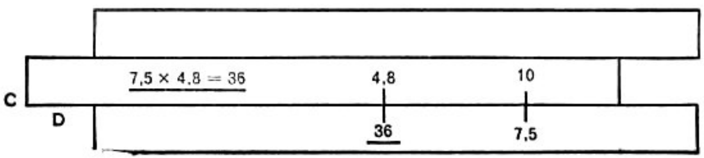


Fig. 3

When calculations are carried out on the lower scales, it will be found that the second factor of a multiplication problem sometimes cannot be selected within the scope of the lower "body" scale. In this case, **C** 10 is placed above the first factor, the cursor line then being placed above the second, after which the result can be read off, as before, beneath the cursor line.

Division

$\frac{a}{b}$

Example: 9·85 ÷ 2·5 = 3·94 (Fig. 4)

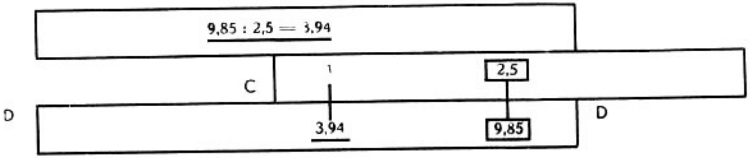


Fig. 4

The denominator 2·5 on the lower slide scale (**C** 25) is placed above the numerator 9·85 on the lower "body" scale (**D** 985) and the quotient (3·94) is read off underneath the beginning of the slide (**C** 1).
This calculatory process can likewise naturally be performed on the upper scales. The result is read off above the right-hand or left-hand extremity of the slide (**B 1** or **B** 100) on Scale **A.**

$\frac{a \times b \times c}{d \times e \times f}$

Compound calculations

Multiplications and divisions in immediate sequence can easily be made with the Calculating Rule. The intermediate results need not be read off if it is not necessary to know them, and, after the last setting, the correct final result will appear. It is best to begin such calculations with a division, then follow with a multiplication, then another division and again a multiplication and so on.

Example: $\frac{13{\cdot}8 \times 24{\cdot}5 \times 3{\cdot}75}{17{\cdot}6 \times 29{\cdot}6 \times 4{\cdot}96} = 0{\cdot}491$; Exercise: $\frac{38{\cdot}9 \times 1{\cdot}374 \times 16{\cdot}3}{141{\cdot}2 \times 2{\cdot}14} = 2{\cdot}883$;

We start by dividing 13·8 by 17·6. Therefore we place **D** 138 and **C** 176 one under the other. Do not read off the answer — approximately 0·8 — but multiply it immediately by 24·5, by placing the cursor-line on **C** 245. Similarly, no reading is taken of the answer — about 19 — and it is simply divided by 29·6. For this purpose, keep the cursor-line firmly in its position and slide **C** 296 under it. Once again, the result (0·65) is not "read" but multiplied at once by 3·75, this being done by placing the cursor-line on **C** 375. The result is merely "retained" by the cursor-line, as before, and divided by 4·96, by sliding **C** 496 under the cursor-line. Only then do we read off the figures of the final answer, 491, above under **C** 10 — and our rough calculation shows us that the actual answer is 0·491.

How to form tables

Example: To convert yards into metres. (82 yds. are 75 m.)

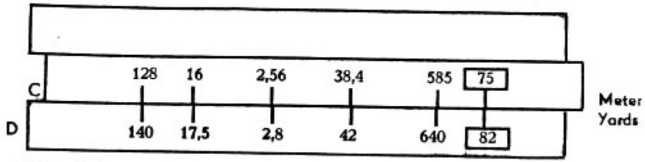


Fig. 5

$$\frac{a}{b} \div \frac{c}{d} \div \frac{e}{f}$$

Place **C** 75 over **D** 82. This automatically produces a comparative Table, from which the following readings can be taken: 42 yds. are 38·4 m.; 2·8 yds. are 2·56 m.; 640 yds. are 585 m.; 16 m. are 17·5 yds.; 128 m. are 140 yds., etc.

Calculations with the CF and DF scales

(1) **How to form tables**

On the **CF** and **DF** scales, "folded" at π, the value 1 is approximately in the middle, so that the further calculations can advantageously be carried out on these scales when tables are being formed, it being thus unnecessary to push the Slide through (transposing right index line to left or vice versa) when making computations on **C** and **D**.

Example: 75 lbs. = 34 kgs. We place **C** 3-4 above **D** 7-5 and thus obtain the conversion from lbs. into kgs. No further readings can be taken, however, beyond 50 kgs. (C 5). At this point we turn the Slide Rule round and can set **CF** and **DF** to the desired values with the aid of the cursor line.

If the equivalent value in each case (e.g. 75 lbs. : 34 kgs.) is not known, but only the general relationship 1 lb. = 0 kgs. 454 gr, then **C** 1 (the beginning of the slide) is placed above D 4-5-4, and this provides the conversion from lbs. into kgs.

(2) **Multiplication**

$a \times b$

If **C** and **D** cannot be set to the second factor in a multiplication problem, and if the slide has to be pushed through, this can be avoided by setting **CF** to the second factor and reading the result on **DF.**

Example: 2·91 × 4 = 11·64. Place **C** 1 above **D** 2-9-1, turn the slide rule over and place the cursor line above **CF** 4. The result, 11·64, is read off underneath it on **DF**.

Examples for practice: 18·4 × 7·42 = 136·2; 42·25 × 3·76 = 158·9; 1·937 × 6 = 11·62.

(3) **Multiplication and division by π** $\boxed{\alpha \times \pi}$

The transition from the **C** scale to the **CF** or **DF** scales can be carried out direct with the cursor and results in a multiplication by π; conversely, when we change over from the **CF** and **DF** scales to the **C** scale, we perform a division by π.

E x a m p l e : $1{\cdot}184 \times \pi = 3{\cdot}72$. With the slide zeroed (**CF** 1 above **DF** 1) place the cursor line above **C** 1-1-8-4, and take a reading of the result (3·72) - likewise underneath the cursor line. The inverse operation results in a division by π.

E x a m p l e : $\frac{18{\cdot}65}{\pi} = 5{\cdot}94$. Place the cursor line above **DF** 1-8-6-5 and read off the result (5·94) on **C.** $\boxed{\frac{\alpha}{\pi}}$

Examples for practice:

Area of an ellipse $= \pi\, a\, b = 5{\cdot}25 \times 2{\cdot}22 \times \pi = 36{\cdot}6$. Place **C** 10 above **D** 5-2-5, slide the cursor until its line is above **C** 2-2-2; a reading of the intermediate result (11·65) on **D** can be dispensed with, and the final result (36·6) is read off on **DF** after the slide rule has been turned over.

Length of an arc of a circle $= \frac{\pi\ \alpha\ r}{180} = \frac{26{\cdot}2 \times 352 \times \pi}{180} = 161.$

We start with the division, thus placing **C** 1-8 and **D** 2-6-2 opposite each other with the aid of the cursor line. No reading need be taken of the intermediate result 0·1455 (underneath **C** 1). We multiply by 352 by placing the cursor line above **C** 3-5-2.

(Intermediate result 51·2 on **D**). Here again, the multiplication by π can be carried out by turning the slide rule over and reading off the result 161 on **DF**, underneath the cursor line.

Calculations with the reciprocal scale CI

1. In order to find the reciprocal value $1 \div a$ for any given number a, find a on **C** (or **CI**) and read above it on **CI** (or below it on **C**) the reciprocal value. Reading off is done therefore without any movement of the slide and entirely by setting the cursor line. $\boxed{\frac{1}{a}}$

   E x a m p l e s : $1 \div 8 = 0{\cdot}125$; $1 \div 5 = 0{\cdot}2$; $1 \div 4 = 0{\cdot}25$; $1 \div 3 = 0{\cdot}333$.

2. To find $1 \div a^2$ move the cursor to a on scale **CI** and read above it on **B** the result.

$$\frac{1}{a^2}$$

Example: $1 \div 2{\cdot}44^2 = 0{\cdot}168$.

Estimated answer — less than $\frac{1}{5} = 0{\cdot}2$.

Example: Find the resistance R of an appliance having a power of 1320 Watts and drawing a current of 6 A.

Solution: $R = P \times \frac{1}{J^2} = 1320 \times \frac{1}{6^2} = 36{\cdot}7\ \Omega$

3. To find $1 \div \sqrt{a}$, set the cursor line to a on scale **B** and find below it on **CI** the result.

$$\frac{1}{\sqrt{a}}$$

Example: $1 \div \sqrt{27{\cdot}5} = 0{\cdot}191$.

Estimated answer — less than $\frac{1}{5} = 0{\cdot}2$.

Example: Change a single phase alternating current of 120 V into direct current by means of a transformer.

Solution: $V_d = \frac{2V}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 2V = \frac{1}{\sqrt{2}} \times 240$

Result above D 240 = 169·6 V or B 2 above D 240, under C 1 = 169·6 V.

4. **Products of three factors** can generally be reached with one setting of the slide. One sets, by means of the cursor, the first two factors against each other on **D** and **CI** respectively, moves the cursor to the third factor on **C** and reads below it on **D**, the final product.

$$a \times b \times c$$

Example: $0{\cdot}66 \times 20{\cdot}25 \times 2{\cdot}38 = 31{\cdot}8$.

Estimated answer — more than $0{\cdot}6 \times 20 \times 2{\cdot}5 = 30$.

Example: What is the area of an ellipse with semi-axes of 15·4 inches and 6·2 inches?

Solution: $A = a b \pi = 15{\cdot}4 \times 6{\cdot}2 \times \pi = 300$ square inches.

5. **Compound multiplication and division**

$$\frac{a}{b \times c}$$

Example: $\frac{36{\cdot}4}{3{\cdot}2 \times 4{\cdot}6} = 2{\cdot}472$

One sets by means of the cursor the figures 3-6-4 on **D** against 3-2 on **C**. It is not necessary to read the intermediary result. Move the cursor line over 4-6 on scale **CI**, which is the same as multiplying by $\frac{1}{4{\cdot}6}$ (= reciprocal value $\frac{1}{c}$)
The result of 2·472 is then found under the cursor line on scale **D.**

Calculations with the reciprocal scale BI

The reciprocal square scale, **BI**, represents the reverse of scale B, operating as a square scale with **CI** and as a reciprocal scale with A and B. This is of advantage in compound calculations.

Here the same computations can be carried out as under 1 to 5 on pages 11 to 13, but scales **A, B, CI, C** and **D** are replaced by scales **D, C, BI, B** and **A.**

Example No. 1 from p. 11: 1 ÷ 8 = 0·125. Place the cursor line above 8 on **BI** or **A** and read off the reciprocal (0·125) - above it, on **A**, or below it, on **BI.**

Here is an example of a compound calculation in which one has the special advantage of starting from **A** and **B** and continuing on **BI.**

Example: $(2{\cdot}45 \times 3)^2 \times 2{\cdot}27 = 122{\cdot}6$.

Place **C** 1 above **D** 2-4-5, move the cursor until its line is above **C** 3: no reading need be taken, on **D**, of the intermediate result 7·35, and the square (50·42) can be found on **A**, again underneath the cursor line. (For squaring, see below). This is multiplied by 2·27 by moving **BI** 2-2-7 into position under the cursor line, above **B** 1, the result (122·6) is found on **A.**

Example: To find the area of a sphere in which $r = 7{\cdot}2$ cm.
$A = 4\pi r^2 = 652$ cm². Place **BI** π above D 7·2 and take the reading of the area (652 cm²) above **B** 4, on **A.**

Calculations with the reciprocal scale CIF

The CIF graduation operates in conjunction with CF and DF just as CI does with C and D scales.

Examples for multiplication by a number of factors:

$2{\cdot}23 \times 16{\cdot}7 \times 1{\cdot}175 \times 24{\cdot}2 = 1059$. Solution: CI-2·23 placed above D-16·7 by the aid of the cursor line; the latter is placed above CF-1·175. CIF 24·2 under the cursor line. Read result, 1059, on DF, above CF 1.

$0{\cdot}53 \times 0{\cdot}73 \times 39{\cdot}1 \times 0{\cdot}732 = 11{\cdot}07$. Solution: CI-0·53 placed above D-0·73 with the aid of the cursor line; the latter is placed above CF-39·1. CIF 0·732 underneath cursor line. Read the result 11·07, on DF, above CF 1.

Squares and square roots

Both the upper scales are graduated to half length. The change over from **D** to **A** (or from **C** to **B**) gives the **square** of the number to which **D** (or **C**) has been set. **Square roots** are extracted by reversing this procedure (Fig. 6).

a^2

Example:
Given the side of a square (47 inches).
Find the area.

$A = 47^2 = 2209$ sq. in.

$\sqrt{b}$

Example:
What is the diameter of a shaft if P = 50 HP and V = 400 r.p.m.?

$$d = 12 \cdot \sqrt[4]{\frac{P}{V}} = 12 \cdot \sqrt{\sqrt{\frac{50}{400}}} = 7{,}138$$

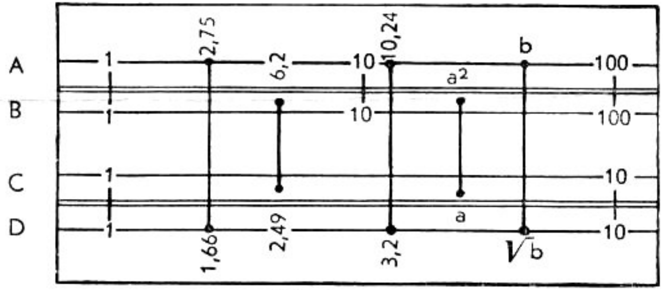


Fig. 6

In extracting a square root it is essential to set the number on the correct half of the upper scale, for $\sqrt{x}$ and $\sqrt{10x}$ do not differ merely in the position of the decimal point. If the figures 6..2 be set to the left, the root of 6·2 appears below, while if they are set to the right the root of 62 is obtained. One must thus proceed in accordance with the numbers as shown (1...10...100). If the number lies outside the scale range 1 to 100, it should be factorised by hundreds to bring the significant figures within these limits.

Example: $\sqrt{1922} = \sqrt{100 \times 19{\cdot}22} = 10 \times \sqrt{19{\cdot}22} = 10 \times 4{\cdot}38 = \mathbf{43{\cdot}8}$

$\sqrt{0{\cdot}000\,071} = \sqrt{71 : 1\,000\,000} = \sqrt{71} : 1\,000 = 8{\cdot}43 : 1000 = 0{\cdot}008\,43$

Cubes and cube roots (Cube scale K' see next page)

For the calculation of cubes and cube roots our slide rules are provided with a cube scale, **K**, which is used in combination with the **C** scale on the back of the rule. In reading both cubes and cube roots it is only necessary to use the cursor.

The scale **K** is made up of three similar portions, each one third the length of the **D, C** and **CI scale.** Scale **K** is so placed that each number on **C** is in line with its cube on **K.** The three portions of scale **K** run from 1 to 10, 10 to 100, and 100 to 1,000.

Examples: $1{\cdot}67^3 = 4{\cdot}66$
$2{\cdot}34^3 = 12{\cdot}8$
$5{\cdot}77^3 = 192$

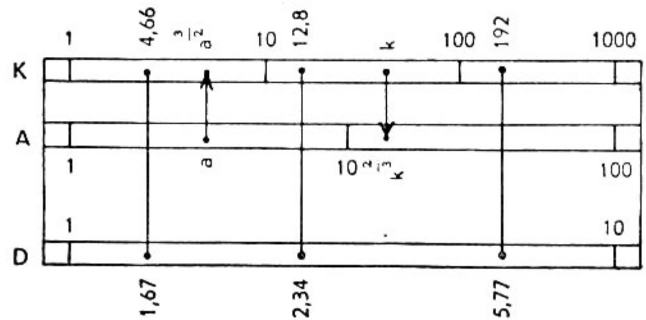


Fig. 7

a^3 $\sqrt[3]{a}$

$a^{\frac{3}{2}}$ $a^{\frac{2}{3}}$

If the number does not lie within the scale range 1 . . . 1000, it must be factorised yb thousands to bring it within these limits.

Example: $\sqrt[3]{1\,260\,000} = \sqrt[3]{1000^2} \times 1{\cdot}26 = 10^2 \times \sqrt[3]{1{\cdot}26} = 100 \times 1{\cdot}08 = 108$

$\sqrt[3]{0{\cdot}32} = \sqrt[3]{320 : 1000} = \sqrt[3]{320} : 10 = 6{\cdot}84 : 10 = 0{\cdot}684.$

If the cube scale be employed with scale **A**, powers having the exponents $\frac{3}{2}$ and $\frac{2}{3}$ may be found (Fig. 7).

The movable cube scale K'

has the advantage of again enabling compound calculations eminating from **K**, to be continued here.

Example: $\frac{3.09^3}{2 \cdot 1} = 14{\cdot}05$. Place the cursor line above **C** 3-0-9, and find the cube (29·5) beneath the cursor line and on **K**; this cube is immediately divided by 2·1 by placing **K'** 2-1 beneath the cursor line. The result (14·05) is read off on **K**, above **K'** 1.

Examples for practice:
Given: Q = 4·4 m³/sec; Cq = 4·37. n = 1460 r.p.m.
To find: Diam. of rotor-wheel.

Solution: $D = Cq \sqrt[3]{\frac{Q}{n}} = 4{\cdot}37 \sqrt[3]{\frac{4 \cdot 4}{1460}} = 0{\cdot}63$ metres.

The Q values are divided by n by placing **K'** 1-4-6 under **K** 4-4 with the aid of the cursor line. The cube root of the quotient appears underneath **C** 1 and need not be read off. This setting, however, immediately provides the basis for the operation of multiplying by 4·37: in other words, all we need to do is to slide the cursor into position so that its line is above **C** 4-3-7, after which the result (0·63) can be read off below it, on **D.**

Given: h = 18·5; b = 10·7.
To find: Moment of inertia of a rectangular cross section $I = \frac{b \times h^3}{12}$

Place the cursor line above **D** 1-8-5, turn the slide rule over, then place **K'** 1-2 underneath the cursor line; the value of I, i.e. 5650 cm³, can be immediately found above **K'** 1-0-7.

Calculations with the Pythagorean scale P

$\sqrt{1 - x^2}$

This scale represents the function $y = \sqrt{1-x^2}$; it operates in conjunction with D (= x), of which the values have to be read from 0·1 to 1. The graduations run in the opposite direction and for distinction are coloured red.
Example: x = 0·8. y = 0·6. sin α = 0·134. cos α = 0·991.

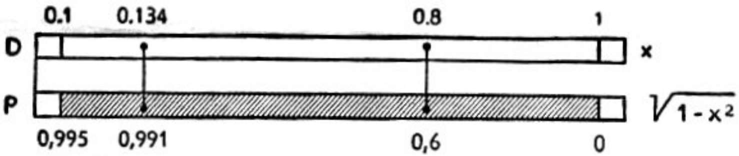


Fig. 8

Example:
Calculate effective current and wattless current of a circuit absorbing 35 A with a cos φ of 0·8:
$I_P = I \times \cos\varphi = 35 \times 0{\cdot}8 = 28$ A.
$I_Q = I \times \sin\varphi = 35 \times 0{\cdot}6 = 21$ A.

Place **C** 35 above **D** 10; at **D** 8 (since cos φ = 0·8) read off on **C** the value 28 for I_P; underneath **D** 8 we also find, on **P**, the value 0·6 (for sin φ). If the cursor is now moved to **D** 6, the value 21 for I_Q can be read off above it, on **C**.

Calculations with the trigonometric scales S, T_1 and T_2

sin x
cos x
tan x
cot x

The trigonometric scales T_1, T_2 and **S** are subdivided decimally; in conjunction with the basic scale **D** they show the angular functions or (with reverse readings) the angles.
When the scales T_1, T_2 and **S** are used in conjunction with scale **D as a trigonometric table,** the following must be borne in mind:

The **S scale,** in conjunction with the **D scale,** provides a **Sine table.**
The **S scale** with the values of the complementary angles (increasing from right to left) provides — in conjunction with the **D scale** — a **Cosine table.**

The **two T scales** — in conjunction with the **D scale** — provide a **Tangent table** up to 84·28°.
The **two T scales** with the values of the complementary angles (increasing from right to left) provide — in conjunction with the **D scale** — a **Cotangent table.**

sin 13° = cos 77° = 0·225.	S 13° — D 0·225.	Only the long cursor line is required for these settings.
sin 76° = cos 14° = 0·97.	S 76° — D 0·97.	
cos 28° = sin 62° = 0·883.	S 62° — D 0·883.	
cos 78° = sin 12° = 0·208.	S 12° — D 0·208.	
tan 32° = cot 58° = 0·625.	T_1 32° — D 0·625.	
tan 57° = cot 33° = 1·54.	T_2 57° — D 1·54.	
cot 18° = tan 72° = 3·08.	T_2 18° — D 3·08*	
cot 75° = tan 15° = 0·268.	T_1 75° — D 0·268*	
* or		
cot 18° = tan 72° = 3·08.	T_1 18° — CI 3·08.	Set with long cursor line, with slide zeroed.
cot 75° = tan 15° = 0·268.	T_2 75° — CI 0·268.	

When changing over from tangent to cotangent the angle need not be read off, as these two values are to be found on **C** and **CI**, one below the other. It is only when changing over from sine or cosine to tangent or cotangent that an intermediate reading of the angle has to be taken. Since when taking readings of the functions, these can be obtained either on **D** or on **CI**, multiplication and division can in many cases follow immediately.

Examples for the use of the trigonometric scales in a **right-angled triangle:**

Example 1: given: a = 2, b = 3. To find: ca and α. Formula: $a \times \frac{1}{b} = \tan \alpha$, $a \times \frac{1}{c} = \sin \alpha$.

Beginning of slide, C 1, above D 2; cursor on CI 3; value of α (33·7) to be read off on the tan scale. Move cursor to 33·7 on the sine scale and read off value of c (3·6) on CI.
Example 2: given: a = 8, b = 20. To find: c and α.
C 10 above D 8, cursor on CI 20 read off value of α (21·8) from tan scale.
Place cursor to 21·8 on sine scale, read off value of c (21·55) from CI.

Example 3: given: $a = 20$, $b = 8$.
to find: c and α.

C 1 above D 20, cursor on CI 8, read off value of α (68·17) on tan scale (T_2).
Cursor on 68·17 on sine scale, read off value of c (21·54) from CI.

Example 4: given: $c = 5$, $\alpha = 36{\cdot}87^0$.
to find: a and b.

Formula: $a = c \times \sin \alpha$, $b = c \times \cos \alpha$.

CI 5 above S $36{\cdot}87^0$ (sine scale), and read off value 3 under C 1 (beginning of slide). Then place cursor line to T_1 $36{\cdot}87^0$ and read off value of b (4) from CI.

Example 5: given: $c = 21{\cdot}54$; $b = 20$
to find: a and α.

Set C 2154 above D 10, cursor on C 2 (for $b = 20$), read off the value for α ($21{\cdot}8^0$) on cos-scale and simultaneously the value 0·372 on scale P. Reset the slide using the right index of the slide in place of the left. Move the cursor to 0·372 on scale D and you read on scale C the value 8 for a.

For scalene triangles we have the ratio $\dfrac{a}{\sin \alpha} = \dfrac{b}{\sin \beta} = \dfrac{c}{\sin \gamma}$

Example 1: given: $a = 38{\cdot}3$, $\alpha = 52^0$, $\beta = 59^0$, $\gamma = 69^0$.
to find: b and c.

Place C 383 above S 52^0. With the aid of the cursor line the results (41·7 and 45·4 cm) can be read off above S 59^0 and 69^0 on C.

Example 2: Given: $a = 195$, $b = 169$, $\alpha = 67{\cdot}5^0$.

Place 195 and $67{\cdot}5^0$ one above the other, move cursor-line onto 169 and find $53{\cdot}2^0$ under it for β. We now have $\gamma =$ 59·3 and can read off $c = 181{\cdot}5$ above it.

Example 3: Given $= 41{\cdot}1$, $\alpha = 53^0$, $\beta = 19^0$; this gives us $\gamma = 108^0$.

Place 41·1 and 53^0 one under the other, find 16·75 (for side b) above 19^0 and (for side c) 48·9 above 72^0 ($\sin 108^0 = \sin 72^0$).

The ST-scale for small angles and The mark ϱ on the C and D scales

For the **functional values of small angles** from 0·58 to 5·7 the face of the slide bears the ST scale ($\sphericalangle$ arc 0·01 x)

with the equation: $\sin\alpha \approx \tan\alpha \approx \text{arc}\,\alpha = \frac{\pi}{180}\alpha = \rho \times \alpha$

The ST scale operates in conjunction with the D or C scale. All subsequent calculations in this column are carried out solely with the aid of the cursor line (or with scale C, when the slide rule is set to zero).

Te **functional values of small angles** can also be ascertained with the aid of the mark $\varrho = \frac{\pi}{180} = 0{\cdot}01745$,

in accordance with the equation $0{\cdot}01745\,\alpha = \varrho \times \alpha$.

Example: $\sin 3^0 \approx \tan 3^0 \approx \text{arc } 3^0 = 0{\cdot}0524$.

Place the index C 1 of the slide, above D 3, the result (0·0524) can then be read below ϱ.

Example for practice: $\sin 2{\cdot}5^0 \approx \tan 2{\cdot}5^0 \approx \text{arc } 2{\cdot}5^0 = 0{\cdot}04365$;
$\sin 0{\cdot}4^0 \approx \tan 0{\cdot}4^0 \approx \text{arc } 0{\cdot}4^0 = 0{\cdot}00698$; $\sin 0{\cdot}0052^0 \approx \tan 0{\cdot}0052^0 \approx \text{arc } 0{\cdot}0052^0 = 0{\cdot}0000908$

The angle-values are set on the Arc-scale ST and the functional values are read on scale D or, when slide rule set to zero, on C.

In "series calculations", set C 1 above ϱ on D; the result can be read on D, underneath the angle value on C.

For calculating the cosine and cotangent functions of angles over 84·5⁰

Example: $\cos 88^0 = \sin 2^0 \approx \text{arc } 2^0 = 0{\cdot}0349$
$\cot 86{\cdot}5^0 = \tan 3{\cdot}5^0 \approx \text{arc } 3{\cdot}5^0 = 0{\cdot}0611$

The cursor line is placed above the angle value on the ST scale and the result is read on D or (if the slide rule is set to zero) on C, underneath the cursor line.

Example: $\cos 88^0\ \sin 2^0 \approx \text{arc } 2^0 \approx \varrho \times 2 = 0{\cdot}0349$
$\cot 86{\cdot}5^0 = \tan 3{\cdot}5^0 \approx \text{arc } 3{\cdot}5^0 \approx \varrho \times 3{\cdot}5 = 0{\cdot}0611$

This is a simple multiplication; the beginning of the slide, C 1, is thus placed above ϱ on D, the cursor line being placed above the second factor on C; the result is then read on D, underneath it.

For conversion of circular measurements into degrees

Examples for practice: $\widehat{6{\cdot}28} = 360^0$; $\widehat{1{\cdot}11} = 63{\cdot}5^0$; $\widehat{0{\cdot}04} = 2{\cdot}29^0$; $\widehat{0{\cdot}007} = 0{\cdot}402^0$; $\widehat{0{\cdot}64} = 36{\cdot}7^0$; $\widehat{0{\cdot}32} = 18{\cdot}35^0$.

Set C or D scale to the circular measurement in question and read the angular value on the arc scale ST (with the help of the cursor line).

The mark ϱ is set above D 1 or D 10, the cursor line then being placed above the circular measurement on C; the angular degrees can be read underneath it, on D.

Calculations with the mantissa scale L

lg a

This operates in conjunction with D or (in the zero position) with C and enables readings of the common logarithms to be obtained.

In these cases the characteristic is found in the usual manner, e.g. —

Characteristic:

log 1·35 (1-1 = 0).

log 57·3 (2-1 = 1).

log 1938 (4-1 = 3).

Examples for Practice:

Setting:				
	C	1·35	57·3	1938
	L	0·1303	(1 +) 0·758	(3 +) 0·287
Result:		0·1303	1·758	3·287

Detailed explanation of the example given in the 2nd column, for finding log 57·3 (= 1·758):

To find the common logarithm of 57·3, the cursor is moved to D 573, the mantissa (0·758) being read on L, on the face of the slide. As the number 57·3 has two integers, the characteristic of the logarithm will be 1 (2 = 1), the logarithm thus being 1 + 0·758 = 1·758.

To find the number of which 1·758 is the logarithm, disregard the characteristic (1) and move the cursor to 0·758 on L. The reading on the scale C is now 5-7-3, which, taking the characteristic into account, leads to the value 57·3.

The use of the logarithms enables the types of calculation to be reduced by one stage, multiplication and division operations thus becoming addition and subtraction operations respectively, while involution and evolution becomes multiplication and division.

Examples: $245^{3\cdot24} = 3\cdot24 \times \lg 245 = 3\cdot24 \times 2\cdot389 = 7\cdot741$; $245^{3\cdot24} = 55\,000\,000$

$420^x = 10\,000$; $x \times \lg 420 = \lg 10\,000$; $x = \frac{\log 10000}{\log 420} = \frac{4\cdot0}{2\cdot623} = 1\cdot525$

The exponential scales LL_1 LL_2 LL_3 for positive exponents LL_{01} LL_{02} LL_{03} for negative exponents

The back of the slide rule presents two three-stage groups of scales for the exponential functions in reference the basic scale C (when the slide rule is set to zero). The scales for positive exponents (black) extend from 1·0101 to 22000, while those for negative exponents (red) extend from 0·00005 to 0·99. The e^{-x} scales are reciprocal scales to the e^x scales. In this connection it must be noted that the numerical values shown against the exponential scales are invariable as regards the decimal place, so that 1·04, for example, always denotes 1·04 and not 10·4, 104 etc.

The exponential scales, when we proceed from an inner scale to the next one in an outward direction, provide powers of ten, e.g.:

$0{\cdot}955^{10} = 0{\cdot}631;\ 0{\cdot}631^{10} = 0{\cdot}01;\ 0{\cdot}924^{10} = 0{\cdot}454;\ 0{\cdot}454^{10} = 3{\cdot}7 \times 10^{-4} = 0{\cdot}00037$

$1{\cdot}0472^{10} = 1{\cdot}586;\ 1{\cdot}586^{10} = 101;\ 1{\cdot}08^{10} = 2{\cdot}16;\ 2{\cdot}16^{10} = 2{\cdot}2 \times 10^3 = 2200$

$\boxed{a^{10}}$

Proceeding from an inner scale to the next scale but in an outward direction, we obtain powers of one hundred, e.g.: $0{\cdot}955^{100} = 0{\cdot}01;\ 1{\cdot}472^{100} = 100;\ 0{\cdot}924^{100} = 3{\cdot}7 \times 10^{-4} = 0{\cdot}00037;\ 1{\cdot}08^{100} = 2200$

$\boxed{a^{100}}$

When proceeding from an outer to an inner scale we obtain the corresponding roots, e.g.:

$\sqrt[10]{0{\cdot}25} = 0{\cdot}8705;\ \sqrt[10]{0{\cdot}8705} = 0{\cdot}98623;\ \sqrt[100]{0{\cdot}25} = 0{\cdot}98623;\ \sqrt[10]{0{\cdot}00007} = \sqrt[10]{7 \times 10^{-5}} = 0{\cdot}384;\ \sqrt[10]{0{\cdot}384} = 0{\cdot}9087;\ \sqrt[100]{0{\cdot}00007} = 0{\cdot}9087;$

$\sqrt[10]{4} = 1{\cdot}1488;\ \sqrt[10]{1{\cdot}1488} = 1{\cdot}01396;\ \sqrt[100]{4} = 1{\cdot}01396;$

$\sqrt[10]{15000} = \sqrt[10]{1{\cdot}5 \times 10^4} = 2{,}616;\ \sqrt[10]{2{,}616} = 1{\cdot}1009;\ \sqrt[100]{15000} = 1{\cdot}1009.$

$\boxed{\sqrt[10]{a};\ \sqrt[100]{a}}$

Note: Above 100 on scale LL_3 you find $\frac{1}{100} = 0{\cdot}01$ on scale LL_{03}.
Above 1·25 on scale LL_2 you find $\frac{1}{1{\cdot}25} = 0{\cdot}8$ on scale LL_{02}.
} as $e^{-x} = \frac{1}{e^x}$

The natural logarithms

ln a

The natural logarithms are found when proceeding from the LL Scales to the movable Basic Scale C. The foregoing applies, **mutatis mutandis,** to the numerical range provided by the Basic Scale.

Examples: ln 25 = 3·22; ln 145 = 4·97; ln 1·3 = 0·262; ln 0·04 = —3·22; ln 0·66 = —0·416; ln 0·98 = —0·0202.

Powers of e

e^n

The powers of e (base of Natural logarithm, e = 2·71828 . . .) are obtained by setting the C scale to the exponent, with the aid of the cursor. The power of e is then read off from the LL scale. In this connection, the range 1-10 on the C scale applies to LL_3, 0·1-1 to LL_2 and 0·01-0·1 to LL_1.

Examples: $e^{1{\cdot}61} = 5$; $e^{0{\cdot}161} = 1{\cdot}1747$; $e^{0{\cdot}0161} = 1{\cdot}01622$.

$$e^{-1{\cdot}61} = \frac{1}{e^{1{\cdot}61}} = 0{\cdot}2; \quad e^{-0{\cdot}161} = 0{\cdot}8513; \quad e^{-0{\cdot}0161} = 0{\cdot}98402.$$

$$e^{-6{\cdot}22} = \frac{1}{e^{6{\cdot}22}} = 0{\cdot}002; \quad e^{-0{\cdot}622} = 0{\cdot}537; \quad e^{-0{\cdot}0622} = 0{\cdot}9397;$$

$$e^{12{\cdot}5} = e^{10 + 2{\cdot}5} = e^{10} \times e^{2{\cdot}5} = 22000 \times 12{\cdot}2 = 268400.$$

To determine the **hyperbolic functions** mark the argument x on the **C** scale with the cursor line.

The powers of e can then be read off the scales LL_1, LL_2, LL_3 (e^x) and LL_{01}, LL_{02}, LL_{03} (e^{-x}) respectively. The sum divided by 2 (or the difference also divided by 2) results in the cosh (and the sinh accordingly) e.g.:

sinh x
cosh x

$$\cosh 35^0 = \cosh 0{\cdot}61 = \frac{e^x + e^{-x}}{2} = \frac{1{\cdot}84 + 0{\cdot}543}{2} = 1{\cdot}1915$$

$$\sinh 35^0 = \sinh 0{\cdot}61 = \frac{e^x - e^{-x}}{2} = \frac{1{\cdot}84 - 0{\cdot}543}{2} = 0{\cdot}6485$$

Roots of e

$\sqrt[n]{e}$

The root is expressed as a power with a reciprocal exponent; we then proceed as before.

Examples: $\sqrt[4]{e} = e^{0{\cdot}25} = 1{\cdot}284$; $\sqrt[0{\cdot}25]{e} = e^4 = 54{\cdot}6$; $\sqrt[8]{e} = e^{0{\cdot}125} = 1{\cdot}133$; $\sqrt[0{\cdot}125]{e} = e^8 = 2980$

$\sqrt[12{\cdot}5]{e} = e^{0{\cdot}08} = 1{\cdot}0834$; $\sqrt[0{\cdot}06]{e} = e^{16{\cdot}66} = e^{8{\cdot}33} \times e^{8{\cdot}33} = 4146 \times 4146 = 17189000$

Powers of any desired numbers

a^n

Powers of the form a^n can be obtained by placing C-1 above the basic value **a** on the corresponding LL scale and by then moving the cursor to C-n. The value of a^n can then be read off from LL, e.g.:

$3{\cdot}75^{2{\cdot}96} = 50$; place C-1 above LL_3-375 and read off value (50) at C-296 on LL_3.

Further examples:

$4{\cdot}2^{2{\cdot}16} = 22{\cdot}2$; $4{\cdot}2^{0{\cdot}216} = 1{\cdot}364$; $4{\cdot}2^{0{\cdot}0216} = 1{\cdot}0315$

$4{\cdot}2^{-2{\cdot}16} = 0{\cdot}045$; $4{\cdot}2^{-0{\cdot}216} = 0{\cdot}7335$; $4{\cdot}2^{-0{\cdot}0216} = 0{\cdot}9695$

Using the LL_{03}-scale:

$0{\cdot}05^{2{\cdot}16} = 0{\cdot}00155$

$0{\cdot}05^{0{\cdot}216} = 0{\cdot}524$; $0{\cdot}05^{0{\cdot}0216} = 0{\cdot}9374$

$0{\cdot}05^{-0{\cdot}216} = \frac{1}{0{\cdot}05^{0{\cdot}216}} = 1{\cdot}91$ (to be read off the LL_2-scale)

$0{\cdot}05^{-2{\cdot}16} = \frac{1}{0{\cdot}05^{2{\cdot}16}} = 646$ (to be read off the LL_3-scale)

Roots of any desired numbers

$\sqrt[n]{a}$

With the aid of the cursor line, place the exponent on C above the root-number on LL (first finding the root number and placing the cursor line above it); the result is then found underneath C 1 or C 10.

$\sqrt[4{\cdot}4]{23} = 2{\cdot}04$; place C-4·4 above LL_3-23 and find the value 2·04 at C-10, all with the aid of the cursor line.

Examples: $\sqrt[2{\cdot}08]{1{\cdot}068} = 1{\cdot}03215$ (place C-2·08 above LL_1-1·068, take reading on LL_1).

$\sqrt[0{\cdot}6]{15{\cdot}2} = 93{\cdot}5$ (place C-0·6 above LL_3-15·2, take reading on LL_3).

$\sqrt[20]{4{\cdot}41} = 1{\cdot}077$ (place C-20 above LL_3-4·41, take reading on LL_1).

$\sqrt[5]{0{\cdot}5} = 0{\cdot}8705$ (place C-5 above LL_{02}-0·5, take reading on LL_{02}).

$\sqrt[50]{0{\cdot}5} = 0{\cdot}98623$ (place C-50 above LL_{02}-0·5, take reading on LL_{01}).

Further examples: $\sqrt[5]{2} = 1{\cdot}1488$; $\sqrt[5]{20} = 1{\cdot}82$

$\sqrt[0{\cdot}06]{2{\cdot}42} = 2{\cdot}42^{16{\cdot}66} = 2{\cdot}42^{8{\cdot}33} \times 2{\cdot}42^{8{\cdot}33} = 1580 \times 1580 = 2496400$

The common logarithms

lg a

Set the hairline of the cursor to LL_3-10 and move C 1 under the hairline. Then one has a table with logarithms of numbers to the base 10. You may also use the setting C 10 over LL_3-10.

Now you may set and read by the means of the bracket cursor.

$\log 10 = 1$; $\log 100 = 2$; $\log 1000 = 3$; $\log 200 = 2{\cdot}301$.
$\log 20 = 1{\cdot}301$; $\log 2 = 0{\cdot}301$; $\log 1{\cdot}1 = 0{\cdot}0414$.

To avoid turning the slide rule round, the CF scale can also be used, by placing CF-1 above LL_3-10.

Logarithms to any desired base

$^{n}\log a$

Place the beginning of the C scale over the base on the LL scale; this provides a table of the corresponding logarithms; e.g.:

${}^{2}\log 200 = 7{\cdot}64$; ${}^{2}\log 22 = 4{\cdot}46$; place C-10 above LL_2-2; take reading of value 7·64 at LL_3-200 on C and the value 4·46 at LL_3-22 on C.

Further examples: ${}^{2}\log 1{\cdot}2 = 0{\cdot}263$; ${}^{0{\cdot}2}\log 10 = -1{\cdot}431$; ${}^{0{\cdot}8}\log 2 = -3{\cdot}11$
${}^{5}\log 25 = 2$; ${}^{0{\cdot}5}\log 25 = -4{\cdot}65$.

Important: ${}^{a}\log a = 1$; e.g.: ${}^{2}\log 2 = 1$; ${}^{2}\log 4 = 2$; ${}^{2}\log 8 = 3$
${}^{0{\cdot}5}\log 0{\cdot}5 = 1$; ${}^{0{\cdot}5}\log 4 = -2$; ${}^{0{\cdot}5}\log 8 = -3$
${}^{0{\cdot}5}\log 0{\cdot}25 = 2$; ${}^{0{\cdot}5}\log 0{\cdot}125 = 3$

Interpretation of scale marks

On the scales A and B, the marks π and M are provided, while the basic scales C and D have marks ϱ, π, C and C_1.

The meanings of these symbols are as follows:

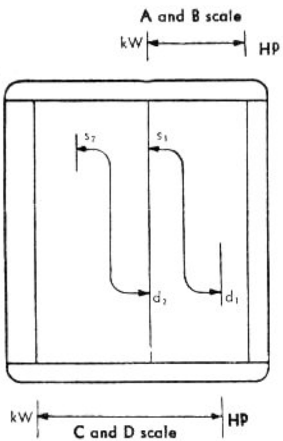
$$\pi = 3{\cdot}14159\ldots; \quad M = \frac{1}{\pi} = 0{\cdot}318; \quad C = \sqrt{4 \div \pi} = 1{\cdot}128; \quad C_1 = \sqrt{40 \div \pi} = 3{\cdot}57; \quad \varrho = \frac{\pi}{180} = 0{\cdot}01745$$

The marks π and $\frac{1}{\pi} = M$, render calculations with π easier to perform. The marks C and C_1 are used for calculating the area of a circle. The mark "C" is placed above the diameter value on the D scale, the area being read off above B-1 on A, e.g.:

For d = 2·5 cm; F = 4·91 cm². (When using the mark "C_1", the reading is taken above B-10).

Table of constants

It may be used as comparative measure for inch and cm. The plastic strip is put under the cursor and placed directly to the left metal strut. Then, inch and cm may be compared by means of the hairline.



The cursor

On the front and back of the double-sided cursor we have the long main line in the centre, in addition to which the right-hand and left-hand edge have red side-lines to provide readings of values at the transition-points no longer reached by the main line.

The accompanying diagram shows the applications and possibilities of the remaining marks. To calculate the areas of circles (d_1, d_2) the D scale is set to the diameter-value, and the corresponding cross section (s_1, s_2) can then be read off from scale A.

The duplicated arrangement adopted for the mark for the area of a circle enables the weights per metre of round steel bars to be calculated. Set D to d_1 above the value for the diameter of the steel and read off the weight per metre at s_2.

The conversion of kW to h.p. and vice versa can be carried out by means of the HP and kW marks on A and B and on C and D.

For direct calculation with the factor 3·6 the lower right-hand mark on the underside of the cursor is used.

Examples: 150 km/h = 41·6 m/sec (mark 3·6 on DF 150 gives 41·6 under the main line on D).

Determine the interest on DM 2420 at 3·75% in 95 days. (Mark 3·6 on DF 2420; CI 3·75 under main line; the interest, i. e. DM 24, can be seen on DF, above CF 95).

The double-sided cursor can be easily removed for cleaning purposes without interfering with the accurate adjustment of the slide rule.

The two white side-parts on the lower edge of the cursor are prised apart at the recess. It is important that the thumb should first of all press the wide part downwards, thus opening the cursor frame. The two plexiglass "windows" are then bent apart just far enough to allow the cursor to be slid off from the top. When replacing the cursor, please see that the "window" with its five index-marks is on the front of the slide rule.

The Care of the Slide Rule:

CASTELL Slide Rules are valuable precision implements and require careful handling.
They are made of an ideal material known as Geroplast. This is highly elastic and thus unbreakable provided it is competently handled. It will stand up to climatic conditions; it is moisture-proof and non-inflammable and will resist the majority of chemicals. Geroplast slide rules should nevertheless not be allowed to come in contact with corrosive liquids or powerful solvents, which are at all events liable to attack the colouring-agents applied to the graduation-marks even if they do not actually harm the material itself. If necessary, the smooth movement of the slide can be improved by the use of vaseline or silicon oil. In order not to detract from the accuracy of the readings, the scales and the cursor should be protected from dirt and scratches and should be cleaned with the special cleaning agents CASTELL No. 211 (liquid), or No. 212 (cleaning paste).