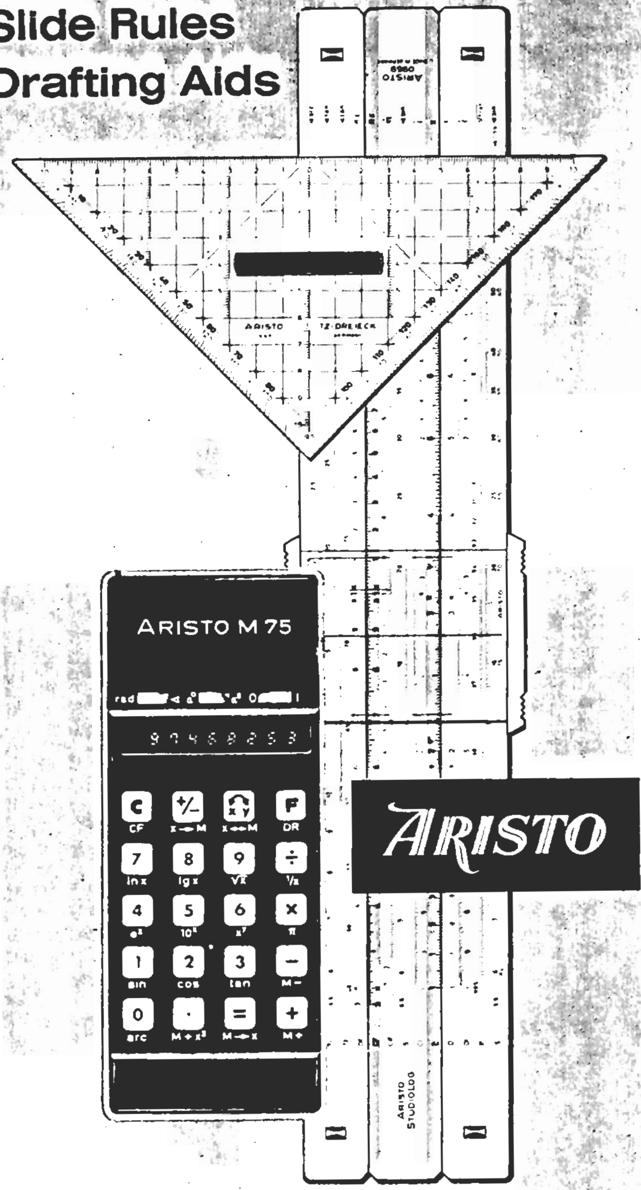
Mini Calculators
Slide Rules
Drafting Aids



INSTRUCTION

FOR USE

ARISTO

STUDIO
STUDIOLOG

868 · 0968 · 01068
869 · 0969

Scale of Preferred Numbers [illegible]

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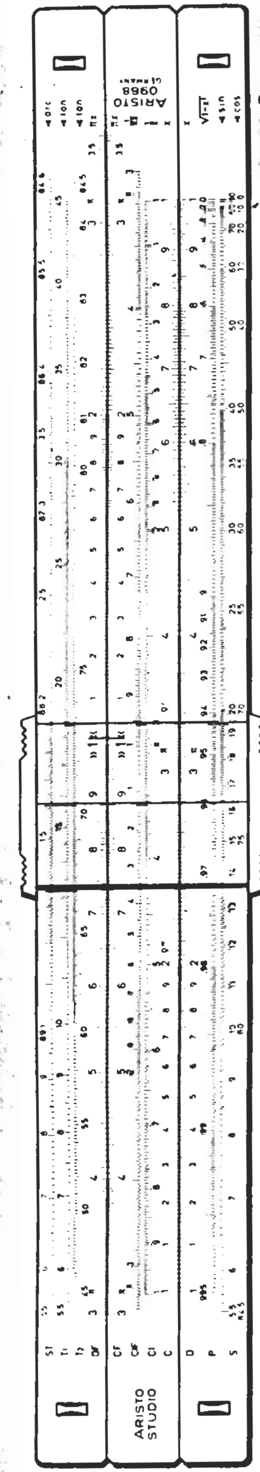

CONTENTS

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1. General introductions

In this instruction book information is given concerning the scales of the slide rule, their range and purpose. Calculations are explained, together with the interrelationships of the scales. To clarify principles involved, examples of use are given for each scale, and guidance in arranging the most important factors in complex formulae.

Expertise in slide rule manipulation comes with practice. Further exercises and detailed explanations are to be found in the textbooks recommended:

Ellis, J. P.: The Theory and Operation of the Slide Rule

1.1 Manipulation of the slide rule

When using the slide rule it is best so to hold it that the light, falling on the cursor, does not throw a shadow of the cursor line. The most precise movement of the slide results from pressure and counter pressure. The projecting end of the slide should be held by the index finger and thumb, close to the body of the rule. Movement of the finger and simultaneous pressure against the rule body achieves the desirable smooth pulling and pushing action. The other hand holds the body of the rule by the upper body panel, so that the thumb can be used to press against the end of the slide.

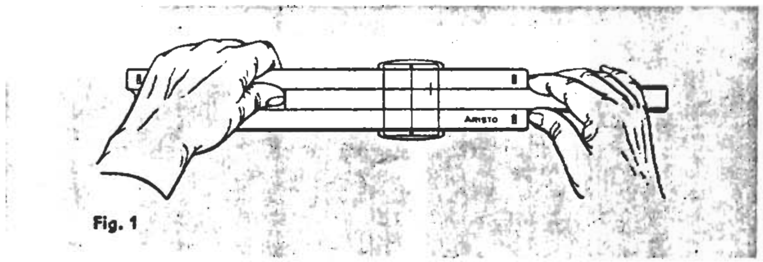

Fig. 1

Setting the cursor is possible, using either hand, but is more speedily and more accurately accomplished by using the thumb and index finger of both hands. By lightly pressing the bearing edge of the cursor, opposite the cursor spring, against the edge of the rule body, tilting the cursor is avoided and the cursor hairline is maintained perpendicular to the scales.

1.2 Personal identification tab

In the case of the slide rule, under the ARISTO scale of preferred numbers 1364, will be found a transparent insert, which can be used to identify ownership of the slide rule. The card contained can be removed, after bending the transparent flap upwards and the name of the owner of the rule can then be written on the card.

1.3 Treatment of the ARISTO slide rule

The slide rule, is a valuable calculating aid and deserves careful treatment. Scale faces and cursor should be protected from dirt and scratches, so that reading accuracy may not suffer.

It is advisable to give the rule an occasional treatment with the special cleansing fluid, DEPAROL, followed by dry polishing. The use of chemical substances of any description should be avoided as they may damage the scales.

Protect the slide rule from plastics erasers and their abrasive dusts, which can cause damage to the ARISTOPAL rule faces. Do not place the rule on hot surfaces such as radiators, or expose it to full sunlight. Deformation is likely to occur at temperatures above 140° F (60° C). Rules so damaged will not be replaced free of charge.

1.4 The slide rule support stands
(Model 0968 and 0969 only)

The supports for attachment to the ends of the ARISTO Studio 0968 or ARISTO-StudioLog 0969 set the rule in an inclined position — either face can be uppermost — raised above the desk top. This is of great convenience when, as for example in tabulating, the rule is used lying on the desk. The raised position of the rule is especially helpful when free movement of a magnifying cursor is required.

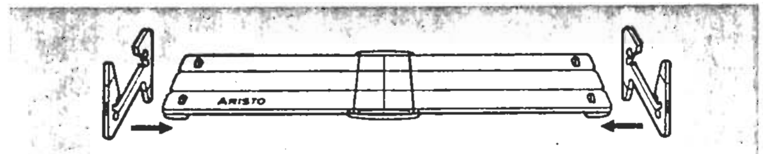

Fig. 2

When mounting the slide rule supports the trigonometrical face of the ARISTO Studio should be uppermost. The supports can then be pushed, endwise, on to the welded and bars of the rule, with the recesses visible, and the lobes of the supports engaged with the slots in the end bars.

1.5 Working diagrams used in the solution of examples

In what follows a method of representation will be used to show, in a form more easily followed than in the more usual slide rule diagrams, the process of solution and sequences of setting. The scales are represented by parallel lines, at the ends of which the scale identifications are given. The undermentioned symbols aid interpretation of the diagrams.

Initial setting

Each subsequent setting

Final result

Setting or reading an intermediate result

Reversing the rule

Arrowhead showing sequence and direction of movement

Cursor line shown by a perpendicular.

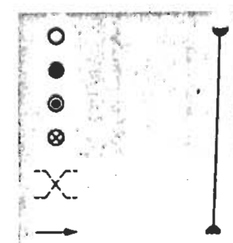

Fig. 3

THE ARISTO STUDIO SLIDE RULE 0968

The ARISTO Studio is a universal LogLog slide rule for scientists, engineers and students.

2. Scale arrangement

Trigonometric side

ST	Scale of tangents, sines and radian measure for angles of 0.55° to 6°	∢ arc	Upper panel of body
T1	Scale of tangents for angles of 5.5° to 45°	∢ tan	
T2	Scale of tangents for angles of 45° to 84.5°	∢ tan	
DF	Fundamental scale folded by π	πx	
CF	Fundamental scale folded by π	πx	On slide
CIF	Scale of reciprocals of CF	$1/\pi x$	
CI	Scale of reciprocals of C	$1/x$	
C	Fundamental scale	x	
D	Fundamental scale	x	Lower panel of body
P	Pythagoras scale	$\sqrt{1-x^2}$	
S	Scale of sines 5.5° to 90°; figured in red,	∢ sin	
	counter-clockwise, between 0° and 84.5°, as scale of cosines	∢ cos	

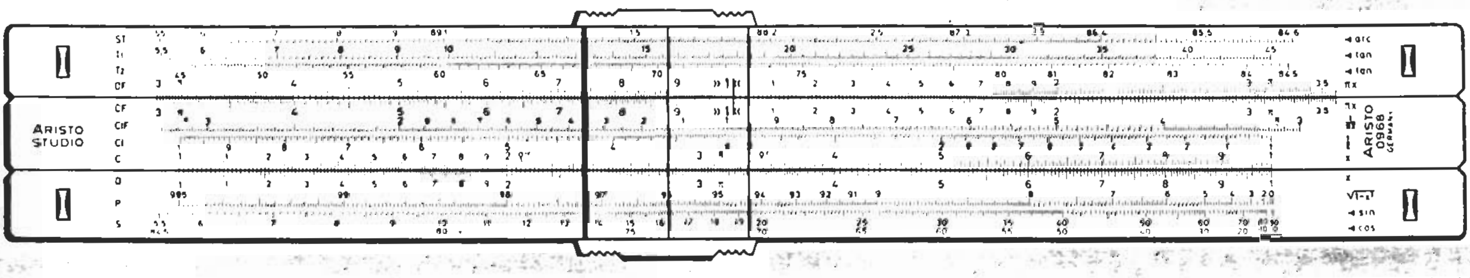

Fig. 4 Trigonometric side

LogLog side

LL01	LogLog scale, range: 0.99 — 0.9	$e^{-0.01x}$	Upper panel of body
LL02	0.91 — 0.35	$e^{-0.1x}$	
LL03	0.4 — 10^{-5}	e^{-x}	
A	Scale of squares	x^2	
B	Scale of squares	x^2	On slide
L	Mantissa scale	lg x	
K	Scale of cubes	x^3	
C	Fundamental scale	x	
D	Fundamental scale	x	Lower panel of body
LL3	LogLog scale, range: 2.5 — 10^5	e^x	
LL2	1.1 — 3.0	$e^{0.1x}$	
LL1	1.01 — 1.11	$e^{0.01x}$	

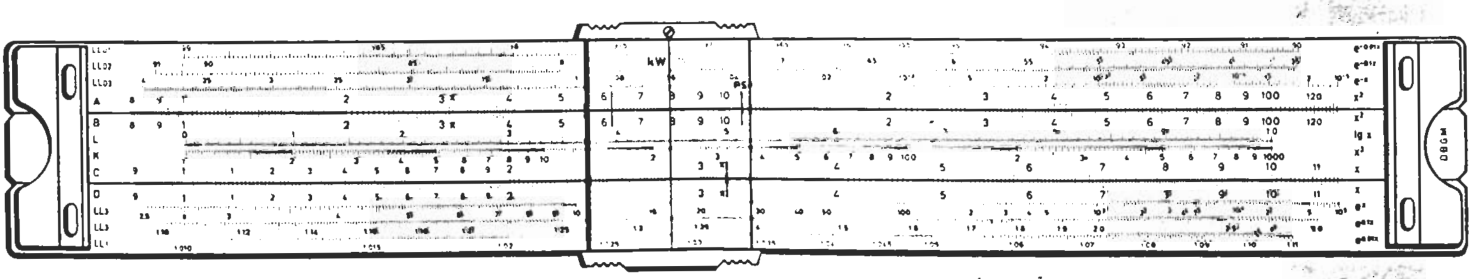

Fig. 5 LogLog side

THE ARISTO STUDIOLOG SLIDE RULE 0969

The trigonometrical face

ST	Scale of tangents, sines and radian measure for angles of 0.55° to 6°	∢ arc	Upper panel of body
T1	Scale of tangents for angles of 5.5° to 45°	∢ tan	
T2	Scale of tangents for angles of 45° to 84.5°	∢ tan	
DF	Fundamental scale, folded at π	πx	
CF	Fundamental scale, folded at π	πx	On slide
CIF	Scale of reciprocals of CF	$1/\pi x$	
S	Scale of sines for angles of 5.5° to 90°	∢ sin	
CI	Scale of reciprocals of C	$1/x$	
C	Fundamental scale	x	
D	Fundamental scale	x	Lower panel of body
DI	Scale of reciprocals of D	$1/x$	
P	Pythagoras scale	$\sqrt{1-x^2}$	
S	Scale of sines of angles 5.5° to 90°. Figured in red, counter clockwise, between 0° and 84.5°, as scale of cosines	∢ sin ∢ cos	

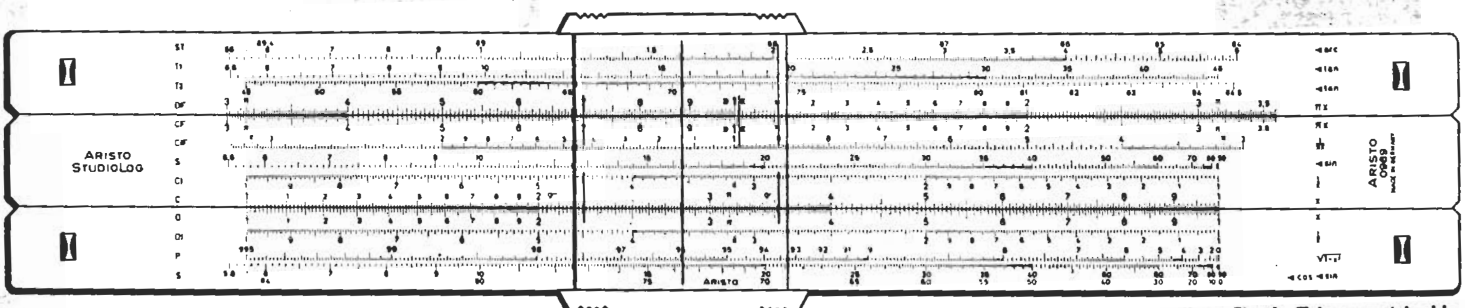

Fig. 6 Trigonometric side

The LogLog face

LL00	LogLog scale, range 0.999	to 0.989	$e^{-0.001x}$	Upper panel of body
LL01	LogLog scale, range 0.99	to 0.9	$e^{-0.01x}$	
LL02	LogLog scale, range 0.91	to 0.35	$e^{-0.1x}$	
LL03	LogLog scale, range 0.4	to 0.00001	e^{-x}	
A	Scale of squares		x^2	
B	Scale of squares		x^2	On slide
BI	Scale of reciprocals of B		$1/x^2$	
K	Scale of cubes		x^3	
L	Mantissa scale		lg x	
CI	Scale of reciprocals of C		$1/x$	
C	Fundamental scale		x	
D	Fundamental scale		x	Lower panel of body
LL3	LogLog scale, range 2.5	to 100 000	e^{x}	
LL2	LogLog scale, range 1.1	to 3.0	$e^{0.1x}$	
LL1	LogLog scale, range 1.01	to 1.11	$e^{0.01x}$	
LL0	LogLog scale, range 1.001	to 1.011	$e^{0.001x}$	

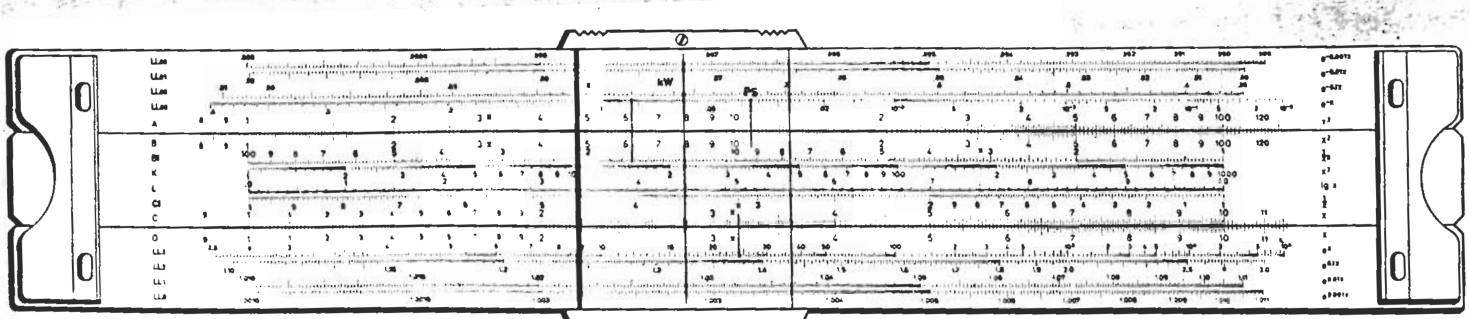

Fig. 7 LogLog side

3. Reading the scales

To use the slide rule efficiently for rapid calculations is essentially a matter of learning to read the scales quickly and correctly. The figures 8—11 show examples referred to the most frequently used fundamental scales C and D. The principal intervals, marked by long strokes, are figured from 1 to 10 (fig. 8). The end mark 10 is, on the trigonometrical face, repeated as 1, because this graduation can be regarded as the beginning of another and identical scale.

Fig. 8 The main intervals

In the range between figured graduations 1 and 2 the scale resembles the graduation of a millimeter scale, the difference consisting only in the reduction of interval width, progressively from left to right and in the use, on the slide rule, of the initial mark 1 in place of 0.

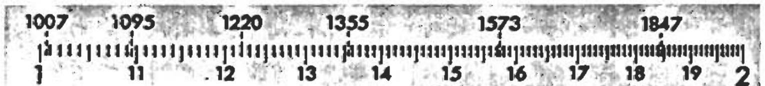

Fig. 9 Reading in the range 1 to 2

The graduation marked 2 of a millimeter scale can be considered as indicating 2 cm, 20 mm, 0.2 dm, 0.02 m and so on. In other words, the dimension, marked 2, can be thought of in association with various powers of 10. Similarly, the figures on the slide rule scales are independent of the position of the decimal point. It is therefore advisable to read a series of figures without regard to the decimal point, expressing them as simple numbers, e. g., 1 — 0 — 4 and not as one hundred and four. This will avoid omitting figures. For practice, move the cursor slowly to the right, from the value marked 1 and read at each graduation line the series of numbers: 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, etc.

The cursor hairline is so thin, by comparison with the width of the intervals, that the midpoint of a subdivision (between two graduations) can easily be located. Indeed, smaller fractions of subdivisions can be distinguished by eye. With practice, even one tenth of a subdivision can be estimated and thus the fourth digit obtained.

For practice, move the cursor slowly still further to the right. Between the graduations 1310 and 1320 estimates can be made, e. g., as 1310, 1311, 1312, 1313, 1314, 1315, etc.

Between a numbered graduation and that immediately following it, especially at the beginning of the scale, observe that a zero is to be read, e. g., 1000, 1001, 1002, 1003, etc. (note 1007 in fig. 9).

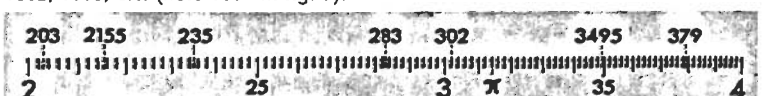

Fig. 10 Reading in the range 2 to 4

Because the intervals to the left of the figure 2 are already very narrow, in the following range between figures 2 and 4, only every second interval is marked. This yields a new graduation pattern, in which from mark to mark the even values are to be counted off: 200, 202, 204, 206, 208, 210, 212, 214, etc. The midpoints of the intervals give the odd numbers: 201, 203, 205, 207, 209, 211, 213, etc. Fig. 10 shows some examples.

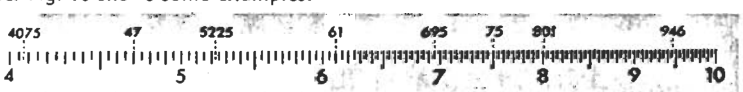

Fig. 11 Reading in the range 4 to 10

In the range 4 to 10, the intervals are marked in subdivisions of 5 units and the successive graduations are read as: 400, 405, 410, 415, 420, 425, 430 etc.

Intermediate values must be estimated. Midway between the marks 400 and 405 is the value 4025; a little to the left of this the value 402, a little to the right 403. In like fashion, at the midpoint of the next pair of subdivision marks is found the value 4075. Fig. 11 shows a series of such points.

4. Reading the scales of pocket rules

(model 868 and 869 only)

Because of the reduced base length, the scales of the pocket rule are divided differently from those of the 10 in. slide rule. The three separate basic interval division patterns are shown in fig. 10.

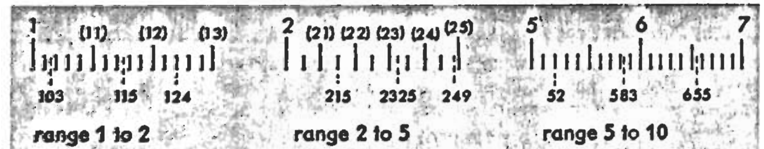

Fig. 12

In the range 1 to 2, only the graduations 1, 1.5 and 2 are figured. The second digit of a number can be counted off, using the longer graduations, as is shown by the numbers in parenthesis — e. g. (12). The interspersed short graduations give the (even numbered) third digit, e. g. 124. These third places are the even numbers 0, 2, 4, 6, and 8. The odd numbers lie in the middle of the space between two graduations, e. g. 103.

In the range 2 to 5 the second digit is again indicated by the longer of the graduations, e. g. (23), whilst the short intermediate lines indicate the third digit, in units of 5, e. g. 215. All other values of the third place must be estimated.

In the range 5 to 10, only the first place is figured. The second digit, as with a millimeter scale, is given by the short graduations, for example 52. The third digit is estimated, as between two adjacent graduations, e. g. 583.

5. Making approximations

It was explained, in chapter 3, that when using the slide rule, numbers are set or read as a simple series of digits. The correct position of the decimal point is determined by approximation. By this means, a check is at the same time imposed on the order of magnitude of the slide rule result.

Rules for approximation:

Values strongly rounded off!

Examples: $3.43 \approx 3$ $9.51 \approx 10$ $7.61 \approx 8$

When multiplying, round up one factor, round down the other!

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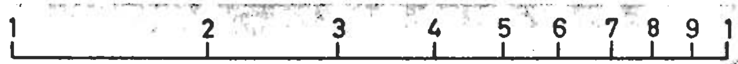

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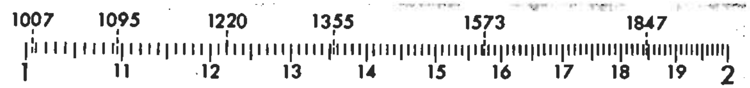

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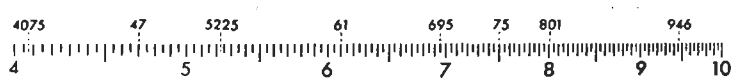

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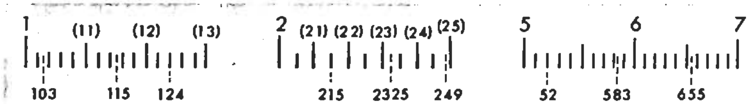

range 1 to 2 range 2 to 5 range 5 to 10

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When dividing, simplify!

Numerator and denominator are rounded off in the same direction.

Examples: $\frac{725}{539} = \frac{7.25}{5.39} \approx \frac{7}{5} = 1.4$

$$\frac{640 \times 15.3}{51 \times 0.8} \approx \frac{60 \times 20}{5 \times 1} = 240$$

Very large or very small numbers are simplified by separation of powers of 10.

Examples: $73215 \approx 7 \times 10^4$ $\quad 0.0078 \approx 8 \times 10^{-3}$

$89 \approx 9 \times 10^1$ $\quad 0.706 \approx 7 \times 10^{-1}$

Separation of powers of 10, when multiplying or dividing with very large numbers, gives a clearer appreciation of quantity.

Examples: $0.07325 \times 0.000513 \approx 8 \times 10^{-2} \times 5 \times 10^{-4} = 40 \times 10^{-6} = 4 \times 10^{-5}$

$$\frac{2950}{0.00598} \approx \frac{3 \times 10^3}{6 \times 10^{-3}} = 0.5 \times 10^6$$

6. The slide rule principle

Calculations are carried through by the mechanical addition or subtraction of scale lengths. The process can be very simply explained by considering two abutting millimeter scales, sliding one upon the other. Fig. 13 shows the example 2 + 3 = 5. If the initial mark 0 of the upper scale is moved over the value 2 of the lower scale, then immediately under 3 of the upper scale is found the sum 5 on the lower scale. In addition the sum 2 + 1 = 3 or 20 + 15 = 35 can be read from fig. 13, if the millimeters are counted off.

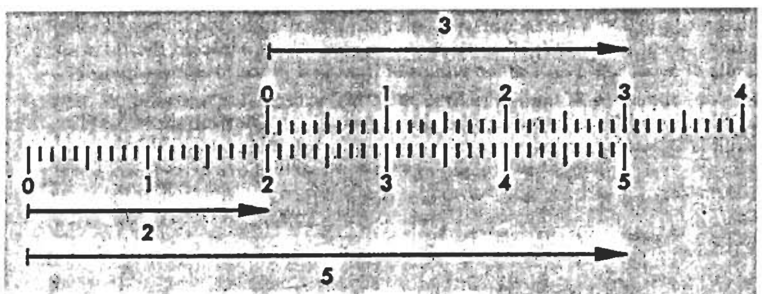

Fig. 13 Graphic addition by use of two ordinary scales

The subtraction 5 — 3 = 2 can also be read from fig. 13, by reversing the process described above. From the length 0 — 5 on the lower scale the length 0 — 3 on the upper scale is subtracted by setting the values 3 and 5 of the upper and lower scales, respectively, the one over the other and reading the result 2 from the lower scale under the initial mark 0 of the upper scale.

In the slide rule the graduations are disposed upon a rigid rule body and on a slide moving therein. The scales of the slide rule are, however, logarithmically divided and so the addition of two scale lengths performs a multiplication and a subtraction of two lengths carries out a division.

7. Multiplication

(Two scale lengths are added.)

The initial value 1, the left hand index of scale C of the slide, is brought over the value 18 on scale D. By moving the cursor to the value 13 on scale C we add the length 13 to the length 18. The product 234 can be read under the cursor hairline on scale D. The position of the decimal point can be located by an approximation, (20 × 10 = 200).

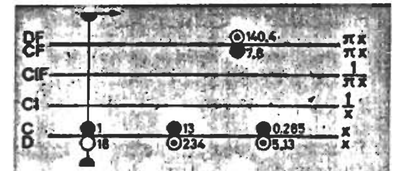

Fig. 14 18 × 13 = 234
18 × 0.285 = 5.13
18 × 7.8 = 140.4

To read the product of 18 × 7.8, the slide would have to be traversed, that is, the terminal index 10 of scale C would be brought over the factor 18 on D. With the ARISTO Studio or the ARISTO StudioLog, this additional slide setting can be avoided, if the upper pair of scales CF/DF is used for the multiplication.

Scales CF and DF make this simplification possible, because they are a repetition of the fundamental scales C and D, with the difference that the initial index 1, is placed approximately in the middle of the rule. It is often advantageous to make the first setting with the index of the CF scale placed opposite the multiplier on DF, because in this case, there is no need to decide whether to start with the right or the left index. Furthermore, in all settings made with the upper pair of scales no more than half a slide length will ever project beyond the body scales.

It will be noted that, when for example index mark 1 of the scale C of the lower pair is set to 18 on D, the setting of 1 on CF (upper scale pair) is simultaneously beneath 18 on scale DF. As a product 3.98 × 2.38 will be calculated by setting 1 on scale CF under 3.98 on DF. The cursor hairline is then moved over 2.38 on scale CF and the result 9.47 read on scale DF.

8. Division

(Subtraction of one scale length from another. This is the reverse of multiplication.)

The cursor hairline is brought over the value 2620 on scale D and the value 17.7 on scale C moved under the cursor line. The two values are then in juxtaposition. The quotient 148 is read on D under the left hand index of C of the slide. In other cases, the quotient may be read under the right hand index of the slide.

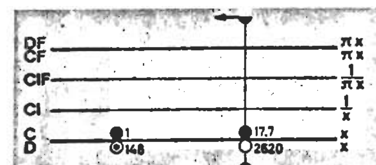

Fig. 15 2620 ÷ 17.7 = 148
Roughly: 3000 ÷ 20 = 150

Naturally, over 1 on CF the quotient can be read on DF, because the division 2620 ÷ 17.7 has also been set on this scale pair. In division with scales CF/DF, the factors are in the same relative position as written in a vulgar fraction.

The slide setting is identical with that for the multiplication 148 × 17.7 = 2620. The difference between multiplication and division consists only in the order of setting and reading. After setting up the division, the quotient will in any case be read on the body scale, under the left hand or right hand index; slide traversing will not be necessary. This characteristic feature will be used in the following chapters.

9. The folded scales CF and DF

In graduation pattern, scales CF and DF are identical with the fundamental scales C and D, but are laterally displaced, with respect to the fundamental scales, by the scale length corresponding to the value of $\pi = 3.142$. The value, figured 1, of these folded scales lies near the middle of the rule, producing an overlapping of the fundamental scales by half the rule length. The two pairs of scales, C/D and CF/DF, constitute a working assembly achieving advantages in multiplication, division, tabulation and proportion problems.

Index 1 of scale CF stands opposite, on DF, the same value as is matched with index 1 or 10 of scale C on D. Any of the multiplications discussed earlier can be begun on the scale pair CF/DF, with advantage, since the initial setting can always be chosen at once. It is not then necessary to decide whether the initial or final scale index should be used. If a division is set with the upper scale pair, the numerator and denominator are in their customary relative position, with the parting line between the scales corresponding to the division line in the fraction as written.

If the result of a problem cannot be read from one scale pair, it is always possible to find it from the other pair and slide traversing is avoided. The yellow strips on the slide are a reminder that factors taken on the moveable slide scales C or CF yield results to be read on D under C or on DF over CF.

9.1 Tabulation without slide traversing

$$y = 29\,x$$

x	1.7	3.45	5.0	10
y	49.3	100	145	290

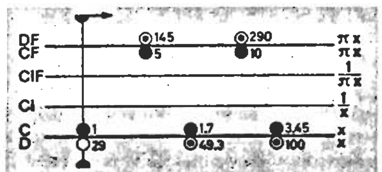

Fig. 16

For x = 5 the upper pair of scales CF and DF provides the answer without resetting the slide.

$$y = \frac{28.2}{x} = 28.2 \times \frac{1}{x}$$

x	7.43	2.92	1.567
y	3.795	9.66	18.0

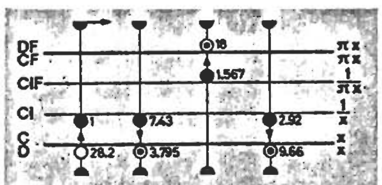

Fig. 17

$$y = \frac{x}{18.2} = \frac{1}{18.2} \times x$$

x	3.17	112.1
y	0.1742	6.16

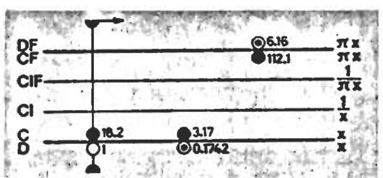

Fig. 18

9.2 Direct reading of multiplications and divisions involving π

A further advantage issues from the displacement of scales CF and DF by the value $\pi = 3.142$. By switching from D to DF or from C to CF, multiplication by π is performed automatically. Conversely, a division by π is accomplished by changing from DF to D or CF to C. If, for example, a diameter d is set with the cursor on D, the circumference π d can be read at once on DF. Similarly, the angular velocity $\omega = 2\,\pi\,f$ is found on DF when 2 f is set on D.

The possibility of taking the final reading by switching scales should always be considered when dealing with problems involving the factor π. Fig. 19 shows a range of results incorporating π, demonstrating the possibilities of a single cursor setting.

Calculations with reciprocals see chapter 11.

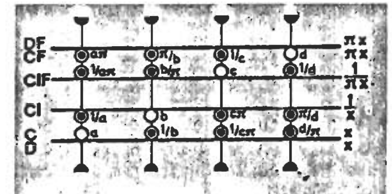

Fig. 19 Calculations with π

10. Combined multiplication and division

In solving expressions of the form $\frac{a \times b}{c}$ the rule to apply is:

First divide, then multiply.

The intermediate result of the division of 345 by 132 in fig. 20 need not be read. The slide rule scales are positioned ready for the final multiplication. The cursor is moved over the value 22 on scale C and the result read on scale D, viz. 57.7.

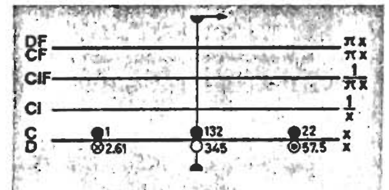

Fig. 20

$\frac{345}{132} \times 22 = 57.5$ Roughly: $\frac{300}{100} \times 20 = 60$

If, in this example $\frac{345 \times 22}{132 \times 19.5} = 2.95$, the further factor 19.5 is introduced in the denominator, the solution obtained in fig. 20 is divided by moving the value 19.5 on scale C under the cursor hairline, thus dividing 57.5 by 19.5. Should there be, in examples of this type, yet more factors in numerator and denominator, simply divide and multiply alternately. The rhythmical alternation between slide and cursor positioning leads to smooth flowing calculation with minimal setting.

In such problems, it can happen that the slide, following a division, projects too far out of the rule body to permit a setting. To perform multiplication, the slide must be traversed. By careful choice of setting for division, between scales C/D or CF/DF, the necessity for slide traversing can often be avoided.

11. Scales of reciprocals CI and CIF

Scale CI is divided exactly as the fundamental scales C and D, but the intervals progress in the opposite direction, i. e., from right to left. To obviate errors the figuring of the graduations is in red. If the cursor is set to any value x on scale C, the reciprocal 1/x can be read from CI, as indicated by the scale identification symbol at the right hand end of the scale.

Over 5 on C is 1/5 = 0.2 on CI. Of more importance, however, is the fact that the reciprocal scale can be used in the reverse direction. By changing from CI to C we find, e. g. under 4 on CI the value 1/4 = 0.25 on C.

The occasional use of scale CI to find reciprocals would not justify its provision; its real value lies in the fact that it can be used to avoid many settings in complex examples.

$$\frac{4}{5} \text{ can be written as } 4 \times \frac{1}{5} \text{ and } 4 \times 5 \text{ as its equivalent } \frac{4}{1/5}.$$

Whilst these expressions are perhaps unusual, they offer the advantage, for slide rule working, of converting a division into a multiplication or, conversely, a multiplication to a division. This advantage will best be displayed by a "game" with simple numbers.

1. With the cursor set to 6 on D, bring 2 on C under the hairline. We then have the usual setting for the division $6 \div 2$ (fig. 21). If, however, the cursor is left in place and by a slide movement 2 on CI is brought under the hairline, we have the multiplication 6×2 and read the product 12, as for a division, under the index of the slide — (fig. 22). Actually, we have found the quotient of $6 \div 0.5$, because with bringing 2 on CI, simultaneously the reciprocal 0.5 on C was set under the hairline.

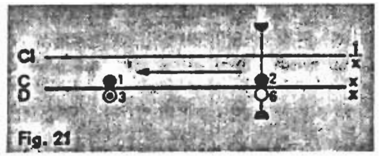

Fig. 21

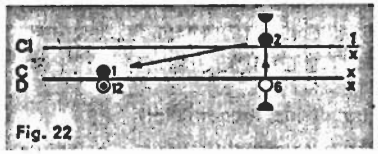

Fig. 22

2. Now, letting the index 1 of C remain over 12 on D, move the cursor to 4 on C, establishing the normal setting for the multiplication $12 \times 4 = 48$ (fig. 23). By moving the cursor to 4 on CI, however, we can read the quotient of $12 \div 4$ on D (fig. 24). In other words, because under 4 on CI stands its reciprocal $1/4 = 0.25$ on C, we actually calculate $12 \times 0.25 = 3$.

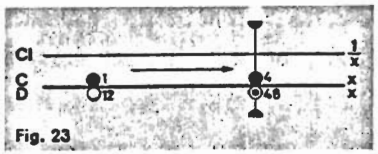

Fig. 23

There are thus two setting possibilities in multiplication and division and the experienced operator will chose the best, in the solution of a complex example by alternate division and multiplication.

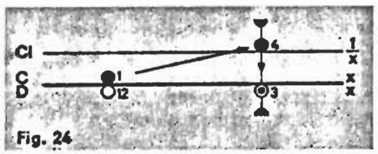

Fig. 24

The stated relationship between scales C and CI holds similarly between scales CF and CIF. To show that this is so, the "number game" can usefully be replayed with the scale group CF/DF/CIF.

Anyone who thoroughly studies the foregoing will at once recognise that scale CIF is the logical complement of the scale system. Whoever properly exploits the advantages of the folded scales will use scale CIF as often as scale CI.

Expressions of the form $a \times b \times c$ or $\frac{a}{b \times c \times d}$ etc. will be solved by alternate multiplication and division, as shown in chapter 10 on combined multiplication and division. In the course of the calculation with scales C, D and CI switching to the scale group CF, DF and CIF will avoid slide traversing in multiplication.

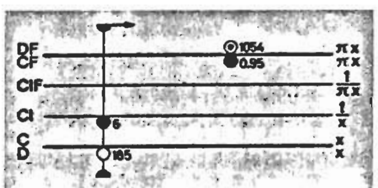

Fig. 25 $185 \times 6 \times 0.95 = 1054$
Roughly: $200 \times 6 \times 1 = 1200$

In the example of fig. 25, the factors 185 on scale D and 6 on scale CI are set in opposition, as for a division. Multiplication by 0.95 is then carried out with the upper scale CF and the result 1054 read on DF over 0.95 on CF.

11.1 Scale of reciprocals DI

The reciprocal scale DI offers advantages to the experienced operator, who may on occasion exchange the functions of body and slide scales, for example when dealing with proportions.

12 Proportions

Proportions of the form $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ are particularly simple to calculate with the slide rule because, after setting one ratio, all other equal ratios can be found by moving the cursor. The parting line between body- and slide scales can be regarded as the line in a common fraction, as written. Proportions should preferably be expressed in this form.

Example: 9.5 lb of a given material cost \$ 6.3. What will be the cost of 8.4 lb?

The solution by "rule of three" follows from $\frac{6.30}{9.5} \times 8.4 = 5.57$

The calculation can be more conveniently made if the ratio of weight and price is set up as a proportion. If the given weight on DF is brought over the corresponding price \$ 6.30 on CF, all equivalent weight/price ratios will be shown on scales DF/CF and D/C. On scales DF and D are all weights, in accordance with the initial setting and on scales CF and C are the corresponding prices. Opposite the weight 8.4 lb is found the price \$ 5.57. Other weight/price relationships are shown in fig. 26.

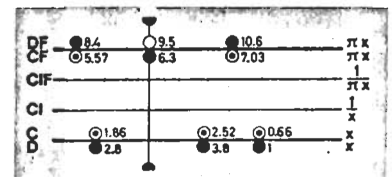

Fig. 26 Proportions

10.6 lb cost \$ 7.03 (scales CF/DF)
3.8 lb cost \$ 2.52 (scales C/D)
2.8 lb cost \$ 1.86 (scales C/D)
1 lb cost \$ 0.66 (scales C/D)

The proportion can be extended at will:

$$\frac{\text{lb}}{\$} = \frac{9.5}{6.3} = \frac{8.4}{5.57} = \frac{10.6}{7.03} = \frac{3.8}{2.52} = \frac{2.8}{1.86} = \frac{1}{0.66} = \ldots$$

Calculation by proportions proceeds independently of the earlier mentioned rule. It is of no consequence, when and how the weight/price ratio is set up, the only difference arising is that weights are looked for on the scale on which the first weight was set and the corresponding prices on the adjacent scale. In the example above 6.3 could have been set on scale DF and 9.5 on CF. The price 8.4 would then be found on CF and the required proportion read on DF, as 5.57.

This principle of direct proportion, $a : b = c : d$ applies with equal force to indirect proportion, which leads to the identity $a \times b = c \times d$, to be solved with the aid of the reciprocal scales (see section 11). Finally, the principle can be seen applicable to the "mixed" proportions $a \times b = c : d$ and $a : b = c \times d$.

13. The scales A, B and K

If the cursor line is brought over any value x on scale C, the value x^2 can be found on scale B (the scale of squares) or x^3 on scale K. Conversely, the square root or the cube root can be obtained.

a) $2^2 = 4$ $\qquad$ $2^3 = 8$

b) $32{,}7^2 = 3{,}27^2 \times 10^2 = 1070$

$\quad 32{,}7^3 = 3{,}27^3 \times 10^3 = 35000$

c) $\sqrt[2]{9} = 3$ $\qquad$ $\sqrt[3]{27} = 3$

d) $\sqrt[2]{51} = 7.14$ $\qquad$ $\sqrt[3]{364} = 7.14$

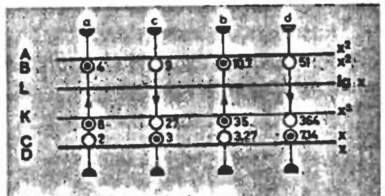

Fig. 27 Powers and roots

The position of the decimal point is best found by approximation. In calculating powers and roots it is of advantage to work in powers of ten, to obtain numbers in which the position of the decimal point is easily seen. To this end, the scale of squares is figured 1 to 100 and the cube scale 1 to 1000. The range in which the cursor is to be set follows from the figuring of the scale.

Examples:

$$\sqrt{3200} = \sqrt{32 \times 100} = 10 \times \sqrt{32} = 10 \times 5.66 = 56.6$$

$$\sqrt[3]{0.1813} = \sqrt[3]{\frac{181.3}{1000}} = \frac{1}{10} \times \sqrt[3]{181.3} = \frac{1}{10} \times 5.66 = 0.566$$

13.1 Calculation with the scales A and B

Scales A and B, like the fundamental scales C and D, are identically divided, with the difference that they consist of two scale segments, each half the length of the fundamental scales C and D. The left hand segment is figured 1 to 10 and the right, 10 to 100. All examples so far discussed can be solved with the scales A and B, by methods described for the fundamental scales. The reading will be somewhat less, because the graduations are disposed over only half the length of the rule.

The juxtaposition of the scales offers the advantage that the slide resetting is avoided, on basic principles.

In many cases it is convenient, if a problem begins with a squared factor, to continue the calculation on the scales of squares.

13.2 Scale of reciprocals BI
(869 and 0969 only)

Scale BI is the reciprocal scale of B. The scale BI offers advantages to the experienced operator, who may use on occasion use the scales of squares as the fundamental scales and their reciprocals.

14. The Pythagoras scale P

In a right triangle, with hypotenuse 1, the Pythagoras relationship with the other two sides holds.

$$y = \sqrt{1 - x^2}$$

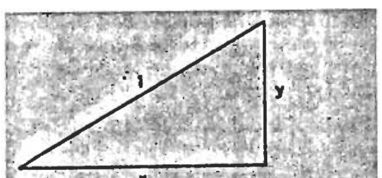

Fig. 28

For any setting of x on the fundamental scale D we find the value of y on scale P and, conversely, $x = \sqrt{1 - y^2}$ on D if y is set on P. In the example of fig. 29 it is clear that 0.6 could equally well be set on D or on P. In either case the required value 0.8 is found on the corresponding adjacent scale.

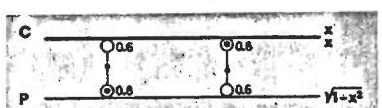

Fig. 29

The choice of initial setting should be made with due regard to maximum accuracy of reading. In the example $\sqrt{1 - 0.15^2} = 0.9887$ the factor 0.15 will be set on D.

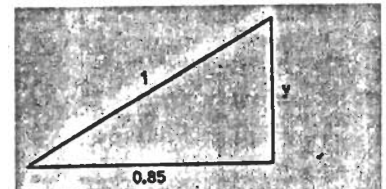

Fig. 30

Example in electrical engineering:

apparent load $\triangleq 1.0$

effective load $\triangleq 0.85$

wattless load $\triangleq \sqrt{1 - 0.85^2} = 0.527$

This method of solution is suitable, however, only when the hypothenuse is 1, 10 or 100 and especially when converting sin ⟷ cos, using the relationship $\sin^2 \alpha + \cos^2 \alpha = 1$. With reference also to the scale DI at the ARISTO StudioLog, the relationships shown in fig. 31 are available. More generally, right triangles are solved with greater elegance by trigonometrical methods (see chap. 18).

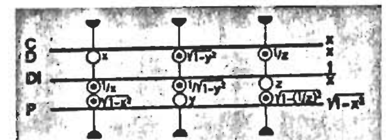

Fig. 31

To achieve greater accuracy in calculations with the scales of squares, rearrangement of the data is useful. For example:

$$\sqrt{0.91} = \sqrt{1 - 0.09} = 0.9540$$

The factor 0.09 is taken on the left hand portion of scale A. On D is then found $\sqrt{0.09} = 0.3$ and the value of $\sqrt{1 - 0.3^2} = 0.9540$ is seen on P. Greater accuracy is obtained, in this way, for roots greater than about $\sqrt{0.65}$ and is always convenient when the radicand is close to 0.01, 1,100, etc.

15. Trigonometrical functions

All angle functions are referred to the fundamental scale D and the angular scales, in the 360° system, are decimally divided.

If an angle is set with the cursor on scales S, T1 or T2, the corresponding function value can be found under the cursor line on scale D. Conversely, for a function value set on scale D, the corresponding angle can be read on scale S, T1 or T2.

The figuring of the decimally divided scales S, T1 and T2 applies uniquely to the inscribed angle values.

The slide rule gives the function value for angles in the first quadrant only. The relationships for any angle, with those of an angle in the first quadrant, are tabulated below.

	$\pm\alpha$	$90° \pm \alpha$	$180° \pm \alpha$	$270° \pm \alpha$
sin	$\pm \sin\alpha$	$+ \cos\alpha$	$\mp \sin\alpha$	$- \cos\alpha$
cos	$+ \cos\alpha$	$\mp \sin\alpha$	$- \cos\alpha$	$\pm \sin\alpha$
tan	$\pm \tan\alpha$	$\mp \cot\alpha$	$\pm \tan\alpha$	$\mp \cot\alpha$
cot	$\pm \cot\alpha$	$\mp \tan\alpha$	$\pm \cot\alpha$	$\mp \tan\alpha$

15.1 The sine scale S

The scale of sine S is figured between 5.5° and 90° in black and also in red from right to left, for cosines between 0° and 84.5°. All sines and cosines read on D are prefixed with 0 before the decimal point.

Sines of angles $\alpha > 45°$ are read with enhanced accuracy on the red figured scale P, using the identity $\sin\alpha = \sqrt{1 - \cos^2\alpha}$. To set the angle, the red figures of scale S are used, hence the colour rule for sine functions: set and read sine functions in like colours.

For cosines of angles $\alpha < 45°$, an analogous colour rule follows from $\cos\alpha = \sqrt{1 - \sin^2\alpha}$. For every setting on scale S read in the contrasting colour the function value on scale D or P.

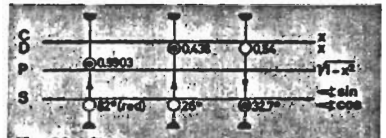

Fig. 32 Sine

Examples:

sin 26°	$= 0.438$
sin 82°	$= \sqrt{1 - \cos^2 82°}$
	$= 0.9903$
arc sin 0.54	$= 32.7°$
cos 75°	$= 0.2588$
cos 7°	$= \sqrt{1 - \sin^2 7°}$
	$= 0.99255$
arc cos 0.9852	$= 9.87°$

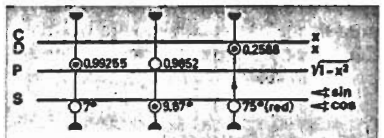

Fig. 33 Cosine

15.2 The sine scale on the slide

(869 and 0969 only)

With the ARISTO StudioLog, a scale of sines is available on both rule body and the slide, providing the facility of a fixed ore moveable scale, either of which may be used according to convenience when dealing with a particular problem. In multiplication or division of multi-term functions, e. g., in spherical trigonometry, both scales are advantageous. The slide scale of sines is helpful, too, in problems involving $\frac{a}{\sin\alpha}$ and $\frac{b}{\cos\beta}$ and in optics, when refraction formulae such as $\frac{n}{n'} = \frac{\sin i'}{\sin i}$ must be used.

Examples:

1. $\cos\varphi = \dfrac{\sin 44.3° \times \sin 16.7°}{\sin 14.6°}$

   $\varphi = 37.2°$

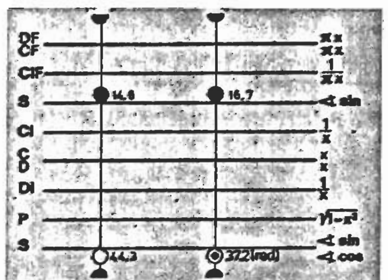

Fig. 34

2. Calculate the trigonometrical altitude h if the length of the basic is c = 18.6 cm and the angles of elevation are $\alpha = 28°$ and $\beta = 25°$ (fig. 35).

$$h = \frac{c \times \sin\alpha \sin\beta}{\sin(\alpha - \beta)} \quad \text{(fig. 35)}$$

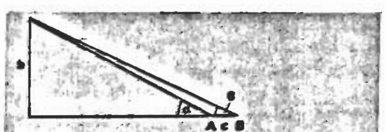

Fig. 35

For slide rule calculation the formula is written:

$$\frac{1}{h} = \frac{\sin(\alpha - \beta) \times \frac{1}{c}}{\sin\alpha \times \sin\beta}$$

$$\frac{1}{h} = \frac{\sin(28° - 25°) \times \frac{1}{18.6}}{\sin 28° \times \sin 25°}$$

h = 70.5 m (fig. 36)

With the aid of the cursor bring 3° in ST over 28° in S. Then move the cursor over 18.6 in CI. Without moving the cursor the slide is displayed, that 25° in S on the slide gets under the cursor. Read the result h = 70.5 m in C over the end of D.

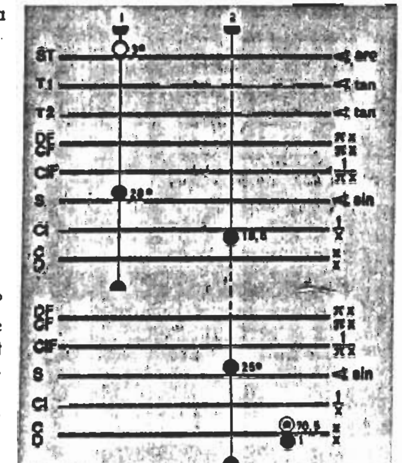

Fig. 36

15.3 The tangent scales T1 and T2

The tangent scale is in two parts. T1 covers the range 5.7° to 45° and T2 45° to 84.3°. For any angle set on the twice, tangents scales the function values of the twice tangents are read on D. Angles set on T1 have function values between 0.1 and 1.0, whilst angles set on T2 have function values between 1 and 10.

The cotangent is the reciprocal of the tangent and consequently the relationship $\cot\alpha = \frac{1}{\tan\alpha}$ is used. For any angle set on T1 or T2 the function values of the cotangent can be read on the scale CI. At the ARISTO StudioLog the function value of the cotangent can be read on the scale of reciprocals DI. Angles set on T1 have function values of cotangents between 1 and 10, whilst angles set on T2 have function values between 0.1 and 1.

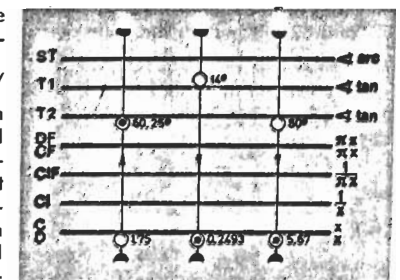

Fig. 37 Tangent

Examples:

tan 14°	$= 0.2493$
tan 23.6°	$= 0.437$
tan 41.1°	$= 0.872$
tan 51.2°	$= 1.244$
tan 73.4°	$= 3.35$
tan 80°	$= 5.67$
arc tan 1.75	$= 60.25°$
arc tan 2.0	$= 63.43°$

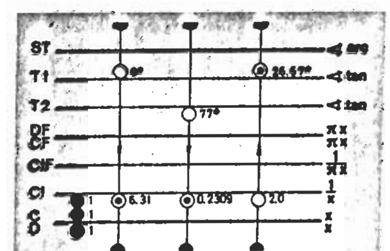

Fig. 38 Cotangent

cot 9° = 6.31
cot 23.6° = 2.289
cot 41.1° = 1.146
cot 51.2° = 0.804
cot 73.4° = 0.298
cot 77° = 0.2309
arc cot 2.0 = 26.57°
arc cot 1.75 = 29.74°

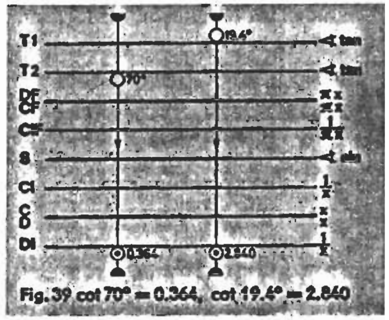

Fig. 39 cot 70° = 0.364, cot 19.6° = 2.840

16. The ST scale

This scale is an extension of scales S and T for angles having function values between 0.01 and 0.1, read on scale D. Its construction satisfies also the requirements for converting between radian and circular measure, again with reference to scale D.

16.1 Small angles — large angles

If sin α and tan α for α < 5.5° or cos α and cot α for α > 84.5° are to be found, use the relationship:

$$\sin\alpha \approx \tan\alpha \approx \cos(90° - \alpha) \approx \cot(90° - \alpha) \approx \frac{1}{180}\alpha° \approx 0.01745\,\alpha$$

Scale ST is figured between 0.55° and 6° but is subdivided in radian measure. This makes possible accurate reading on scale D in radians for sine and tangent functions of small angles. The red figuring of scale ST, from right to left, between 84° and 89.45° enables the scale of small angles to be used for the cosines and cotangents of large angles.

The agreement in value between sin α, tan α, and arc α is very good up to 4°; for example sin 4° = 0.0698, tan 4° = 0.0699, and arc 4° = 0.0698. For larger angles between 4° and 6° more accurate values can be obtained from the relationship:

$$\sin\alpha = \alpha \times \frac{\sin 6°}{6} \qquad \text{or} \qquad \tan\alpha = \alpha \times \frac{\tan 6°}{6}$$

Examples:

$$\sin 4.7° = 4.7° \times \frac{\sin 6°}{6} = 0.0819$$

$$\sin 5.3° = 5.3° \times \frac{\sin 6°}{6} = 0.0924$$

$$\tan 4.7° = 4.7° \times \frac{\tan 6°}{6} = 0.0822$$

$$\tan 5.3° = 5.3° \times \frac{\tan 6°}{6} = 0.0928$$

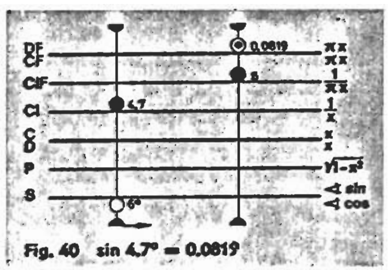

Fig. 40 sin 4.7° = 0.0819

To work the foregoing examples, the data are re-arranged as follows:

$$\sin\alpha = \frac{\sin 6°}{\frac{1}{\alpha} \times 6} \qquad \tan\alpha = \frac{\tan 6°}{\frac{1}{\alpha} \times 6}$$

With the aid of the cursor sin 6° on scale S or tan 6° on scale T are brought under the value of the angle α on scale CI. The cursor is then moved over 6 on scale CIF and the result read on scale DF.

Values cos α for α < 5.7° and sin α for α > 84.3° can only be read rather inaccurately from the rule. In such cases, the first terms of an expansion provide a helpful approximation.

$$\cos\alpha \approx 1 - \frac{\alpha^2}{2} \text{ (in radians)}$$

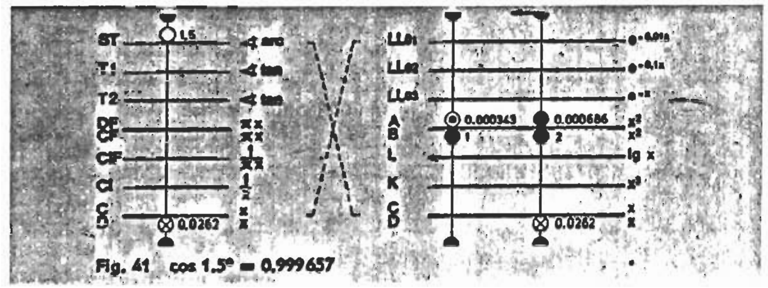

Fig. 41 cos 1.5° = 0.999657

Example: $\cos 1.5° = 1 - \frac{0.0262^2}{2} = 0.999657$ (fig. 41)

To calculate the second term of the expansion the angle 1.5 is set on scale ST with the cursor. On scale D is then found the angle in radian measure and the square of this value 0.000686 is seen on scale A. To divide by 2, bring 2 on B under the hairline and read the result 0.000343 on scale A. Finally, the subtraction 1 — 0.000343 gives the result 0.999657.

$$\sin 86.5° = \cos 3.5° = 1 - \frac{0.0611^2}{2} = 0.99813$$

16.2 Conversion circular ⟷ radian measure

Conversion from circular to radian measure follows a cursor setting and transfer from scale ST to scale D, because ST is a repetition of scale D displaced by the factor $\frac{\pi}{180}$. Conversely, radian measure can be converted to circular measure. This facility applies not only for angles marked on ST, but for all angles, because the scale is decimally divided and therefore 1 can be read as 0.1°, 10° and so forth, with an appropriate shift in the position of the decimal point in the radian measure (see fig. 42). The figure 1 of scale ST corresponds to the value $\frac{\pi}{180}$ on D.

Examples:

a) 0.1° = 0.001745 rad

b) 10° = 0.01745 rad

c) 0.5° = 0.008725 rad

d) 5° = 0.08725 rad

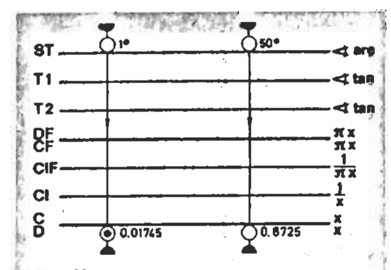

Fig. 42

Should the small angle be given in minutes or seconds of arc, it can be transformed into a decimal fraction of a degree: $1' = \frac{1}{60}°$ and $1'' = \frac{1}{3600}°$ (see also chap. 16.3 and 21.1).

By setting 6 or 36 of scale CF under 1° on scale ST a useful tabulation is produced, for conversions of this type.

16.3 The marks ϱ' and ϱ''

These gauge marks simplify conversion when the angle is given in minutes or seconds of arc. They indicate the factors:

$$\varrho' = \frac{180}{\pi} \times 60 = 3438 \quad \text{for minutes}$$

$$\varrho'' = \frac{180}{\pi} \times 60 \times 60 = 206\,265 \text{ for seconds}$$

Hence, converting by division: $\text{arc}\,\alpha = \frac{\alpha'}{\varrho'} = \frac{\alpha''}{\varrho''}$

For example:

$\text{arc } 22' = \frac{22'}{\varrho'} = 0.00640$ rad (fig. 43)

$\text{arc } 400' = \frac{400'}{\varrho'} = 0.1163$ rad

$\text{arc } 17'' = \frac{17''}{\varrho''} = 0.0000824$ rad

$\text{arc } 380'' = \frac{380''}{\varrho''} = 0.001843$ rad

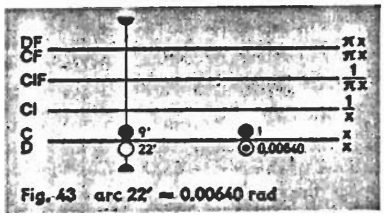

Fig. 43 arc 22' = 0.00640 rad

These marks are of great use when finding small angles or lengths of arc for given radii:

$\alpha = \frac{b}{r} \times \varrho$ when the angle is to be found.

$b = \frac{\alpha \times r}{\varrho}$ when the length of arc is required.

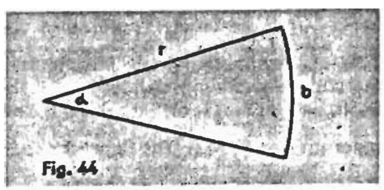

Fig. 44

Examples:

$\alpha = \frac{0.6}{45} \times \varrho' = 45.8'$

$b = \frac{48'' \times 67}{\varrho''} = 0.0156$

17. ARISTO Studio, 400^g system

The trigonometrical scales S, T1, T2 and ST are in the ARISTO Studio 0968/400^g divided in the 400^g system. Calculations with these scales follow the routines described in chap. 15 to 16.3. The previous examples and the relationships quoted must however be modified to conform with a right angle of 100^g. When finding cofunctions note:

$$\cos\alpha = \sin(100^g - \alpha)$$

$$\cot\alpha = \frac{1}{\tan(100^g - \alpha)}$$

In the 400^g system the examples of chap. 15.1 to 16.3 appear as:

17.1

$\sin 26^g = 0.397$

$\sin 82^g = \sqrt{1 - \cos^2 82^g} = 0.9063$

$\text{arc sin } 0.54 = 36.3^g$

$\cos 75^g = 0.383$

$\cos 7^g = \sqrt{1 - \sin^2 7^g} = 0.99396$

$\text{arc cos } 0.9852 = 10.97^g$

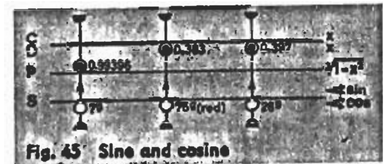

Fig. 45 Sine and cosine

17.2

$\tan 14^g = 0.2235$

$\tan 80^g = 3.078$

$\tan 85^g = 4.17$

$\text{arc tan } 1.75 = 66.95^g$

$\cot 77^g = 0.378$

$\text{arc cot } 2.0 = 87.44^g$

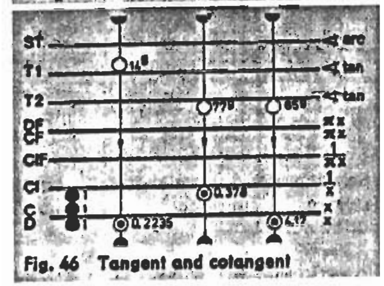

Fig. 46 Tangent and cotangent

17.3 $\sin\alpha \approx \tan\alpha \approx \cos(100^g - \alpha) \approx \cot(100^g - \alpha) \approx \frac{\pi}{200^g}\alpha^g = 0.01571\,\alpha$

For the sines of large angles and the cosines of small angles use the first terms of the series expansion.

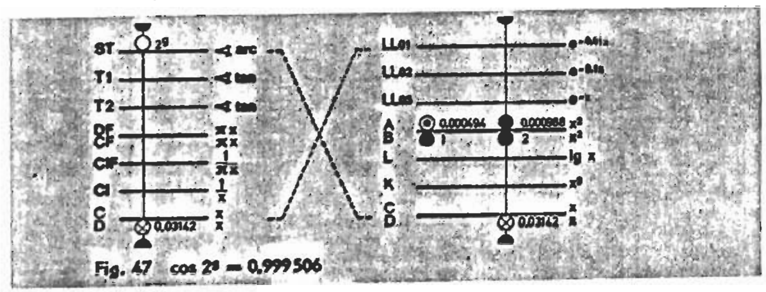

Fig. 47 cos 2^g = 0.999506

Example: $\cos 2^g = 1 - \frac{0.03142^2}{2} = 1 - 0.000494 = 0.999506$ (fig. 47)

$\sin 95^g = \cos 5^g = 1 - \frac{0.0786^2}{2} = 1 - 0.00308 = 0.99692$

17.4 In the ARISTO Studio 0968/400^g scale ST is displaced by the factor $\frac{\pi}{200}$, with respect to the fundamental scale. The value 1 on this scale is the setting point for $\frac{200}{\pi}$.

a) $0.1^g = 0.001571$ rad
b) $10^g = 0.1571$ rad
c) $0.5^g = 0.007854$ rad
d) $5^g = 0.07854$ rad

17.5

The numerical values of the gauge points conform to the new system, in respect of degrees, minutes and seconds of arc:

$$\varrho^g = 63.66$$
$$\varrho^c = 6366$$
$$\varrho^{cc} = 636\,600$$

18. Trigonometrical solution of plane triangles

The advantage offered by the trigonometrical scales is not simply the availability of function values. Of more importance, function values can be used in calculation without their being read from the scales.

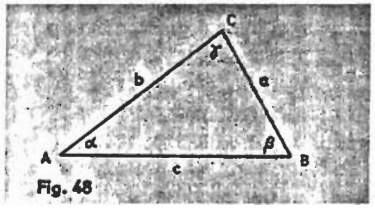

Fig. 48

The law of sines is a convincing example of the efficiency of the slide rule in solving proportions:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

When one of these ratios is set up by bringing the length on scale C opposite the corresponding angle on scale S or ST, all other parts of the triangle can at once be read.

In practice this law is most often applied in the case of right triangles, in which we have $\gamma = 90°$, $\sin\gamma = 1$, angle $\alpha = (90° - \beta)$ and angle $\beta = (90° - \alpha)$. The law of sines is then rearranged as

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = \frac{a}{\cos\beta} = \frac{b}{\cos\alpha}$$

and further $\tan\alpha = \frac{a}{b}$

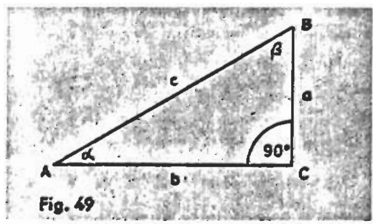

Fig. 49

Depending on the given elements, there follows one of two procedures:

1. Given any two parts (other than those of case 2).
2. Given the two short sides a and b.

Example to case 1:

Given: c = 5, a = 3

Required: α, β, b.

Note that: $\beta = (90° - \alpha)$

$$\frac{c}{1} = \frac{a}{\sin\alpha} = \frac{b}{\cos\beta}$$

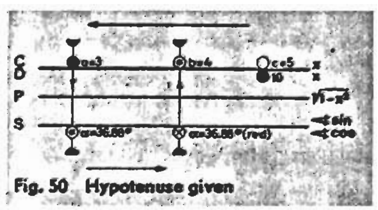

Fig. 50 Hypotenuse given

By setting the hypotenuse 5 on scale C over 1 on scale D (function value of sin 90°), the angle $\alpha = 36.88°$ is at once found on scale S opposite the short side 3 on C. Without moving the slide, bring cursor over $\beta = 36.88°$ (red figures of scale S). The side b, corresponding to the angle β can then be read on scale C, i. e., 4.

The procedure is the same if a short side and an angle are given. The sine ratio is established from the short side and the subtended angle on scales S and C. On occasion it is of advantage to work with scale CF instead of C, to avoid traversing the slide.

Example to case 2:

Given: a = 3, b = 6

Required: α, β, c

Note that: $\tan\alpha = a \times \frac{1}{b}$

$$\sin\alpha = a \times \frac{1}{c}$$

$$\tan\alpha = 3 \times \frac{1}{6} \rightarrow \tan\alpha < 1$$

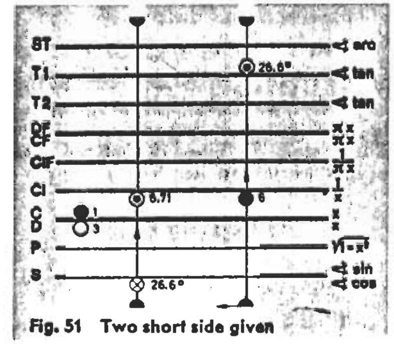

Fig. 51 Two short side given

Setting 1 of scale C over the shorter side 3, $\alpha = 26.6°$ is read on scale T1 over 6 on scale CI. Without moving the slide, move the cursor to 26.6° on scale S and read c = 6.71 on scale CI.

Then from $\sin\alpha = \frac{a}{c}$ there is the proportion $\frac{a}{1} = \frac{\sin\alpha}{1/c}$ · $\beta = 90° - 26.6° = 63.4°$.
If a > b, that is $\alpha > 45°$, the angle is not read on T1 but on T2. The remaining steps in the solution are exactly as described above.

Example to case 2:

Given: a = 2, b = 4.5

Required: α, β, c.

If, in solving a right angles triangle, the shortest side is identified as side a, the required values can be found from the following proportion:

$$\frac{1}{1/a} = \frac{\tan\alpha}{1/b} = \frac{\sin\alpha}{1/c}.$$

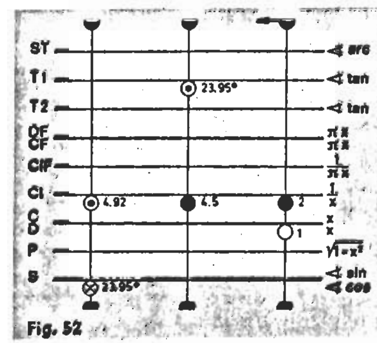

Fig. 52

The shortest side a = 2 is set on scale CI over the right hand index 1 of scale D. Over b = 4.5 on CI the value of $\alpha = 23.95°$ is read on T1. The hairline is then brought over $\sin\alpha = 23.95°$ on scale S and the hypotenuse c = 4.92 read on scale CI.

$\beta = 90° - 23.95° = 66.05°$.

18.1 Complex numbers

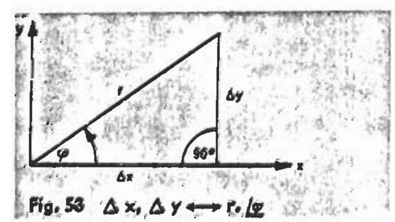

Fig. 53 $\Delta x, \Delta y \longleftrightarrow r, \underline{/\varphi}$

These two cited procedures for the solution of right triangles have special significance in connection with coordinate and vector calculations and in work with complex numbers. They apply in problems of conversion from rectangular coordinates to the polar form and vice versa.

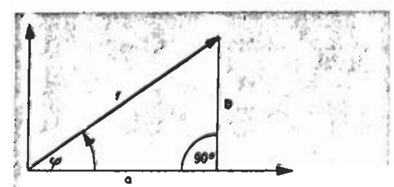

Fig. 54 $Z = a + ib = r\underline{/\varphi}$

Complex numbers in the coordinate form $Z = a + ib$ can easily be added or subtracted. The vector form $Z = r \times e^{i\varphi} = r\underline{/\varphi}$ is better suited to multiplication and division. For this reason the conversion of one form into the other must often be performed.

Examples: $Z = 4.5 + i\,1.3 = 4.68\underline{/16.13^\circ}$

$Z = 6.7\underline{/49^\circ} = 4.39 + i\,5.05$

The process of solution is shown in fig. 54 and follows the description of methods of solving right angled triangles given above.

19. The LogLog scales

The LogLog scales are divided as logarithms of logarithms and are referred to the fundamental scales C and D. In the ARISTO StudioLog the range 10^{-5} to 10^{+5} is displayed in eight sections, four with negative exponents e^{-x}, identified as LL00, LL01, LL,02 LL03 from 10^{-5} to 0.999 and four with positive exponents, e^x, marked LL0, LL1, LL2, LL3 for 1.001 to 10^5. In the ARISTO Studio the scales LL00 and LL0 are not included; the range 10^{-5} to 10^5 is displayed in six sections only, three with negative exponents and three with positive exponents. Readings taken on the LogLog scales are unique values, that is to say, the value, e. g., 1.35 denotes only 1.35 and cannot be read as 13.5 or 135, as on the fundamental scales.

The LogLog scales LL and LL0 are reciprocal one of the other. They permit direct readings of reciprocals of numbers less than 2.5 with greater precision than is possible with scales CI or CIF.

Example: $\frac{1}{1.0170} = 0.98328$

By means of the exponential scales problems of involution or evolution are solved by addition or subtraction, respectively, of scale lengths. Thus, required powers, roots, and logarithms within the scale range can be calculated.

19.1 Powers and roots with exponents 10 and 100

The LogLog scales are so arranged that, by passing from one scale to that next to it, the tenth power or the tenth root of a number set on one scale can be read at once on the other, according to the direction in which the move is made. This relationship is clearly shown by the examples in fig. 55, for a setting of the value 1.015 on LL1.

Examples:	Reading on scale
$1.015^{0.1} = \sqrt[10]{1.015} = 1.00149$	LL0
$1.015^{1} = 1.015$	LL1
$1.015^{10} = 1.1605$	LL2
$1.015^{100} = 4.43$	LL3
$\frac{1}{1.015^{100}} = 1.015^{-100} = 0.2257$	LL03
$\frac{1}{1.015^{10}} = 1.015^{-10} = 0.8617$	LL02
$\frac{1}{1.015^{1}} = 1.015^{-1} = 0.98522$	LL01
$\frac{1}{1.015^{0.1}} = \frac{1}{\sqrt[10]{1.015}} = 0.99851$	LL00

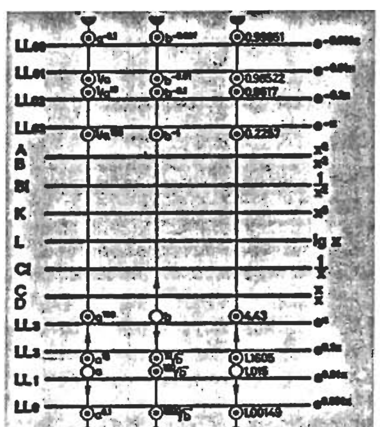

Fig. 55 Arrangement of LL scales

Variations of readings for the number series of fig. 55:

$\sqrt[10]{4.43} = 1.1605 \quad \sqrt[100]{0.2257} = 0.98522 \quad 0.98522^{10} = 0.8617 \quad 1.00149^{1000} = 4.43$

Problems such as the above will seldom arise in practice, but study of them will promote a better understanding of the construction of the LogLog scales.

19.2 Powers $y = a^x$

Just as multiplication is carried out with the fundamental scales, so raising a number to a power is accomplished with the LL scales.

Procedure:

a) Use the cursor to set the initial or terminal index of C to the base "a" on the appropriate LL scale.

b) Bring the cursor hairline over the value of the exponent on scale C.

c) Read the power y under the hairline on the appropriate LL scale.

With the slide set to the value of the base "a" we obtain a complete table of values of the function $y = a^x$. Fig. 56 shows such a setting for the function $y = 3.2^x$, in which the cursor hairline is over the value of the exponent $x = 2.5$ and its decimal variants.

Examples		Reading on scale
$3.2^{2.5}$	= 18.3	LL3
$3.2^{0.25}$	= 1.338	LL2
$3.2^{0.025}$	= 1.02956	LL1
$3.2^{0.0025}$	= 1.002912	LL0
$3.2^{-2.5}$	= 0.0546	LL03
$3.2^{-0.25}$	= 0.7476	LL02
$3.2^{-0.025}$	= 0.97134	LL01
$3.2^{-0.0025}$	= 0.997096	LL00

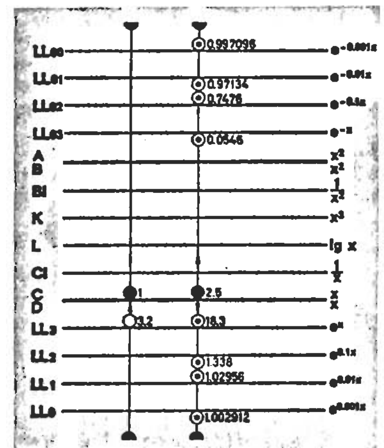

Fig. 56 Powers

Reading rules:

a) When the exponent x is positive, set and read in the same scale group LL0 — LL3 or LL00 — LL03, using figuring of like colour. With negative exponents switch from one scale group to the other, reading in unlike colour.

b) In conformity with the symbols given at the right hand end of each scale, read on the adjoining scale of lower value, for each place that the decimal point in the exponent is moved to the left (see examples in fig. 56).

c) When the base is set with the right hand slide index, all readings must be taken on the adjoining "higher" value LogLog scale.

When $a < 1$ powers with positive exponents are found in the scale group LL00 — LL03; with negative exponents, in scale group LL0 — LL3.

$0.685^{2.7} = 0.36$ (fig. 57)

$0.685^{-2.7} = 2.78$

$1.46^{2.7} = 2.78$

$1.46^{-2.7} = 0.36$

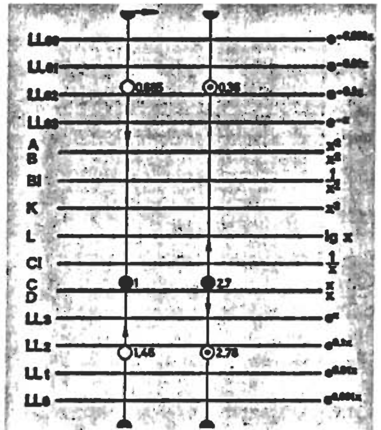

Fig. 57 Left hand index of C over base

Fig. 58 shows the examples of fig. 57, but with setting to the right hand index of the slide. The result is not then to be read on the scale on which the base is set, but on the adjacent scale LL3-LL03.

Should the value of the base lie towards the middle of the scale, as in the examples given, it is of advantage to employ scale CF in the calculation. The whole of this scale (CF) is then available for setting exponents and re-setting the slide is avoided, saving time when tabulating.

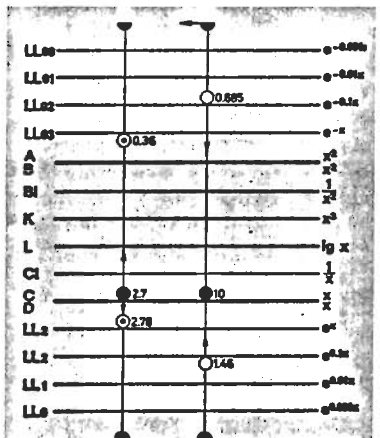

Fig. 58 Right hand index of C over base

19.3 Special cases of $y = a^x$

The possible variation of exponent or base is limited by the range of the LogLog scale sections.

19.3.1 $y > 100000$ and $y < 0.00001$

If the result of raising a number to a power exceeds the range of the LogLog scales, the exponent must be expressed as a sum of two or more terms. The power is similarly factored.

Example:

$3.14^{19} = 3.14^{6+6+7} = (3.14^6)^2 \times 3.14^7 = 0.955^2 \times 10^6 \times 3.02 \times 10^3 = 2.76 \times 10^9$

Analogous procedure is, of course, appropriate for negative exponents.

19.3.2 $0.999 < y < 1.001$
(for ARISTO StudioLog only)

When the exponent is so small that the number raised to a power is less than 1.001 but greater than 0.999, the result cannot be read on the LogLog scales.

The series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a \pm \frac{x^3}{3!} \ln^3 a + \ldots$$

provides in such cases an approximate solution in the form:

$$a^{\pm x} \approx 1 \pm x \ln a \text{ for } |x \ln a| \ll 1$$

When index 1 of C is set to the base a on the LogLog scale by means of the cursor, the value of $\log_e a$ is simultaneously set on the D scale (see section 19.4 and 19.6). Multiplication by x is achieved by moving the cursor along the C scale and reading $x \log_e a$ on D. This intermediate value, with 1 added or subtracted, is the required power $a^{\pm x}$. The smaller the exponent, the closer the approximation secured by this method.

The example of fig. 56 can by this means be carried further, thus:

$$3.2^{0.00025} = 1 + 0.0002908 = 1.0002908$$

$$3.2^{-0.00025} = 1 - 0.0002908 = 0.9997092$$

Should the exponent be still further reduced by change in the position of the decimal point, the answer obtained by the method given above will be varied only in respect of the number of zeros or nines immediately following the decimal point. E. g., $3.2^{0.000025} = 1.00002908$.

19.3.3 $0.999 < a < 1.001$
(ARISTO StudioLog only)

When in the power $y = a^x$ the base is greater than 0.999 but less than 1.001, an approximation is again of service.

From the series previously quoted, $a^{\pm x} \approx 1 \pm x \ln a$. As a approaches 1, we can write $a = 1 \pm n$ and hence:

$$a^x = (1 \pm n)^x \approx 1 + x \ln (1 \pm n)$$

$$\text{Now } \ln (1 \pm n) = \pm n - \frac{n^2}{2} \pm \frac{n^3}{3} - \ldots \approx \pm n \text{ (for } |n| \ll 1)$$

$$(1 \pm n)^x \approx 1 \pm n x \text{ and } (1 \pm n)^{-x} \approx 1 \mp n x \text{ (for } |n x| \ll 1)$$

If the range of the LogLog scales will not permit setting the base a, scale D can be used as an exponential scale. In this event, note a difference in procedure. In place of $a = 1 \pm n$, we must set the value n. When the initial index 1 of C is brought over n on scale D, the setting is for all practical purposes identical with the setting of $1 \pm n$ on an exponential scale which could be regarded as an imaginary LogLog scale covering the range 1.001 to 1.01 or 0.990 to 0.999. The smaller the value of n, the closer the approximation $\ln (1 \pm n) = \pm n$.

The value of the power is obtained, as usual, by a simple multiplication $n \times x$. To complete the result, the value found thus on D must be added to or subtracted from 1, according to the sign of n. With larger exponents, the power will lie within the range of the LogLog scales and the result can then be read directly from them.

Examples:

$1.00023^{3.7} = (1 + 0.00023)^{3.7} = 1.000851$	Reading on scale D added to 1
$1.00023^{37} = 1.00854$	Reading on scale LL0
$0.99977^{3.7} = (1 - 0.00023)^{3.7} = 0.999149$	Reading on scale D subtracted from 1
$0.99977^{37} = 0.99152$	Reading on scale LL00

19.3.4 $0.99 < y < 1.01$
(ARISTO Studio only)

When the exponent is so small that the number raised to a power is less than 1.01 but greater than 0.99, the result cannot be read on the LogLog scales.

The series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a \pm \frac{x^3}{3!} \ln^3 a + \dots$$

provides in such cases an approximate solution in the form:

$$a^{\pm x} \approx 1 \pm x \ln a \quad \text{for } |x \ln a| \ll 1$$

When index 1 of C is set to the base a on the LogLog scale by means of the cursor, the value of $\log_e a$ is simultaneously set on the D scale (see section 19.4 and 19.6). Multiplication by x is achieved by moving the cursor along the C scale and reading $x \log_e a$ on D. This intermediate value, with 1 added or subtracted, is the required power $a^{\pm x}$. The smaller the exponent, the closer the approximation secured by this method.

The example of fig. 56 can by this means be carried further, thus:

$$3.2^{0.0025} \approx 1 + 0.0025 \times \ln 3.2$$
$$\approx 1 + 0.002\,908 = 1.002\,908$$
$$3.2^{-0.0025} \approx 1 - 0.002\,908 = 0.997\,092$$

Should the exponent be still further reduced by change in the position of the decimal point, the answer obtained by the method given above will be varied only in respect of the number of zeros or nines immediately following the decimal point.

E. g. $3.2^{0.00025} = 1.000\,2908$

19.3.5 $0.99 < a < 1.01$
(ARISTO Studio only)

When, in the power $y = a^x$, the base exceeds 0.99 but is less than 1.01, the solution is again obtained by approximation.

In accordance with the series expansion applied to the case reviewed in the preceding paragraph: $a^{\pm x} \approx 1 \pm x \ln a$. Since a, in the present case, is near 1 we can write $a = 1 \pm n$, from which we can further derive:

$$a^x = (1 \pm n)^x \approx 1 + x \ln (1 \pm n)$$
$$\ln (1 \pm n) = \pm n - \frac{n^2}{2} \pm \frac{n^3}{3} - \dots$$
$$\ln (1 \pm n) \approx \pm n \quad (\text{for } |n| \ll 1)$$
$$(1 \pm n)^x \approx 1 \pm nx \quad (\text{for } |nx| \ll 1)$$
$$(1 \pm n)^{-x} \approx 1 \mp nx \quad (\text{for } |nx| \ll 1)$$

If the range of the LL scales will not permit setting the base a, scale D can be used as an exponential scale. In this event, note a difference in procedure. In place of $a = 1 \pm n$, we must set the value $|n|$.

When the initial index 1 of scale C is brought over n on scale D, the setting is for all practical purposes identical with the setting of $1 \pm n$ on an exponential scale which could be looked upon as an imaginary LogLog scale covering the range 1.001 to 1.01 or 0.99 to 0.999. The smaller the value of n, the closer the approximation $\ln (1 \pm n) \approx \pm n$.

The value of the power is obtained, as usual, by a simple multiplication n x. To complete the result, the value found on D must be added to 1 or subtracted from 1, according to the sign of n. With large exponents, the power will lie within the range of the LL scales and the result can then be read directly from the LogLog scales.

Examples:

		Read on scale
$1.0023^{3.7}$	$= (1 + 0.0023)^{3.7} = 1.00851$	D and add 1
1.0023^{37}	$= 1.0888$	LL1
$0.9977^{3.7}$	$= (1 - 0.0023)^{3.7} = 0.99149$	D and deduct from 1
0.9977^{37}	$= 0.9184$	LL01

With the cursor hairline aligned over the left index of D, the amount of displacement relative to the line for 1.01 on LL1 provides a good check on the amount of error in the approximative computation. The maximum degree of error will be introduced into the approximation when both setting and reading take place on scale D in substitution for the LogLog scales.

19.3.6 Improving the accuracy
(ARISTO Studio only)

The precision can be improved when the disparity between reading on the D scale and the actual LogLog scale within the range 1.001 to 1.01 is corrected by also applying both the linear and the quadrature term in the series expansion to the previously discussed procedure.

A) $\ln (1 \pm n) \approx \pm n (1 \mp n/2)$ for settings of the base on D

B) $e^{\pm x} \approx 1 \pm x (1 \pm x/2)$ for readings taken from D

When the result is obtained from a LogLog scale, only formula A need be applied before making the setting on scale D. If, however, scale D is used exclusively in a computation, corrections have to be applied to the setting as well as to the answer (formula B).

Example:

$1.0023^{3.7} = 1.008\,54$

For $n = 0.0023$ substitute the setting
$0.0023\,(1 - 1/2 \times 0.0023) = 0.0023 \times 0.99885 = 0.002\,297$ by slide index on scale D.

The operation required to determine the 3.7th power, viz. $1 + 0.002297 \times 3.7$, gives 1.00850. This reading, because of its taking place on scale D, requires correction by formula B, as follows:

$$0.008\,50\,(1 + 1/2 \times 0.008\,50) = 0.008\,50 \times 1.004\,25 = 0.008\,54$$

After adding the "1", the final answer then is 1.00854 (exactly: 1.0085362). The foregoing computation may at first sight appear rather involved and awkward but will actually be found quite simple after some little practice, so that in time the computer will be able to make the corrections by visual estimate.

Corrections of the kind above reviewed are no longer necessary when the base drops below 1.001, because slide rule accuracy will then be equivalent to that obtainable by approximation.

19.4 Powers $y = e^x$

When the indexes of the slide and body scales are in coincidence, the rule is adjusted to the equation $y = e^x$. Because the base e = 2.718 on scale LL3 is always aligned with the index of scale D, any power of e can be found by moving the cursor to the exponent on D. Using the body scales only we obtain as examples the results for the exponent 1.489 and its decimal variants:

$e^{1.489} = 4.43$	$e^{-1.489} = 0.2260$
$e^{0.1489} = 1.1605$	$e^{-0.1489} = 0.8618$
$e^{0.01489} = 1.015$	$e^{-0.01489} = 0.98522$
$e^{0.001489} = 1.001489$	$e^{-0.001489} = 0.99851$

With further variation, we arrive once more at the equivalence

$$e^{0.0001489} = 1.0001489$$

19.5 Roots $a = \sqrt[x]{y}$

Evolution, with given radicands, can be carried through directly with the exponential scales. The extraction of roots, the converse of raising to a power, follows from division using the LogLog scales and fundamental scale C. If the example of a power, $3.2^{2.5} = 18.3$, given in chapter 19.2, is reversed, it will be seen that by working in the contrary sequence we can read $\sqrt[2.5]{18.3} = 3.2$.

The extraction of roots is more easily understood if the radix is expressed as an exponent. The exponent can then be set on scale CI or, if the base is e, on scale DI.

In the following example the cursor of the ARISTO StudioLog is set to 3.5 on DI and the root read on LL2 and LL00.

$$\sqrt[3.5]{e} = e^{+\frac{1}{3.5}} = 1.3307 \qquad \frac{1}{\sqrt[3.5]{e}} = e^{-\frac{1}{3.5}} = 0.7514$$

Procedure:

a) Set the radicand y on the appropriate LogLog scale over the radix on slide scale C.

b) Read the value of the root under the initial or final index of the slide scale on the appropriate LogLog scale.

The rules for reading the result, given in section 19.2, are again applicable. It should be noted that when a reading is to be taken under the right hand index, reference must be made to the next lower LogLog scale, LL0 — LL3 or LL00 — LL03.

$\sqrt[0.77]{21} = 52.1 \qquad \frac{1}{\sqrt[0.77]{21}} = 0.0192$

$\sqrt[7.7]{21} = 1.485 \qquad \frac{1}{\sqrt[7.7]{21}} = 0.6734$

$\sqrt[77]{21} = 1.0403 \qquad \frac{1}{\sqrt[77]{21}} = 0.96122$

$\sqrt[770]{21} = 1.00396 \qquad \frac{1}{\sqrt[770]{21}} = 0.99605$

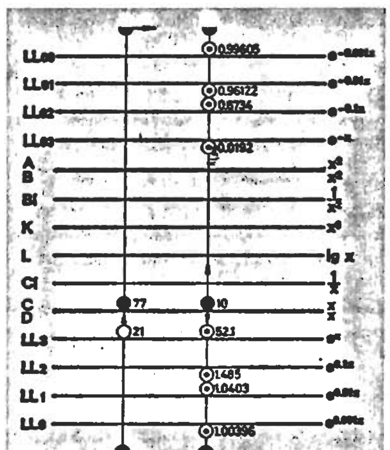

Fig. 59 Roots

19.6 Logarithms

19.6.1 Logarithms to any base

Required logarithms to any base can be found with the LogLog scales. By reversing the process of raising a number to a power, we obtain its logarithm, as is immediately seen if we write:

$$y = a^x \qquad x = \log_a y \text{ (read: logarithm of y to base a).}$$

The finding of a logarithm is thus identical with the problem of a power for which the exponent is required.

Procedure:

a) Set cursor to base a on the appropriate LogLog scale.

b) Bring the initial or final index of the slide to the cursor hairline.

c) Set the value of y, with the cursor, on the LL scale.

d) Read the logarithm on scale C under the hairline.

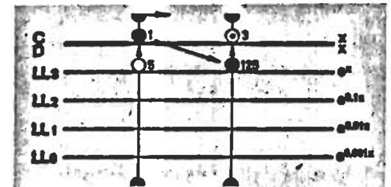
Fig. 60 $\log_5 125 = 3.0$

The position of the decimal point can be determined from the relationship $\log_a a = 1$.

With the initial index of slide over base a, all values to the right of the value a on scale C are greater than one and all values to the left of a on scale C are less than 1.

Reading rules:

a) Passing from one LogLog scale to the adjacent scale, in the sequence LL3, LL2, LL1, LL0 or LL03, LL02, LL01, LL00, results in a shift of the decimal point in the logarithm by one place to the left. A change of scale in the opposite direction calls for a shift of the decimal point to the right.

b) The logarithms will be positive (negative) when their antilogs and bases are set on like (unlike) coloured LogLog scales.

Examples for practice: $\log_2 16 = 4.0$
$\log_2 1.02 = 0.02857$
$\log_2 0.25 = -2$

19.6.2 Logarithms to base 10

If index 1 of scale C is set to base 10 on LL3, then for any number set on an LL scale, the decadal logarithm can be read on scale C (see figs. 61 and 62).

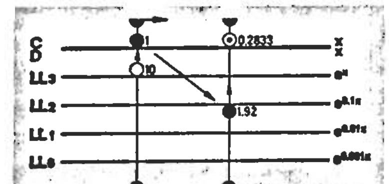

Fig. 61 lg 1.92 = 0.2833

The frequently required decadal logarithms can also be found from the usual log scale L on the slide, if the anti-log is set on scale C. Scale L gives only the mantissa and the characteristic must be added in accordance with the rule "number of places minus 1", as when a table of logarithms is used. For every plain number (anti-log) on scale C, the logarithm is directly available on scale L and conversely, given the logarithm, the anti-log can be read directly from scale C.

To use scale L, it is only necessary to move the cursor and thus the finding of decadal logs is more simple than when the LogLog scales are used. However, within the range of scale LL1, the LogLog scale gives greater precision in reading.

Examples:

lg 1.03 = 0.01283 with scale LL1
lg 1.03 = 0.013 with scale L

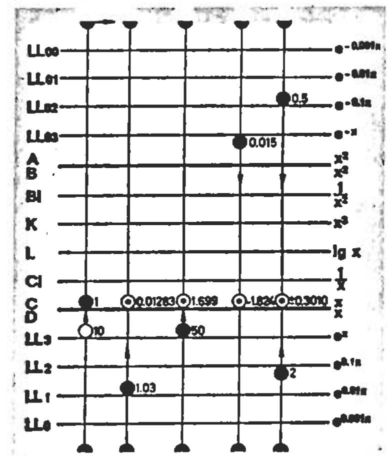

Fig. 62 Logarithms to base 10

Examples for practice:

Refer to fig. 62

$\log_{10}50 = 1.699$
$\log_{10}2 = 0.301$
$\log_{10}1.03 = 0.01283$
$\log_{10}0.015 = -1.824$
$\log_{10}0.5 = -0.3010$
$\log_{10}0.1 = -1$
$\log_{10}6 = 0.778$
$\log_{10}1.14 = 0.0569$
$\log_{10}1.015 = 0.00647$

When setting with the right hand index of scale C, all results will lie on the left of the base and are therefore < 1, e. g., $\lg_{10}9 = 0.954$. Logarithms of numbers < 1 are negative.

19.6.3 Natural Logarithms

Logarithms to base "e" are simply found by transfer from the LogLog scale to the fundamental scale D (fig. 63).

Examples:

$\ln 4.375 = 1.475$
$\ln 0.622 = -0.475$
$\ln 0.05 = -2.994$

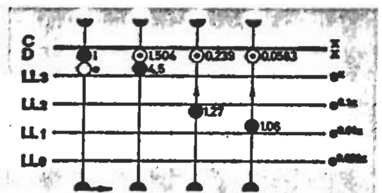

Fig. 63 $\ln 4.5 = 1.504$, $\ln 1.27 = 0.239$, $\ln 1.06 = 0.0583$

20. Further applications of the LogLog scales

So far, of the slide scales, C only has been used in conjunction with the LogLog scales to show the essential relationships. Naturally, however, other slide scales can be used. For example, scale B can be used for a power such as $a^{\sqrt{x}}$ and similarly scale S at the ARISTO StudioLog on the slide is convenient for the term $e^{\sin x}$. The reciprocal scales offer still further possibilities in logarithmic computations. Scale CF, replacing scale C, can be used with the LogLog scales to avoid slide re-setting during tabulation, if the base occurs near the middle of the slide.

20.1 Proportions with the LogLog scales

If the index 1 of slide scale C is set to a base on a LogLog scale, the powers to any exponent and also the logarithms of any number, to this base, can be read. A base, a, set on a LogLog scale, can thus be regarded as a term in a proportion.

20.1.1 $y_1 = a^n \qquad y_2 = a^m$

$$\log y_1 = n \log a \qquad \log y_2 = m \log a$$

$$\frac{\log a}{1} = \frac{\log y_1}{n} = \frac{\log y_2}{m}$$

$$\text{or} \quad \frac{\ln a}{1} = \frac{\ln y_1}{n} = \frac{\ln y_2}{m}$$

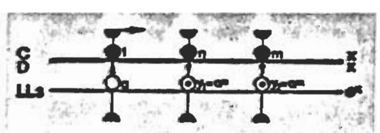

Fig. 64

If three terms of a proportion are known, the fourth proportional can be calculated and with the initial setting, a number of terms in the same ratio can be found. Here again is seen the advantage of the principle of proportionality, a method of calculation for which the slide rule is particularly well adapted, as examples will show.

20.1.2

$$y = a^{\frac{m}{n}} \rightarrow \log y = \frac{m}{n} \log a$$

$$\frac{\log y}{m} = \frac{\log a}{n}$$

$$y = 4.3^{\frac{6.8}{2.7}} \rightarrow \frac{\log y}{6.8} = \frac{\log 4.3}{2.7}$$

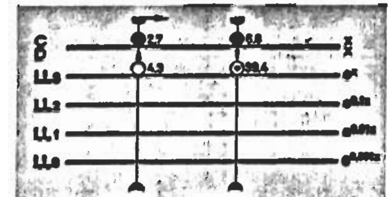

Fig. 65 $\frac{\log 4.3}{2.7} = \frac{\log 39.4}{6.8}$

After setting 2.7 on C over 4.3 on LL3, the result 39.4 will be found under 6.8 on C, and read on LL3.

Transformations of this problem will similarly be solved:

$$y = \sqrt[2.7]{4.3^{6.8}} \quad \text{or} \quad y^{2.7} = 4.3^{6.8}$$

20.1.3

Many natural laws can be expressed in proportional form, if the change or difference in one variable is proportional to the change or difference in the logarithm of the other:

$$\log y_2 - \log y_1 = \text{const}\,(x_2 - x_1)$$

Because $\quad \log a - \log b = \log \frac{a}{b}$

we may write $\quad \log \frac{y_2}{y_1} = \text{const}\,(x_2 - x_1)$

A change from x_1 to x_2, by an amount i, results in a change of y_1 to y_2. If we denote the ratio $\frac{y_2}{y_1}$ by r, representing the residue of the initial quantity, the above equation can be written

$$\frac{\log r}{i} = \text{const} = \frac{\log r_1}{i_1} = \frac{\log r_2}{i_2} = \dots$$

Example: Radioactive decay

The decomposition rate of a substance is known to be 40% in 30 days, i. e., the residue is then 60%. After how many days will the residue be 20%.

Here $i_1 = 30$, $r_1 = 0.6 \quad r_2 = 0.2$

$$\frac{\log 0.6}{30} = \frac{\log 0.2}{x} \quad \text{whence} \quad x = 94.5 \text{ days}$$

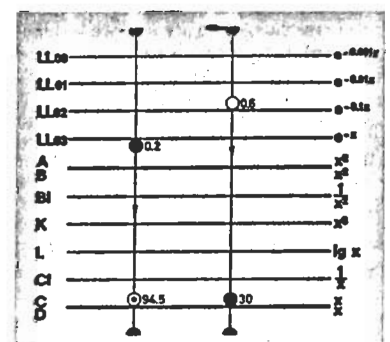

Fig. 66

20.1.4

If a logarithm is to be multiplied by a constant factor, the constant is set on C, over the base of the logarithm on the LogLog scale. A tabulating position for the multiplication is at once set up.

For $x = c \log_a y$, write in proportional form:

$$\frac{x}{\log_a y} = \frac{c}{1} = \frac{c}{\log_a a}$$

$$2 \times \log_{10} 100 = 4$$

$$2 \times \log_{10} 1.8 = 0.511$$

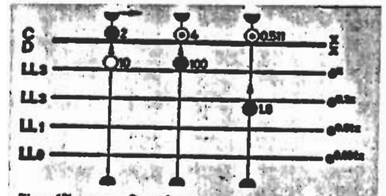

Fig. 67 $x = 2 \times \lg y$

As shown in fig. 67, all logarithms to base 10 can be multiplied by the constant 2. The process applies also to the LL0 scale group, with logarithms of numbers < 1.

In physics and telecommunications, it is often necessary to calculate the decibel (dB) value for a given amplitude ratio.

$$\text{dB} \triangleq 20 \lg \frac{A_1}{A_2}$$

Examples:

20 dB = 20 lg 10
40 dB = 20 lg 100
5.11 dB = 20 lg 1.8

20.2 Hyperbolic functions

The logical arrangement of the LogLog scales makes possible the simple development of hyperbolic functions. Because the values of powers with negative and positive exponents are mutually opposed, a simple movement of the cursor gives e^{+x} and e^{-x}, whence the hyperbolic functions are easily derived.

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$$

21. The cursor and its marks

21.1 The mark 36
(868, 869, 0968 and 0969 only)

The cursor has, on the front face (fig. 68) a short line upper right, corresponding to the value 36 on scales CF/DF, with respect to a value set on C/D under the middle cursor line. This enables multiplication by 36 to be performed by cursor transfer from C/D to CF/DF, a convenience when converting:

1 hour = 3600 seconds
1 m/s = 3.6 km/h
1° = 3600″
100% = 360°
1 year = 360 days
1 kWh = 3.6×10^6 J
$\varkappa_{Al} = 36 \frac{m}{\Omega\, mm^2}$ (conductivity)

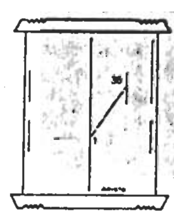

Fig. 68

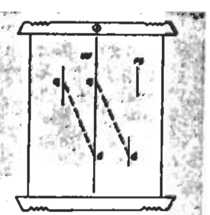

Fig. 69

21.2 The gauge marks 2π
(869 and 0969 only)

In addition to the mark 36, the front face of cursor pattern L 0969 (fig. 68) has at left and right hand sides reference lines for the factor 2π. These lines lie over scales C/D, CF/DF and are interrupted lines, to avoid confusion with the principal hairline. The 2π marks are of especial importance in frequency calculations.

Multiplication by 2π is accomplished by bringing the right hand 2π mark over the factor involved and reading the product under the left hand 2π mark. The converse procedure achieves division by 2π.

Example 1:

Find the frequency f of an oscillator if angular frequency $\omega = 372$ Hz from the relationship $f = \omega/2\pi$.

Move cursor to bring the left hand 2π mark over 3—7—2 on D. Beneath the right hand 2π mark read f = 59.2 Hz.

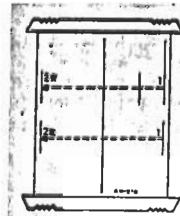

Fig. 70

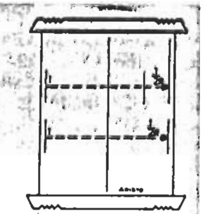

Fig. 71

Example 2:

Find the inductive resistance $X_L = 2\pi f L$ of a coil of frequency f = 59.2 Hz and inductance L=21.5 mH.

Set 1 on CF under 5—9—2 on DF. Then move cursor so that the right hand 2π mark is over 2—1—5 on CF. The inductive resistance X_L = 8 Ω can then be read on DF, under the left hand 2π mark.

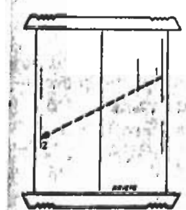

Fig. 72

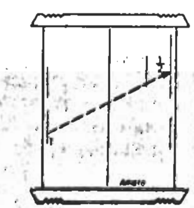

Fig. 73

In conjunction with the 2π marks reference to scale pairs C/D, CF/DF allows multiplication or division by 2 without slide setting. Multiplication by 2 follows when the upper right hand 2π mark is brought over the relevant factor on DF. On D beneath the left hand 2π mark the result is read at once. The converse sequence of cursor setting and reading provides the quotient of a division by 2.

21.3 Marks for circular areas, weights of bar steel

On the reverse face of the cursor (fig. 69) are two short lines, upper left and lower right, displaced from the main cursor line by a distance proportional to the value $\pi/4 = 0.785$, (referred to the scale of squares). These are used in finding circular areas from the formula $A = d^2 \pi/4$. If the main cursor line is brought over the diameter on scale D, the area can be read on scale A under the upper left short line. The same relationship holds between the lower right and main cursor lines.

Where the metric system of measures is in use, these special cursor lines can be used to find the weight of bar steel, because the specific weight of mild steel is 7.85 g/cm³. If the bar diameter is set on D with the main cursor line, the weight of unit length is read at once under the upper left short line. The index 1 of scale B is set to the reading under the upper left line and the cursor moved over the total length of bar stock, to find the total weight.

This facility is not available with model 01068, because in this the doubled base length contains the width corresponding to the factor $\frac{\pi}{4}$ only once, as one passes from the lower right to the upper left cursor lines.

21.4 The marks kW and HP

The distance between the main cursor line and the upper mark gives, with reference to the scale of squares, the factor for converting kW to HP and vice versa (see fig. 69).

If, for example, the main hairline is set to 20 kW on the scale of squares, then under the upper right hand line will be found the equivalent HP, 26.8. Conversely, when the upper right line is set to 7 HP the main hairline will indicate the equivalent kW, 5.22.

On the 20 in. model No. 01068 the kW and the HP mark are attached to the upper left and the upper right cursor hair, respectively.

21.5 Removing the cursor

The cursor hairlines are so adjusted that transfer from one face of the rule to the other is possible at any stage in a calculation. The cursor can be removed, for cleaning, without disturbing this adjustment — provided that the screw on the cursor bridge piece is not lost.

To remove the cursor, hold firmly, with one hand, the screwed cursor bridge piece. The other cursor bridge piece — that without the screw — can then be released by a rotary movement of the screwed cleat and cursor glasses across the face of the rule, as shown in fig. 74. Cursor glasses and bridge pieces can then be removed.

When replacing the cursor, take care to set it with the gauge marks kW and HP over scales A and B. The sprung cursor bridge piece should then be brought over the cursor glasses and the assembly closed by light pressure.

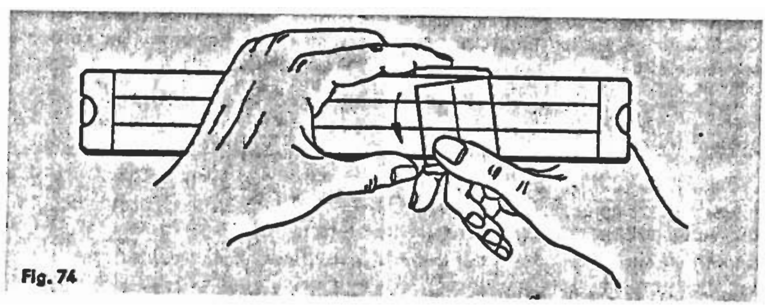

Fig. 74

21.6 Adjustment of the cursor

After loosening the adjustment screw of the cursor, the rule should be turned over so that the cursor hairline can be set to the auxiliary marks on the LL scales. Without moving the cursor, turn the rule over again and place it on the table. The now facing cursor hairline can be set to the right hand index marks of scales A and D. This done, the adjusting screw can be re-tightened.

22. The scale of preferred numbers 1364 (NZ scale)

(Models 0968, 0969 and 01068 only)

22.1 Construction of the NZ scale

Standards and standardisation have become important factors in rationalised production and in this technology preferred numbers assume ever greater significance. Preferred numbers (BS 2045, ISO R 3, R 17) are selected values from a geometric series, developed from the denary number system. There is a very useful relationship between the graduations of the fundamental scale D and the associated mantissa scale L.

Opposite the equal intervals of the mantissa scale L are the corresponding plain numbers on scale D. The principal values tabulated as preferred numbers (BS 2045, ISO R 3, R 17) are these numbers, rounded off.

A scale of preferred numbers can be derived if the D scale is disregarded and the corresponding graduations of the mantissa scale are marked as preferred numbers.

On scale 1364, the ten numbered divisions of the upper mantissa scale are located over the preferred numbers of the R 10 series. The division of the mantissa scale into 20 equal divisions leads to the preferred numbers of the R 20 series and, with division into 40 equal intervals, to the R 40 series.

The preferred numbers are also marked against the mm scale: R 10 series by arrow points, R 20 by graduated lines and R 40 by dots. This provision enables the preferred numbers to be used in drafting.

22.2 Application of the NZ scale

Scale 1364 is, first of all, an aid to memory, serving to exhibit instantly the commonly used preferred numbers. These are also of practical use when constructing single- and double-deck logarithmic graphs on normal squared graph paper. Because the multiplication or division of a preferred number by a preferred number always yields a preferred number, a graph doubly divided in preferred numbers serves for the graphic solution of problems.

The combination of preferred numbers and mantissas in a single scale offers the advantage that logarithmic approximations are simplified. The preferred numbers stand opposite the simplified logarithms of the mantissa scale and the latter can easily be added or subtracted mentally. By prefixing the characteristic, as must be done when using a table of logarithms, the decimal point can be correctly placed and the error in the result is a maximum of 3% if the R 40 series is used in the calculation.

In many cases the preferred number scale can equally well be used, if numbers are strongly rounded off. For example, if we take $\pi = 3.15$, for $\gamma = 7.85$ we take $\gamma = 8$. The mantissas corresponding to the preferred numbers are read from the mantissa scale set over the preferred numbers. It is very important to take into account the characteristic, on the presence of which this method of calculation essentially depends.

With complicated formulae, it is of advantage to write down the logarithms as read, so that a check can be made by addition. Natural numbers less than 1 (e. g., 0.8) are often best expressed as negative logarithms, e. g., $\lg 0.8 = -0.1$ is better put in the form $\lg 0.8 = 0.9 - 1$.

The graduations of L and D offer a more exact method of logarithmic calculation, since they provide, graphically, a three place table of logarithms.

22.3 Logarithmic scales

For the exact setting out of logarithmic scales or chart-nets, the scale 1364 carries logarithmically divided scales of base length 200 mm, 150 mm, 100 mm, 50 mm and 25 mm. Base lengths 125 mm and 250 mm can be taken from the slide scales of the rule.

22.4 Conversion factors for non-metric units

In the study of English, American and Continental literature, differences between anglo-saxon and metric units of measurement give rise to difficulty and relationship between the system must often be laboriously searched for in handbooks. This searching is obviated by the assembly of the most important conversion factors in a table on scale 1364.